

Mechanics II

CHAPTER 4

Rigid Body Dynamics: Part 2

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4

Rigid Body Dynamics: Part 2

INTRODUCTION

Even though the force and torque are different quantities, let us try to discover some similarity between them. As we know, force can change the linear momentum. Following this logic, let us enquire if a torque can change something.

ANGULAR MOMENTUM

Just as the idea of the linear momentum helps us to analyze translational motion, a rotational analog **angular momentum** helps us to analyze the objects undergoing rotational motion.

Consider a particle of mass m located at the vector position $\vec{r}$ and moving with linear momentum $\vec{p}$ as shown in figure.

In describing translational motion, we found that the net force on the particle equals the time rate of change of its linear momentum,

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \dots(i)$$

Let us take the cross product of each side of Eq. (i) with $\vec{r}$, which gives the net torque on the particle on the left side of the equation:

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times \frac{d\vec{p}}{dt}$$

Now let's add to the right side, which is zero because $\frac{d\vec{r}}{dt} = \vec{v}$ and $\vec{v}$ and $\vec{p}$ are parallel.

$$\text{Therefore, } \vec{\tau} = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

We recognize the right side of this equation as the derivative of $\vec{r} \times \vec{p}$.

$$\text{Therefore, } \vec{\tau} = \frac{d(\vec{r} \times \vec{p})}{dt} \quad \dots(ii)$$

which looks very similar in form to Eq. (i). Because torque plays the same role in rotational motion that force plays in translational motion, this result suggests that the combination $\vec{r} \times \vec{p}$ should play the same role in rotational motion that $\vec{p}$ plays in translational motion. We call this combination as the **angular momentum** of the particle.

The instantaneous angular momentum $\vec{L}$ of a particle relative to an axis through the origin O is defined by the cross product of the particle's instantaneous position vector $\vec{r}$ and its instantaneous linear momentum $\vec{p}$:

$$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) \quad \dots(iii)$$

As torque ($\vec{\tau} = \vec{r} \times \vec{F}$) is defined as the 'moment of force', angular momentum is also referred sometimes as 'moment of linear momentum'.

We can now write Eq. (ii) as,

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad \dots(iv)$$

which is the rotational analog of Newton's second law, $\vec{F} = d\vec{p}/dt$. Torque causes the angular momentum $\vec{L}$ to change just as force causes linear momentum $\vec{p}$ to change.

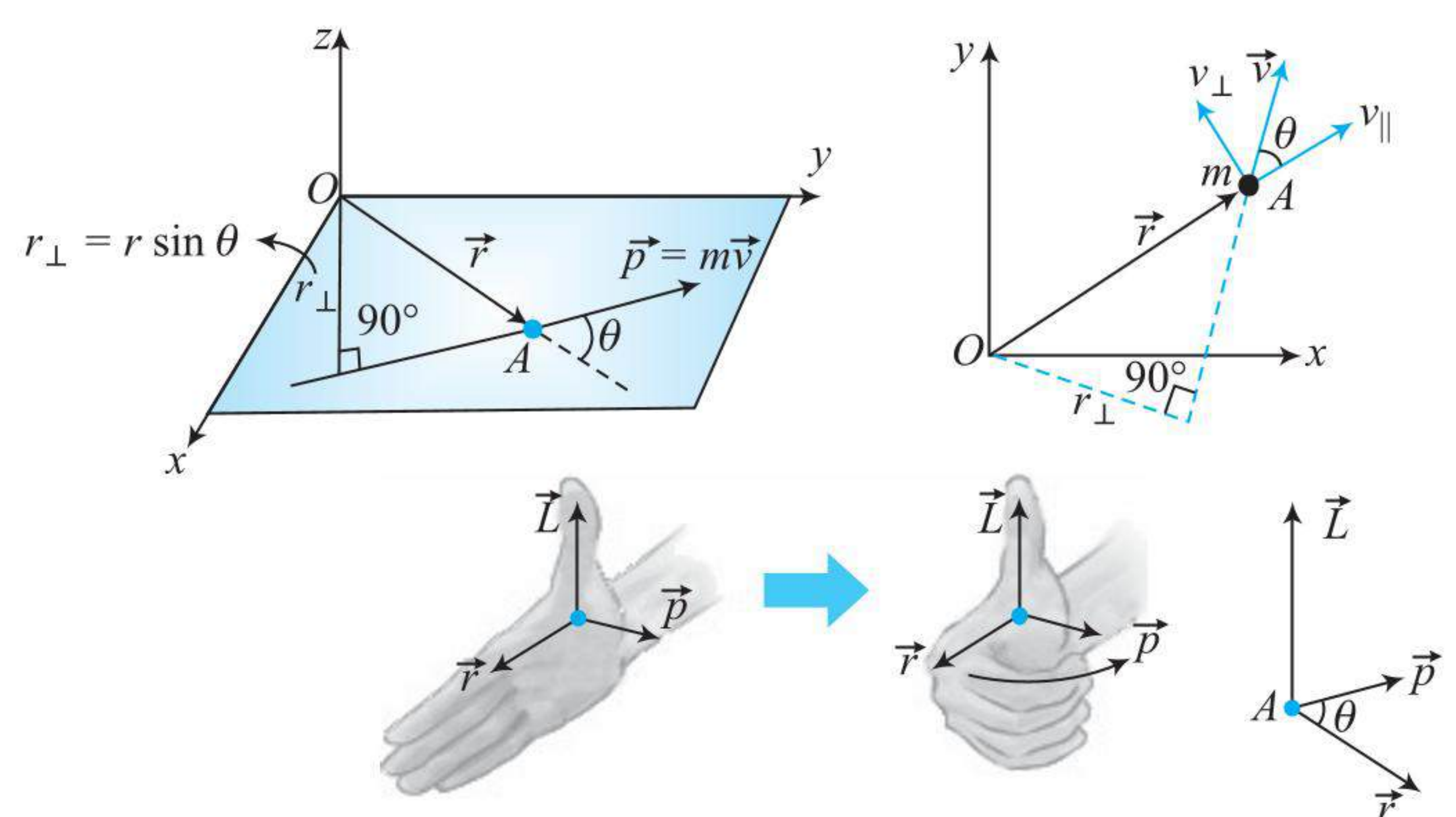
Note: Equation (iv) is valid only if $\vec{\tau}$ and $\vec{L}$ are measured about the same axis. Furthermore, the expression is valid for any axis fixed in an inertial frame.

The SI unit of angular momentum is $\text{kg m}^2/\text{s}$. Note that both the magnitude and the direction of $\vec{L}$ depend on the choice of axis. Regarding angular momentum, it is worth nothing that it is an axial vector (i.e., always perpendicular to the plane of motion), following the right-hand rule, we see that the direction of $\vec{L}$ is perpendicular to the plane formed by $\vec{r}$ and $\vec{p}$. In figure, $\vec{r}$ and $\vec{p}$ are in the x - y plane, so $\vec{L}$ points in the z direction. Because $\vec{p} = m\vec{v}$ and θ is the angle between $\vec{r}$ and $\vec{p}$, we can write the magnitude of $\vec{L}$ as

$$L = mv(r \sin \theta) = mvr_{\perp} \quad \dots(v)$$

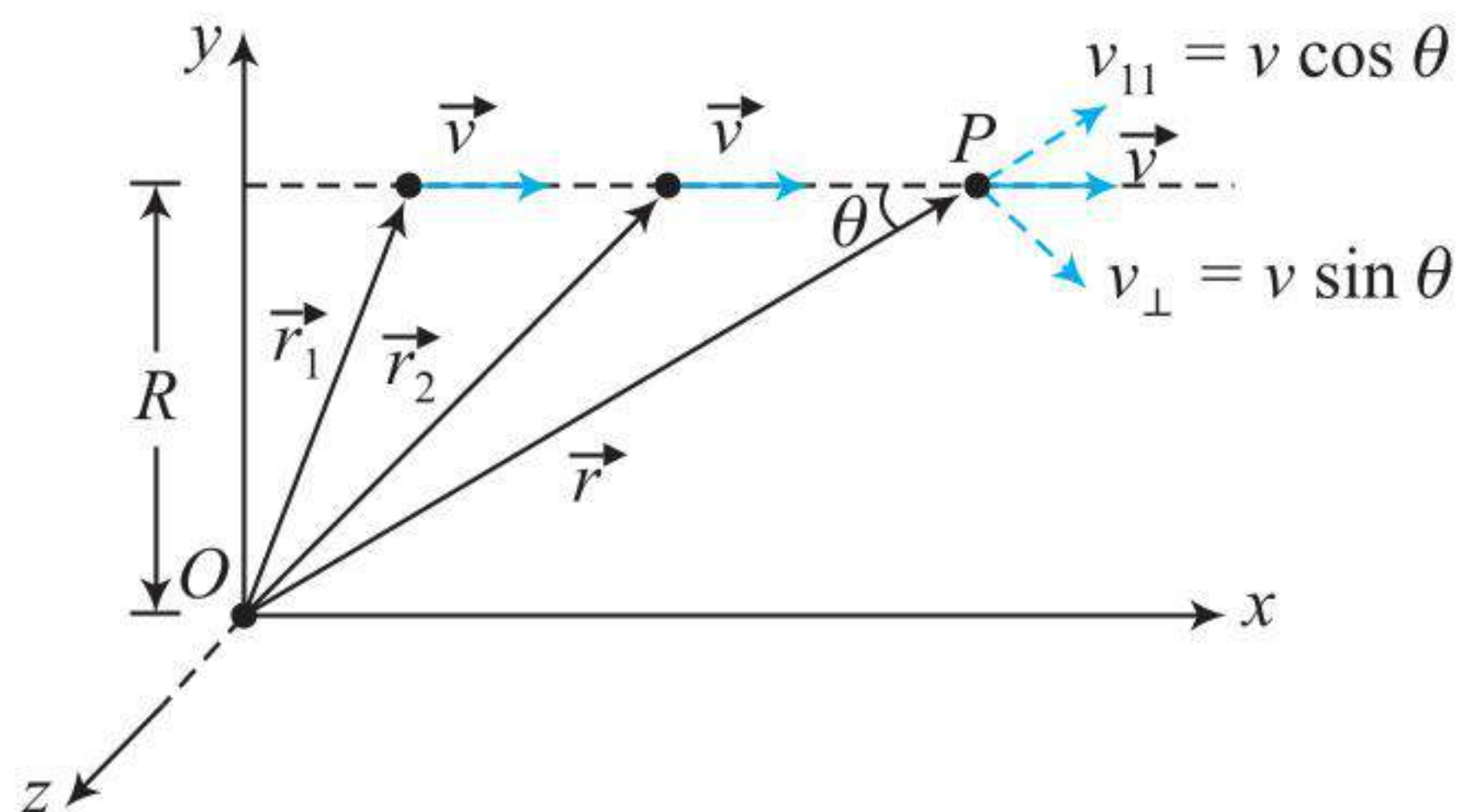
$$\text{Also, } L = m(v \sin \theta)r = mv_{\perp}r \quad \dots(vi)$$

where $r_{\perp}$ is the perpendicular distance from O called lever arm of angular momentum and the direction of $\vec{p}$ and $v_{\perp}$ is the component of $\vec{v}$ perpendicular to $\vec{r}$.



CAN A PARTICLE MOVING IN A STRAIGHT LINE HAVE ANGULAR MOMENTUM?

Let a particle of mass m moving in a straight line with constant velocity as shown in figure. Generally, we think that when a particle moves in a straight line, how can it possess an “angular momentum” without undergoing angular motion or circular motion etc.? In other words, how can we use angular quantity like angular momentum when the particle moves in a straight line? Let us explain the idea behind this.



We can provide a logical interpretation of this by saying that when a particle moves in a straight line or any curve, its position vector rotates about a fixed point. Hence, we can say that the particle is turning about that point.

We can see that in the given figure, the particle P turns about the fixed-point O with an angular velocity given by

$$\omega_{PO} = \frac{v_{PO\perp}}{r} = \frac{v_{\perp}}{r}$$

It means, the turning of the particle about O is decided by the component of its velocity perpendicular to $\vec{r}$, that is $v_{\perp}$. On the other hand, the particle approaches to (or) departs from the point O with a velocity $v_{\parallel}$.

Hence, the angular momentum of the particle

$$|\vec{L}| = mv_{\perp}r = m(v \sin \theta)r = mv(r \sin \theta) = mvR$$

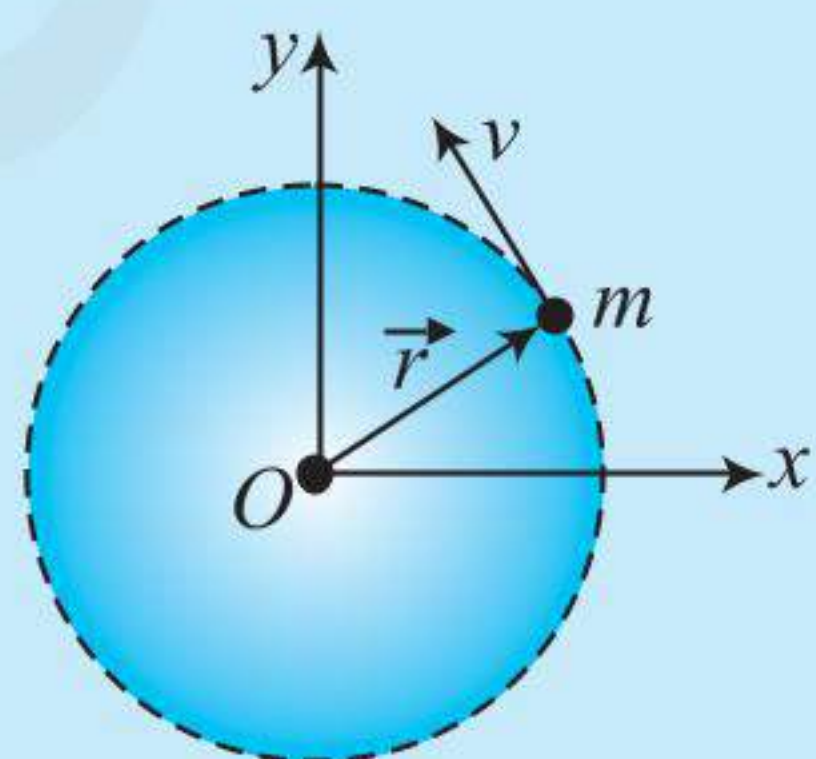
$\therefore |\vec{L}| = \text{constant as } m, v \text{ and } R \text{ all are constants.}$

But the direction of $\vec{r} \times \vec{v}$ also remains the same. Therefore, angular momentum of the particle about the origin remains constant with due course of time. In vector form, we can write

$$\vec{L} = mvR(\hat{k})$$

ILLUSTRATION 4.1

A particle moves in the x - y plane in a circular path of radius r as shown in figure. Find the magnitude and direction of its angular momentum relative to an axis through O when its velocity is $\vec{v}$.



Sol. Let's evaluate the magnitude of the angular momentum of a particle by

$$L = mvr \sin \theta$$

where θ is the angle between $\vec{r}$ and $\vec{p}$. Here in this case it is 90° .

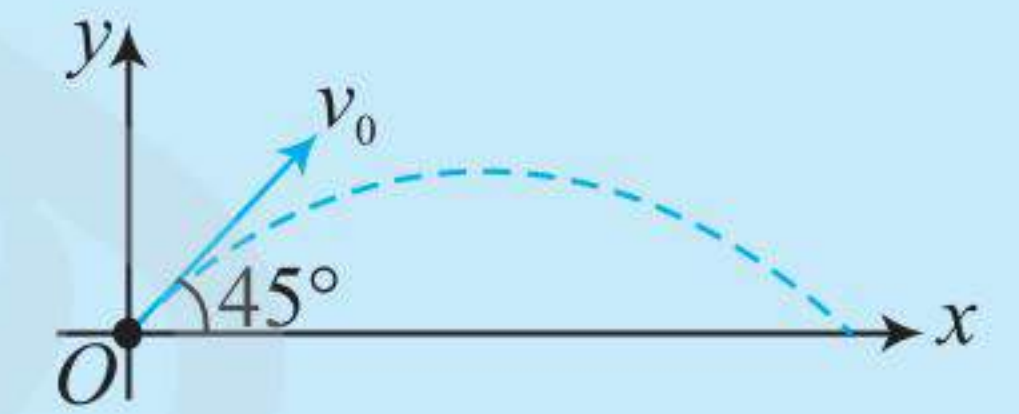
Hence, $L = mvr \sin 90^\circ = mvr$

This value of L is constant because all three factors on the right are constant. The direction of $\vec{L}$ also is constant, even though the direction of $\vec{p} = m\vec{v}$ keeps changing. To verify this statement, apply the right-hand rule to find the direction of $\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v}$. Your thumb points out of the page, so that

is the direction of $\vec{L}$. Hence, we can write the vector expression $\vec{L} = (mvr)\hat{k}$. If the particle were to move clockwise, $\vec{L}$ would point downward and into the page and $\vec{L} = -(mvr)\hat{k}$. A particle in uniform circular motion has a constant angular momentum about an axis through the center of its path.

ILLUSTRATION 4.2

A particle is projected at time $t = 0$ from a point O with a speed v_0 at an angle of 45° to the horizontal. Find the magnitude and the direction of the angular momentum of the particle about the point O at time $t = v_0/g$.

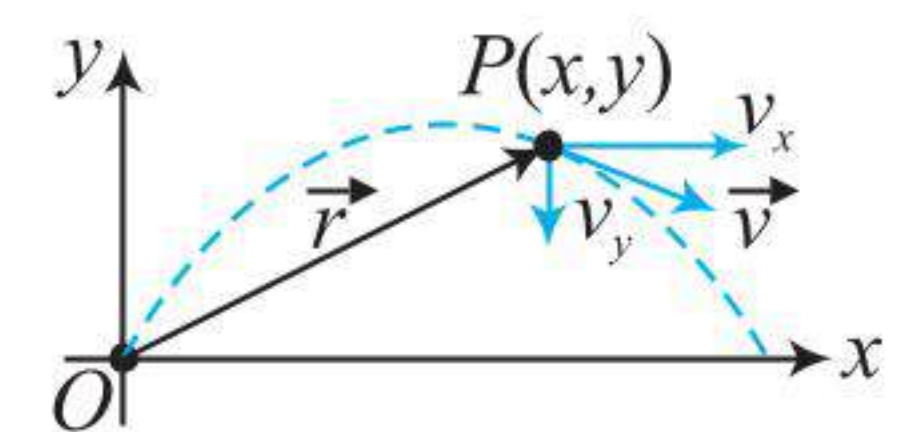


Sol. In this case the angular momentum of the particle will not remain constant as there is always torque of gravity is acting. Let us take the origin at O , x -axis along the horizontal and y -axis along the vertically upward direction as shown in figure.

For horizontal motion during the time 0 to t ,

$$v_x = v_0 \cos 45^\circ = \frac{v_0}{\sqrt{2}}$$

$$\text{and } x = v_x t = \frac{v_0}{\sqrt{2}} \frac{v_0}{g} = \frac{v_0^2}{\sqrt{2}g}$$



For vertical motion, using $v_y = v_{oy} - gt$, we get

$$v_y = v_0 \sin 45^\circ - g \left(\frac{v_0}{g} \right) = \frac{(1 - \sqrt{2})}{\sqrt{2}} v_0$$

$$\text{and } y = (v_0 \sin 45^\circ)t - \frac{1}{2}gt^2$$

$$\Rightarrow y = \frac{v_0}{\sqrt{2}} \left(\frac{v_0}{g} \right) - \frac{1}{2}g \left(\frac{v_0}{g} \right)^2 = \frac{v_0^2}{2g}(\sqrt{2} - 1)$$

The angular momentum of the particle at time t about the origin is

$$\begin{aligned} L &= \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) \\ &= (x\hat{i} + y\hat{j}) \times (v_x\hat{i} + v_y\hat{j}) = m(xv_y\hat{k} - yv_x\hat{k}) \\ &= m \left[\left(\frac{v_0^2}{\sqrt{2}g} \right) \frac{v_0}{\sqrt{2}} (1 - \sqrt{2}) - \frac{v_0^2}{2g} (\sqrt{2} - 1) \frac{v_0}{\sqrt{2}} \right] (\hat{k}) \\ &= \frac{mv_0^2}{2\sqrt{2}g} (-\hat{k}) \end{aligned}$$

Thus, the angular momentum of the particle is $mv_0^2/(2\sqrt{2}g)$ in the negative z -direction, i.e., perpendicular to the plane of motion, going into the plane.

ANGULAR MOMENTUM OF A SYSTEM OF PARTICLES

The total angular momentum of a system of particles about some axis is defined as the vector sum of the angular momenta of the individual particles about that axis:

$$\vec{L}_{\text{tot}} = \vec{L}_1 + \vec{L}_2 + \cdots + \vec{L}_n = \sum_i \vec{L}_i$$

where the vector sum is over all n particles in the system.

Differentiating this equation with respect to time gives

$$\frac{d\vec{L}_{\text{tot}}}{dt} = \sum_i \frac{d\vec{L}_i}{dt} = \sum_i \vec{\tau}_i$$

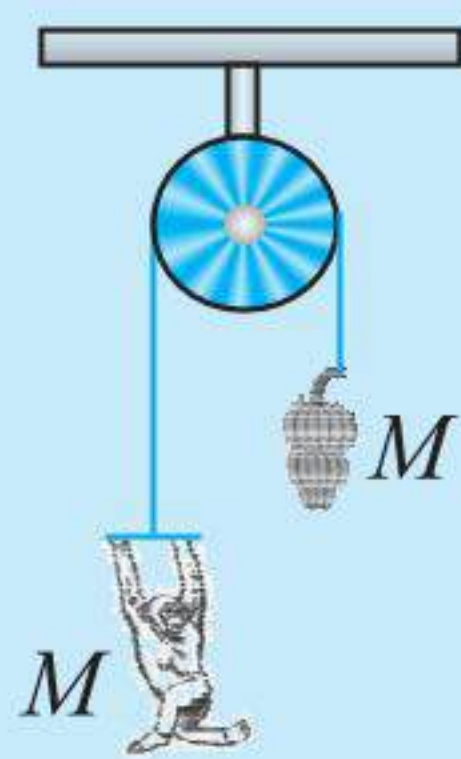
The torques acting on the particles of the system are those associated with internal forces between particles and those associated with external forces. The net torque associated with all internal forces, however, is zero. In the summation, therefore, the net internal torque is zero. We conclude that the total angular momentum of a system can vary with time only if a net external torque is acting on the system. Therefore,

$$\sum \vec{\tau}_{\text{ext}} = \frac{d\vec{L}_{\text{tot}}}{dt}$$

The net external torque on a system equals the time rate of change of angular momentum of the system

ILLUSTRATION 4.3

A monkey of mass M is hanging through a light rope passes over a light, frictionless pulley. Other end of the rope is fastened to a bunch of bananas of mass M . The monkey climbs the rope in an attempt to reach the bananas. Will the monkey reach the bananas?



Sol. Assuming the rope is massless, the tension is the same on both sides of the pulley. Net torque

$$\sum \tau = TR - TR = 0$$

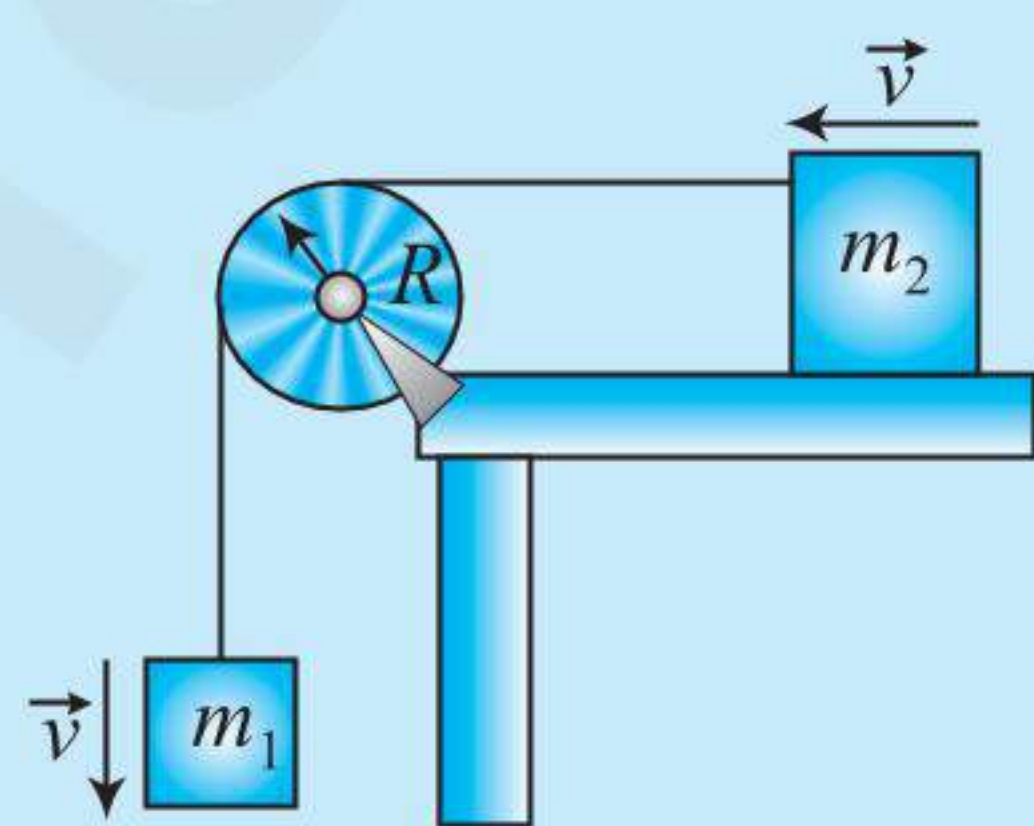
We know $\sum \tau = \frac{dL}{dt}$, and since $\sum \tau = 0$, $L = \text{constant}$.

Since the total angular momentum of the system is initially zero, the total angular momentum remains zero, so the monkey and bananas move upward with the same speed at any instant.

The monkey will not reach the bananas. The motions of the monkey and bananas are identical, so the bananas remain out of the monkey's reach—until they get tangled in the pulley.

ILLUSTRATION 4.4

Two blocks '1' and '2' of masses m_1 and m_2 respectively are connected by a light cord that passes over a pulley as shown in figure. The radius of the pulley is R , and its moment of inertia about its axis of rotation is I . The block '2' slides on a frictionless, horizontal surface. Find an expression for the linear acceleration of the two objects, using the concepts of angular momentum and torque.



Sol. Let us take the block, pulley, and sphere as a system for angular momentum, subject to the external torque due to the gravitational force on the block '1'. Let us calculate the angular momentum about an axis that coincides with the axle of the pulley. The angular momentum of the system includes that of two objects moving translationally (the block '1' and the block '2') and one object undergoing pure rotation (the pulley).

At any instant of time, the block '1' and the block '2' have a common speed v , so the angular momentum of the block '1' about the pulley axle

$$L_1 = m_1 v R \quad \dots(i)$$

We have taken counter clockwise rotation as positive. Similarly, for block '2':

$$L_2 = m_2 v R \quad \dots(ii)$$

At the same instant, all points on the rim of the pulley also move with speed v , so the angular velocity of the pulley is $\omega = \frac{v}{R}$.

The pulley is rotating about a fixed axis, hence the angular momentum of the pulley is

$$L_{\text{pulley}} = I\omega = I \frac{v}{R} \quad \dots(iii)$$

Now let's address the total external torque acting on the system about the pulley axle. The force exerted by the axle on the pulley does not contribute to the torque. Furthermore, the normal force acting on the block '2' is balanced by the gravitational force $m_2 \vec{g}$, so these forces do not contribute to the torque. The gravitational force $m_1 \vec{g}$ acting on the block '1' produces a torque about the axle equal in magnitude to $m_1 g R$, where R is the moment arm of the force about the axle. This result is the total external torque about the pulley axle, that is,

$$\sum \tau_{\text{ext}} = m_1 g R \quad \dots(iv)$$

Write an expression for the total angular momentum of the system:

$$L_{\text{total}} = m_1 v R + m_2 v R + I \frac{v}{R} = \left(m_1 + m_2 + \frac{I}{R^2} \right) v R \quad \dots(v)$$

Now using, $\sum \tau_{\text{ext}} = \frac{dL}{dt}$

$$\text{Hence, } m_1 g R = \frac{d}{dt} \left(m_1 + m_2 + \frac{I}{R^2} \right) v R$$

$$m_1 g = \left(m_1 + m_2 + \frac{I}{R^2} \right) \frac{dv}{dt} \quad \dots(vi)$$

Recognizing that $dv/dt = a$, solve Eq. (vi) for a :

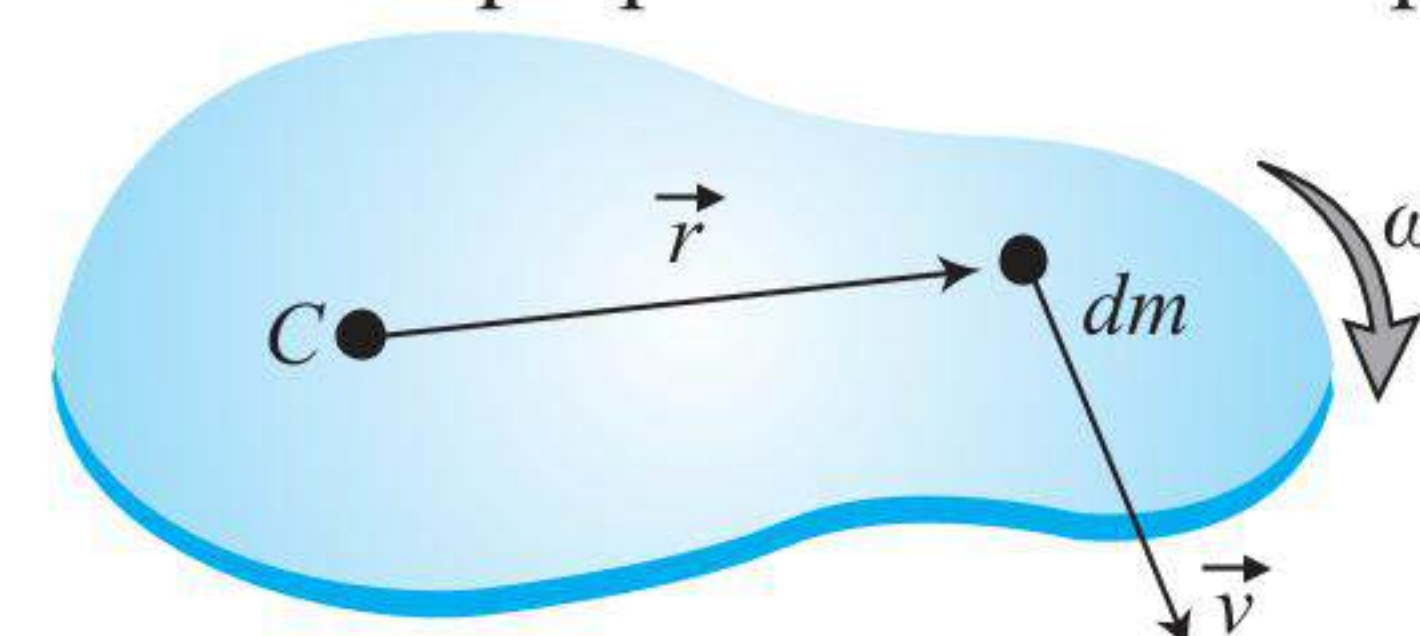
$$a = \frac{m_1 g}{m_1 + m_2 + \frac{I}{R^2}}$$

Important Point:

When we evaluated the net torque about the axle, we did not include the forces that the cord exerts on the objects because these forces are internal to the system under consideration. Instead, we analyzed the system as a whole. Only *external* torques contribute to the change in the system's angular momentum.

ANGULAR MOMENTUM OF A ROTATING RIGID OBJECT ABOUT AN AXIS PASSING THROUGH CENTRE OF MASS

For the sake of simplicity let us consider a plate and try to find its angular momentum about a centroidal axis. Here, we will discuss the cases when the axis is perpendicular to the plane of the plate.



To find the angular momentum, first we consider an elementary mass “ dm ” which moves with a velocity $\vec{v}$ relative to the centre of mass C . Since the body spins with angular speed ω , we have

$$v = r\omega$$

Then, the angular momentum of the mass “ dm ” about C is

$$d\vec{L}_C = dm\vec{r} \times \vec{v}$$

Since $\vec{v} \perp \vec{r}$ and $v = r\omega$, we have

$$d\vec{L}_C = dm r \{r\omega(-\hat{k})\} = (r^2 dm)\vec{\omega}$$

Integrating $d\vec{L}_C$, the total angular momentum is $\vec{L}_C = \int d\vec{L}_C$.

Then, $\vec{L}_C = \vec{\omega} \int r^2 dm$

Substituting $\int r^2 dm = I_C$, we have $\vec{L}_C = I_C \vec{\omega}$

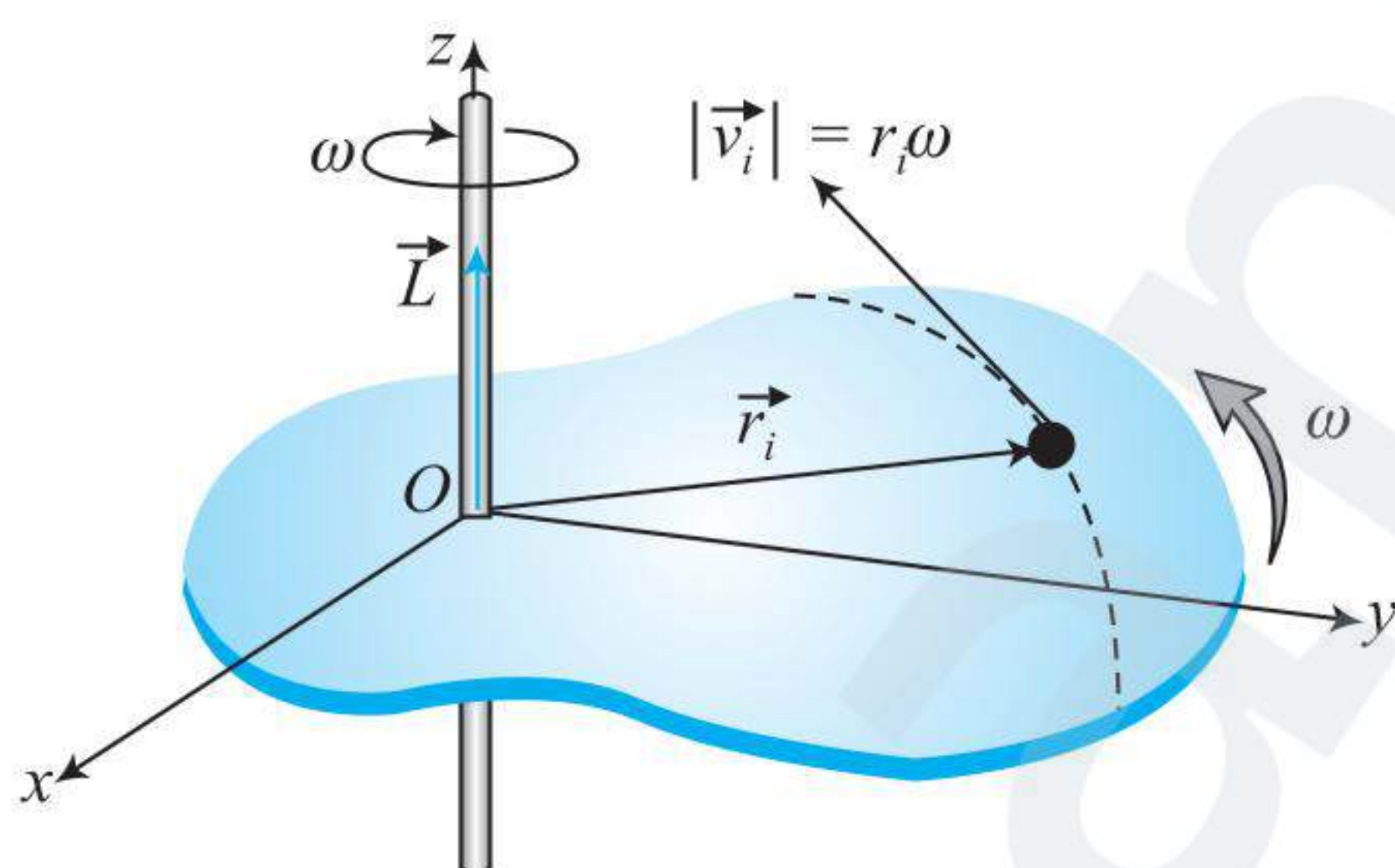
where I_C is called moment of inertia about centroidal axis.

L_C is called spin angular momentum. It is independent of coordinate system. It is an intrinsic property of a rotating body.

ANGULAR MOMENTUM OF A ROTATING RIGID OBJECT ABOUT A FIXED AXIS OF ROTATION

Consider a rigid object rotating about a fixed axis that coincides with the z -axis of a coordinate system as shown in figure. Let's determine the angular momentum of this object. Each particle of the object rotates in the x - y plane about the z -axis with an angular speed ω . The magnitude of the angular momentum of a particle of mass m_i about the z -axis is $m_i v_i r_i$. Because $v_i = r_i \omega$, we can express the magnitude of the angular momentum of this particle as

$$\vec{L}_i = m_i r_i^2 \vec{\omega}$$



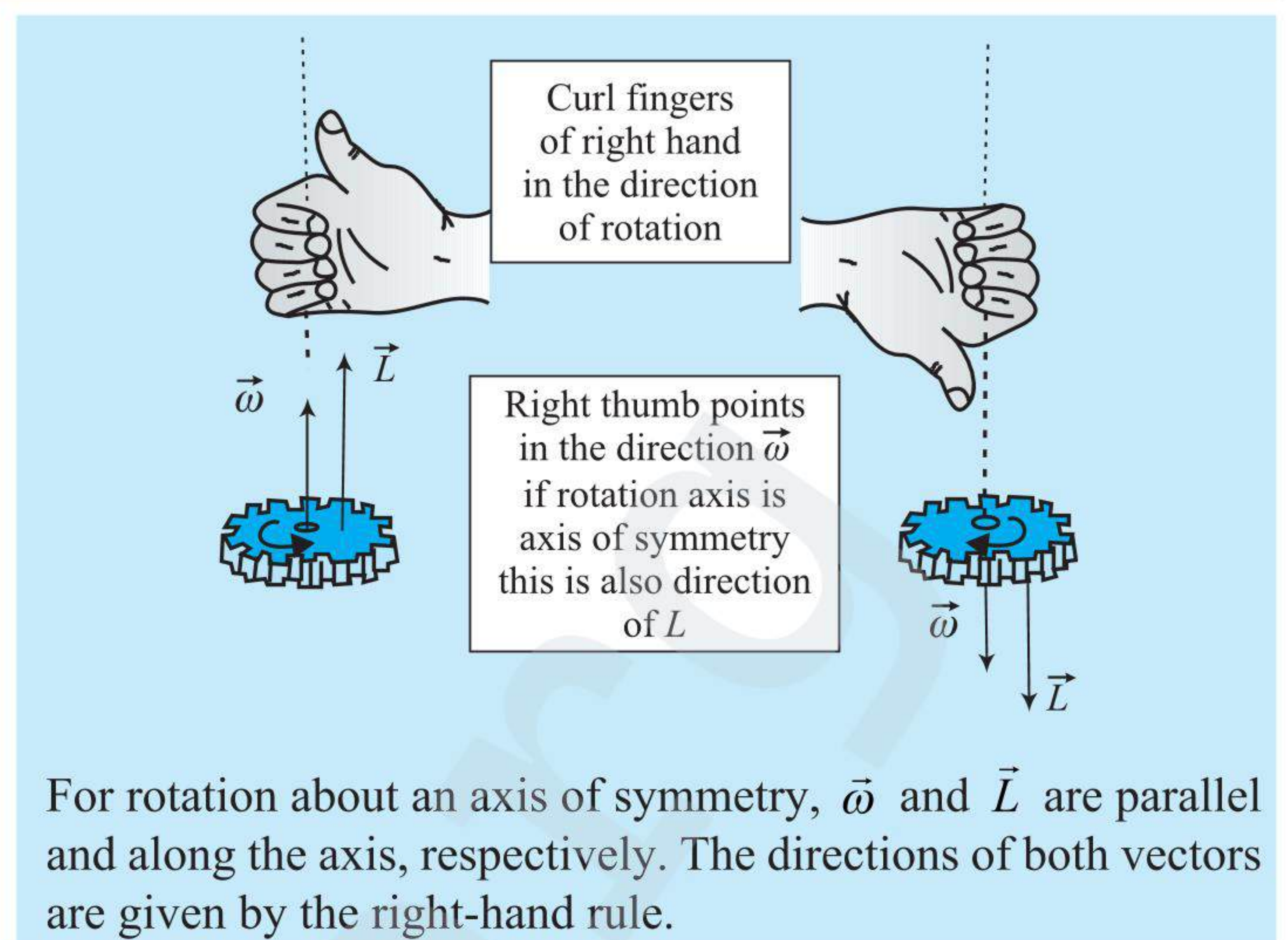
The vector $\vec{L}_i$ for this particle is directed along the z -axis, as is the vector $\vec{\omega}$.

We can now find the angular momentum (which in this situation has only a z component) of the whole object by taking the sum of $\vec{L}_i$ over all particles:

$$\begin{aligned} \vec{L}_z &= \sum_i \vec{L}_i = \sum_i m_i v_i^2 \vec{\omega} = \left(\sum_i m_i r_i^2 \right) \vec{\omega} \\ \vec{L}_z &= I \vec{\omega} \end{aligned} \quad \dots(i)$$

where we have recognized $\sum m_i r_i^2$ as the moment of inertia I of the object about the z -axis. Notice that Eq. (i) is mathematically similar in form for linear momentum: $\vec{p} = m\vec{v}$

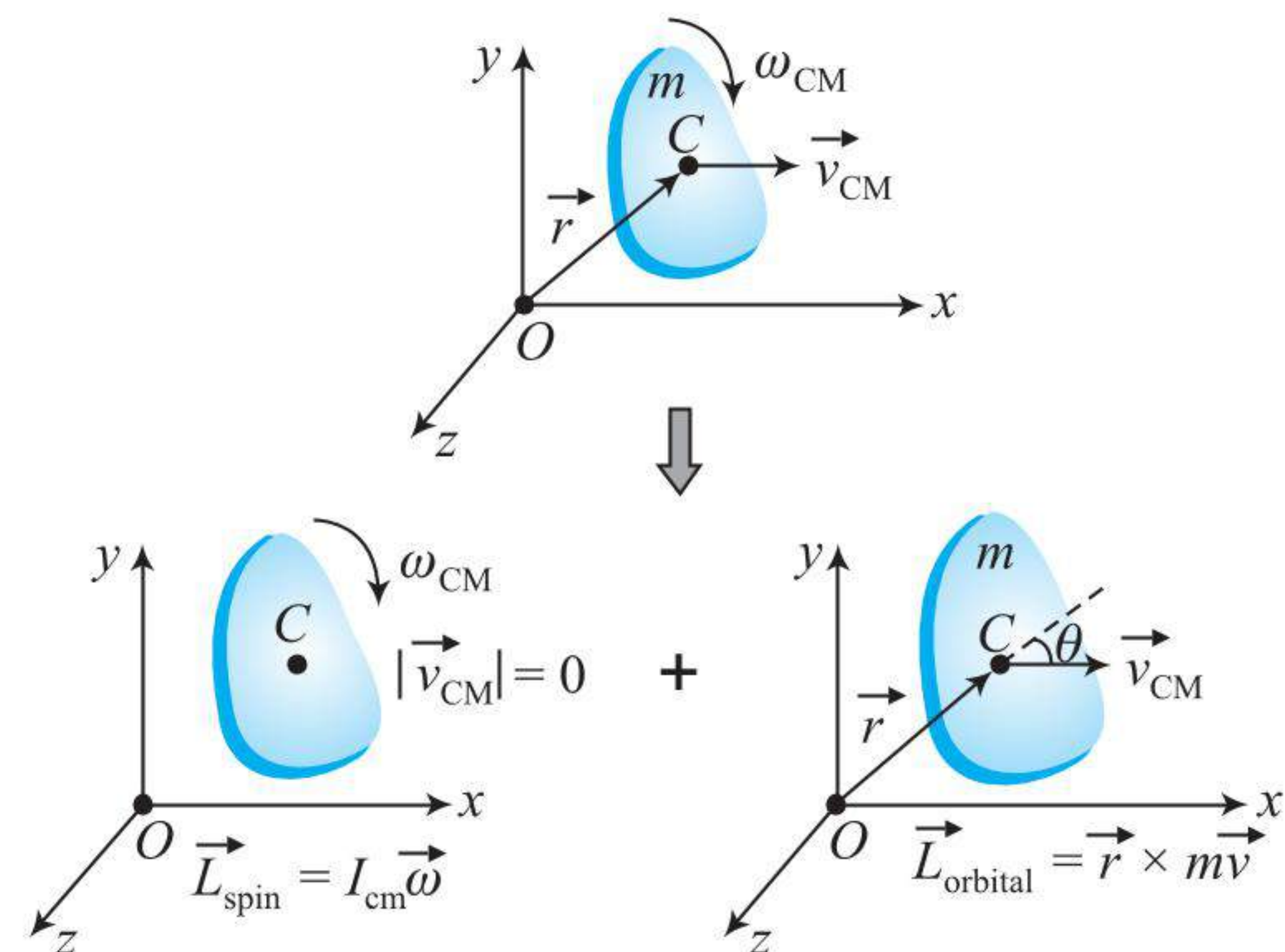
In vector form, we can write Eq. (i) as $\vec{L} = I\vec{\omega}$, where $\vec{L}$ is the total angular momentum of the object measured with respect to the axis of rotation.



For rotation about an axis of symmetry, $\vec{\omega}$ and $\vec{L}$ are parallel and along the axis, respectively. The directions of both vectors are given by the right-hand rule.

ANGULAR MOMENTUM OF A RIGID BODY DUE TO TRANSLATION AND ROTATION BOTH

Consider a body of mass m is rotating with angular velocity ω about CM axis and translating with a linear velocity $\vec{v}$ as shown in figure.



The angular momentum of the body is given by

$$\begin{aligned} \vec{L} &= \vec{L}_{\text{spin}} + \vec{L}_{\text{orbital}} \\ \vec{L} &= \vec{L}_{\text{spin}} + m(\vec{r} \times \vec{v}_{\text{cm}}) = I_{\text{cm}} \vec{\omega} + m(\vec{r} \times \vec{v}_{\text{cm}}) \end{aligned}$$

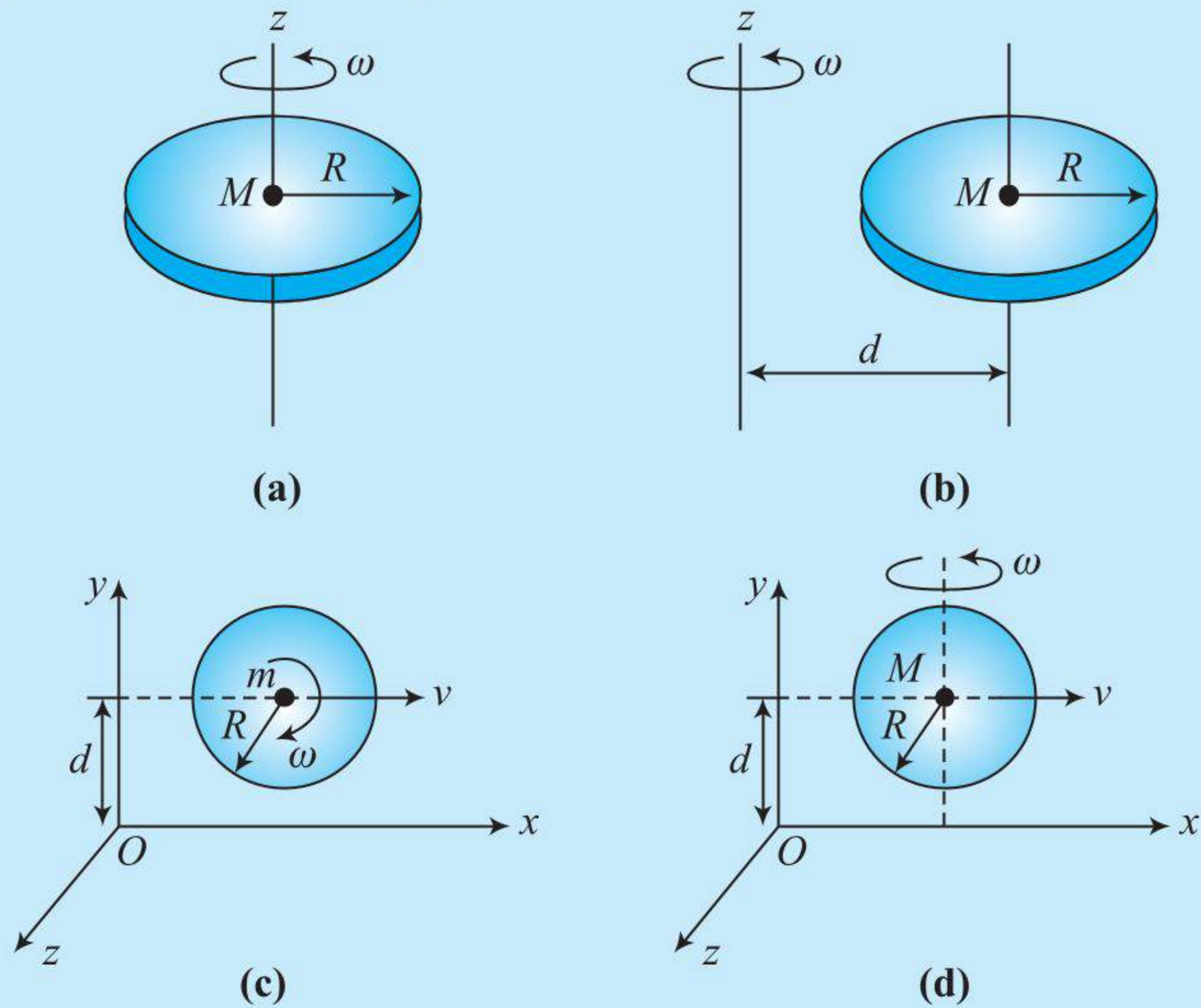
Here $\vec{L}_{\text{spin}}$ represents the angular momentum of the body about CM frame, and $m(\vec{r} \times \vec{v})$ equals the angular momentum of the body if it is assumed to be concentrated at CM translating with velocity $\vec{v}_{\text{cm}}$.

The first term “ $I_{\text{cm}} \vec{\omega}$ ” appears because of spin of the rigid body. Hence, we call it “spin angular momentum” denoted by $\vec{L}_{\text{spin}}$. We can see that $\vec{L}_{\text{spin}}$ does not depend on the reference frame. This can also be termed as “internal angular momentum” because it arises due to the relative motion (velocity) between particles of the rigid body.

The second term “ $\vec{r} \times m\vec{v}$ ” is named as “orbital angular momentum” denoted as $\vec{L}_{\text{orbital}}$ which depends on the position of CM relative to the reference frame.

ILLUSTRATION 4.5

Find the angular momentum of a disc about the axis shown in figure in the following situations.



Sol. Case (a): In this case the disc is rotating about fixed axis which is parallel to the z -axis and passing through its centre and perpendicular to the plane of the disc.

$$\text{Hence, } \vec{L} = I_z \omega (\hat{k}) = \left(\frac{MR^2}{2} \right) \omega (\hat{k})$$

Direction can be determined using right hand rule.

Case (b): In this case also the disc rotates about fixed axis which is not passing through the disc. We can write angular momentum for this situation.

$$\vec{L} = I_z \omega (\hat{k})$$

Using parallel axis theorem, we can find the value of

$$I_z = \frac{MR^2}{2} + Md^2$$

$$\text{Hence, } \vec{L} = \left(\frac{MR^2}{2} + Md^2 \right) \omega (\hat{k})$$

Case (c): In this case the disc is rotating about an axis parallel to the z -axis and centre of mass of disc is also translating.

Hence, angular momentum

$$\vec{L} = \vec{L}_{\text{spin}} + \vec{L}_{\text{orbital}} = I_{\text{cm}} \vec{\omega} + M(\vec{r} \times \vec{v}_{\text{cm}})$$

$$\Rightarrow \vec{L} = \frac{MR^2}{2} \omega (-\hat{k}) + Mvd(-\hat{k}) = \left(\frac{MR^2}{2} \omega + Mvd \right) (-\hat{k})$$

Case (d): In this case also the disc is rotating about an axis passing through its centre and parallel to y -axis and centre of mass of disc is translating with velocity v parallel to x -axis.

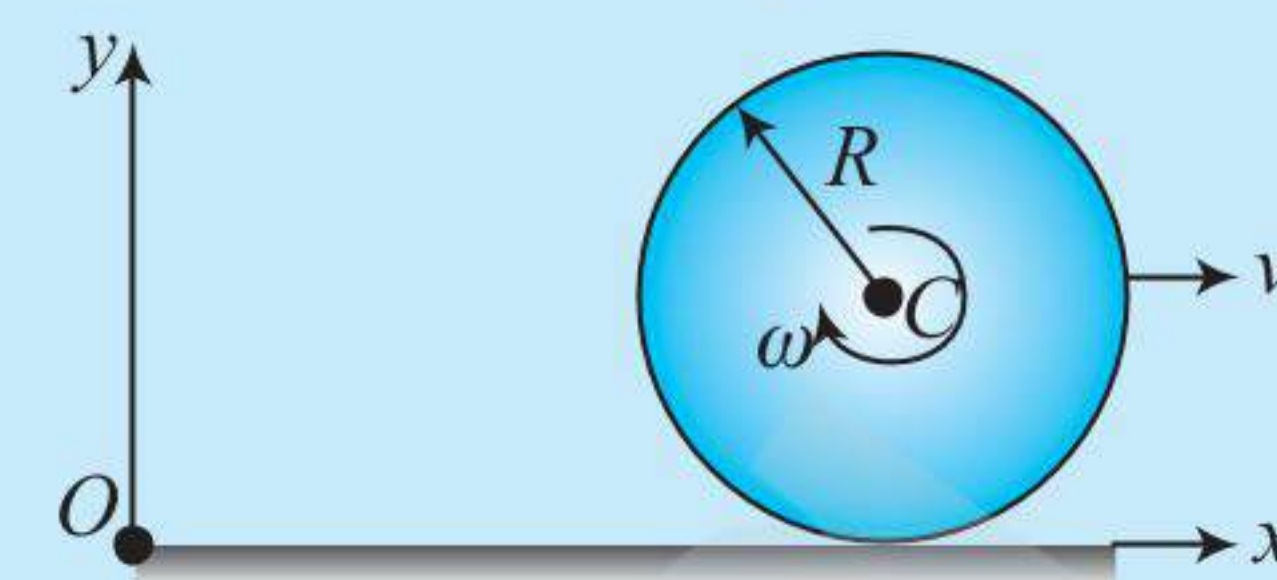
$$\vec{L} = I_{\text{CM}} \omega (\hat{j}) + Mvd(-\hat{k})$$

$$\text{Here, } I_{\text{CM}} = \frac{MR^2}{4}$$

$$\text{Hence, } \vec{L} = \frac{MR^2}{4} \omega (\hat{j}) + Mvd(-\hat{k})$$

ILLUSTRATION 4.6

A cylinder of mass m and radius R is moving on smooth horizontal surface as shown in figure.



- Find its angular momentum about point O on ground.
- Find its angular momentum about point O on if cylinder is rolling without sliding.
- What happens to this angular momentum if the cylinder is rotating in opposite direction but moving translationally in same direction?

Sol. In the given situation, cylinder has both translational and rotational motion. We find its angular momentum about any point by using the equation:

$$\vec{L} = \vec{L}_{\text{spin}} + \vec{L}_{\text{orbital}}$$

$$\vec{L} = \vec{L}_{\text{spin}} + m(\vec{r} \times \vec{v}_{\text{cm}}) = I_{\text{cm}} \vec{\omega} + m(\vec{r} \times \vec{v}_{\text{cm}})$$

- The magnitude of angular momentum of the cylinder about its axis of rotation passing through its centre of mass (called spin angular momentum) is given as

$$L_{\text{spin}} = I_c \omega = \left(\frac{1}{2} mR^2 \right) \omega$$

For direction we can use right hand rule, the cylinder is rotating in clockwise sense, so writing spin angular momentum in vector form,

$$\vec{L}_{\text{spin}} = \left(\frac{1}{2} mR^2 \right) \omega (-\hat{k})$$

the magnitude of angular momentum due to its translational motion

$$L_{\text{orbital}} = mvr_{\perp} = mvR$$

In vector form,

$$\vec{L}_{\text{orbital}} = mvR(-\hat{k})$$

Total angular momentum of cylinder about point O is given as

$$\begin{aligned} \vec{L}_{\text{total}} &= \left(\frac{1}{2} mR^2 \right) \omega (-\hat{k}) + mvR(-\hat{k}) = \left(\frac{1}{2} mR^2 \omega + mvR \right) (-\hat{k}) \\ \Rightarrow \vec{L}_{\text{total}} &= \left(\frac{1}{2} mR^2 \omega + mvR \right) (-\hat{k}) \quad \dots(i) \end{aligned}$$

- If cylinder is rolling, $v = \omega R$ Then from (i) and (ii), we get

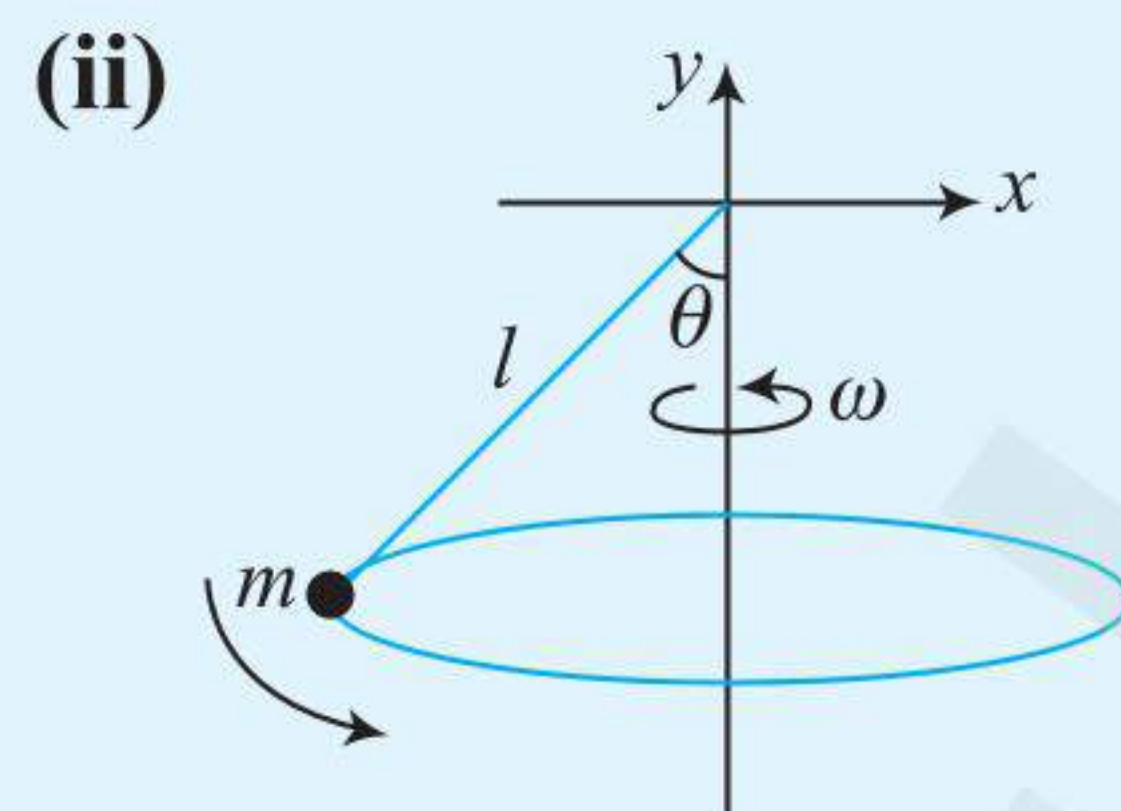
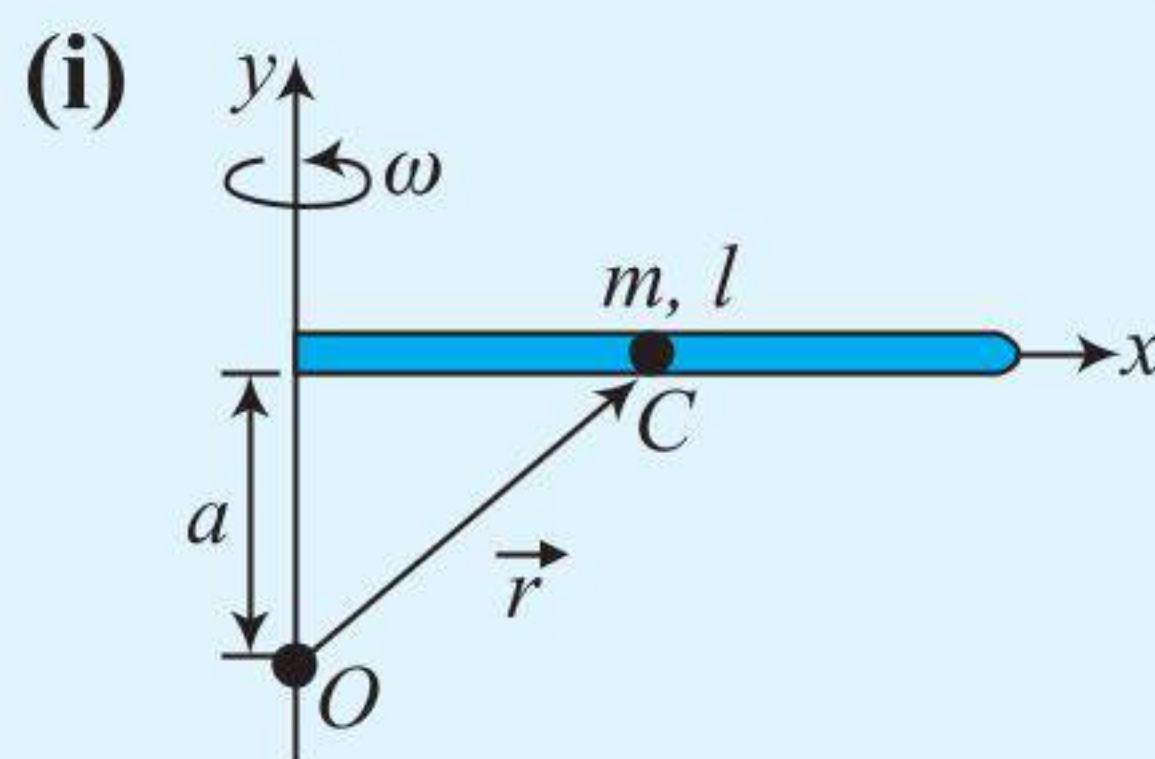
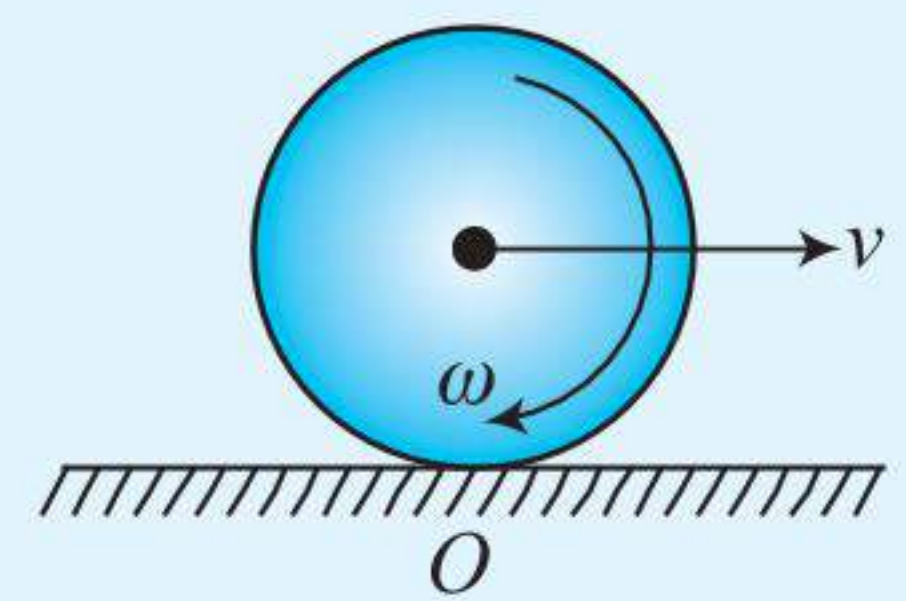
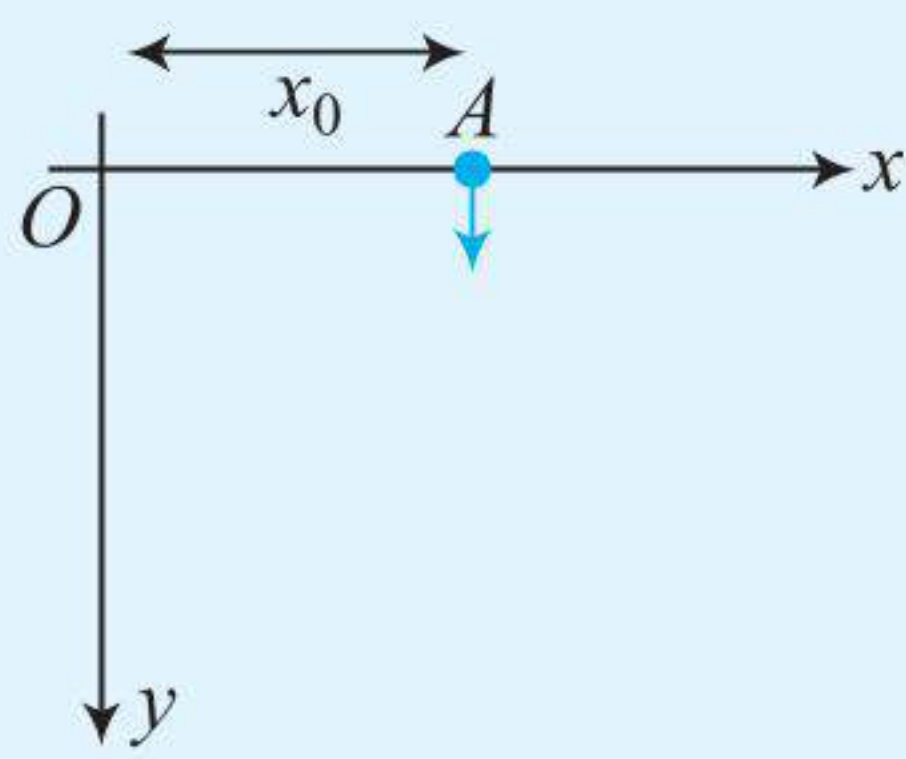
$$\vec{L}_{\text{total}} = \left(\frac{3}{2} mR^2 \omega \right) (-\hat{k})$$

- If cylinder rotates in anticlockwise direction and moving in same direction, its direction of spin angular momentum will change, and other factors will remain same as case (i). Total angular momentum of cylinder about point O is given as

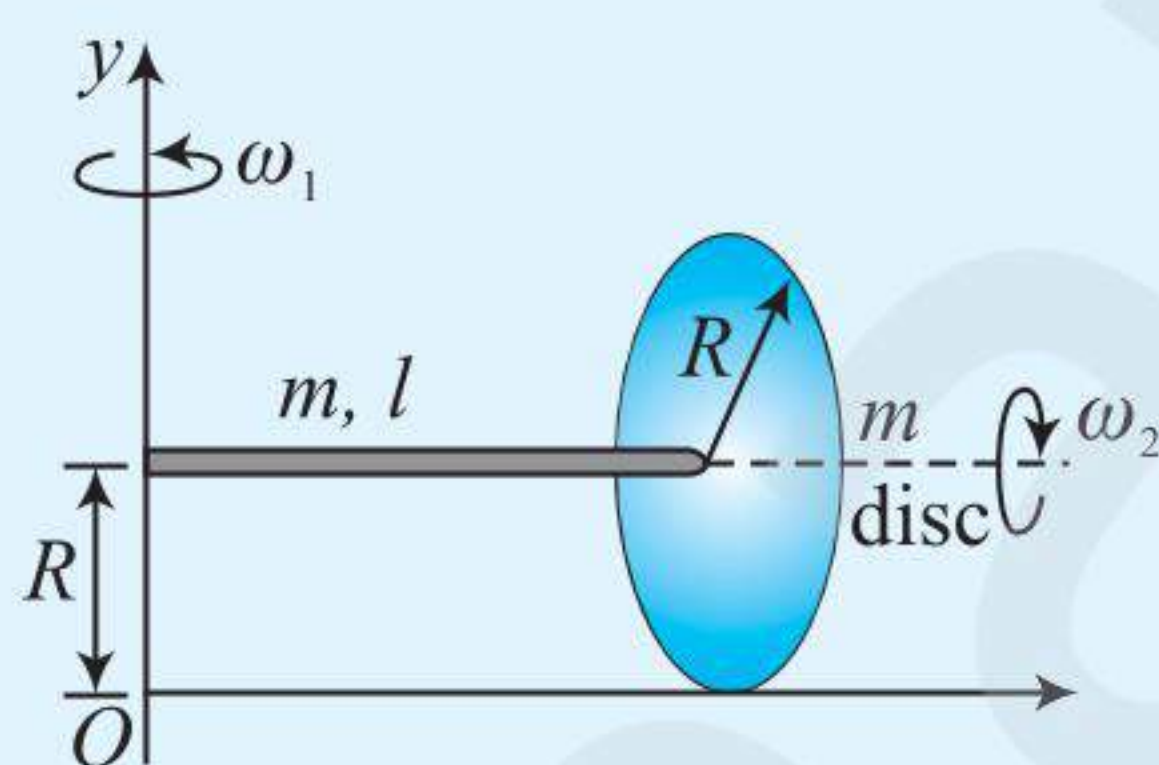
$$\vec{L}_{\text{total}} = \frac{1}{2} mR^2 \omega (\hat{k}) + mvR(-\hat{k})$$

CONCEPT APPLICATION EXERCISE 4.1

1. A particle of mass m is released from rest at point A in figure, falling parallel to the (vertical) y -axis. Find the angular momentum of the particle at any time t with respect to the same origin O .
2. A particle of mass m is projected with a speed u at an angle θ to the horizontal at time $t = 0$. Find its angular momentum about the point of projection O at time t , vectorially. Assume the horizontal and vertical lines through O as X and Y axes, respectively.
3. A circular disc of mass m and radius R is set into motion on a horizontal floor with a linear speed v in the forward direction and an angular speed $\omega = v/R$ in the clockwise direction as shown in figure. Find the magnitude of the total angular momentum of the disc about the bottommost point O of the disc.
4. Calculate the angular momentum about point " O " on the axis of rotation in following cases.



5. Calculate the angular momentum in following situation. A disc is connected with a rod of mass m and length l . The disc is free to rotate about its own axis. If whole system is rotating with $\omega_1 \hat{j}$ and disc also rotate about its own axis with $\omega_2 \hat{i}$ then calculate $\vec{L}$ about origin.



6. A sphere rolls without slipping on a rough surface with centre of mass has constant speed v_0 . If mass of the sphere is M and its radius be R , then find the angular momentum of the sphere about the point of contact.

ANSWERS

1. $mgx_0 t(\hat{k})$ 2. $\frac{mut^2 g \cos \theta}{2}(-\hat{k})$ 3. $\frac{3}{2}mvR$
4. (i) $\frac{ml^2}{3}\omega \hat{j} - \frac{mla\omega}{2}\hat{i}$ (ii) $-2m\omega l^2 \cos \theta \sin \theta \hat{i} + ml^2 \sin^2 \theta \omega \hat{j}$
5. $\frac{4ml^2}{3}\omega_1 \hat{j} - \left(\frac{mR^2}{2}\omega_2 - \frac{3}{2}m\omega_1 lR \right) \hat{i} + \frac{mR^2}{4}\omega_1 \hat{j}$
6. $\frac{7}{5}Mv_0 R(-\hat{k})$

ANGULAR IMPULSE:
IMPULSE-MOMENTUM THEOREM

In complete analogue with the linear impulse, the angular impulse $\vec{J}$ for a rigid body rotating about a fixed axis is defined as $\vec{J} = \int \vec{\tau} dt$.

The impulse-momentum theorem for rotational motion may be stated as

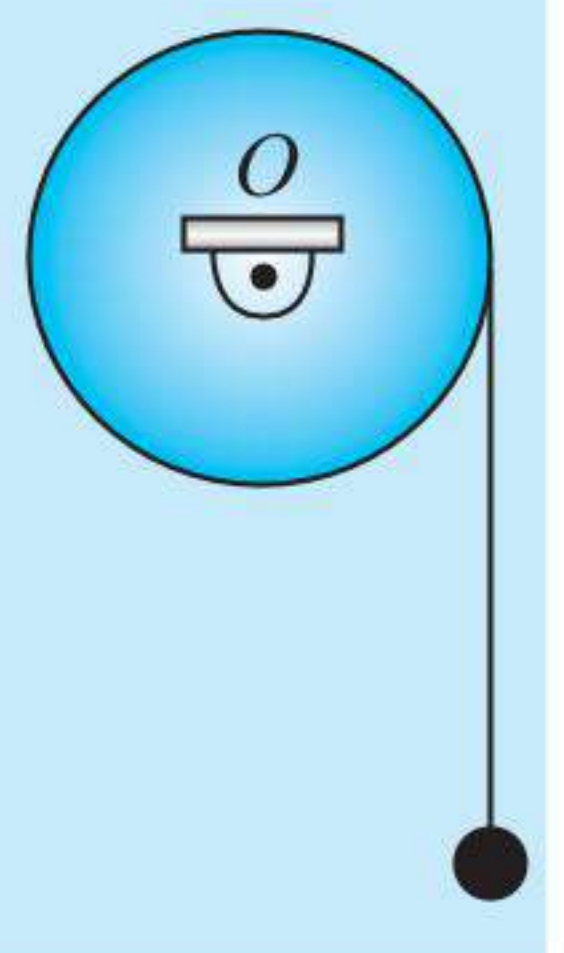
$$\text{Angular impulse, } \vec{J} = \Delta \vec{L} = \vec{L}_f - \vec{L}_i$$

$$\Rightarrow \vec{J} = I\vec{\omega}_f - I\vec{\omega}_i = I(\vec{\omega}_f - \vec{\omega}_i)$$

As linear impulse changes the linear momentum, angular impulse changes the angular momentum. This equation is called Impulse-momentum equation.

ILLUSTRATION 4.7

A uniform disc of mass M and radius R is smoothly pivoted at O . A light inextensible string wrapped over the disc hangs a particle of mass m . If the system is released from rest, assuming that the string does not slide on the disc, find the angular speed of the disc as the function of time using impulse-momentum equation.



Sol. The net torque about O is

$$\vec{\tau} = -mgR\hat{k} \quad \dots(i)$$

The angular momentum of the disc particle system about O is

$$L = I_0 \omega + mvR, \text{ where } v = R\omega$$

$$L = \frac{MR^2}{2}\omega + mR^2\omega = \left(\frac{MR^2}{2} + mR^2 \right) \omega$$

$$\vec{L} = \left(\frac{M + 2m}{2} \right) R^2 \omega (-\hat{k}) \quad \dots(ii)$$

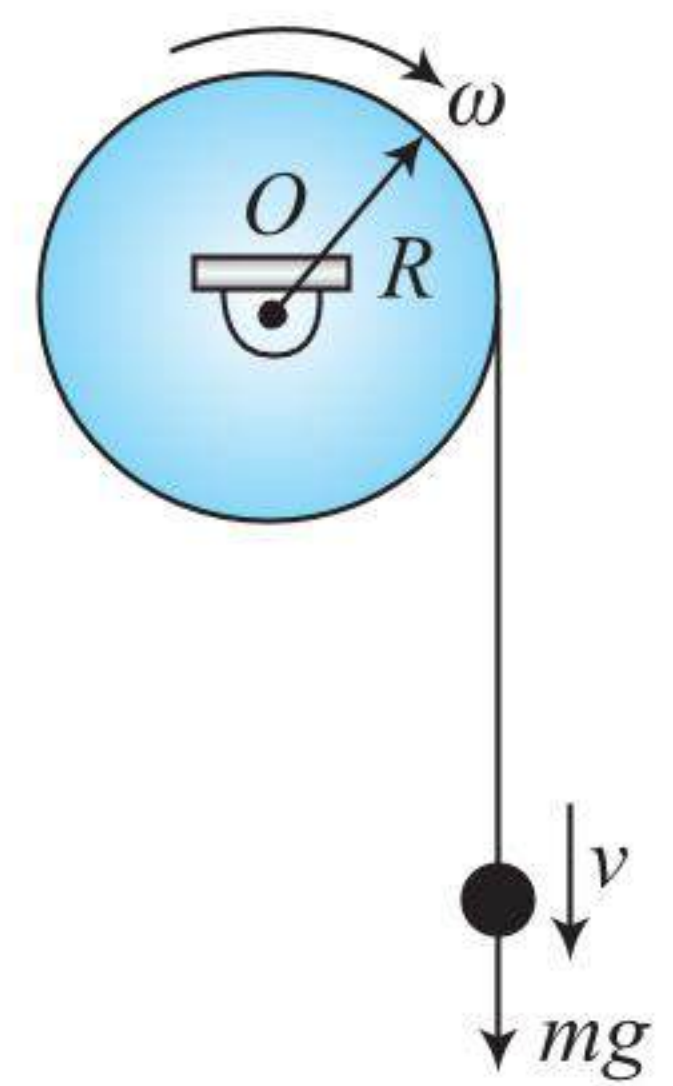
Impulse-momentum equation is

$$\Delta \vec{L} = \int \vec{\tau} dt \quad \dots(iii)$$

Using Eqs. (i), (ii) and (iii), we have

$$\left(\frac{M + 2m}{2} \right) R^2 \omega = mgR \int_0^t dt$$

$$\text{or } \omega = \frac{2mgt}{(M + 2m)R}$$

LAW OF CONSERVATION OF
ANGULAR MOMENTUM

We know that the total linear momentum of a system of particles remains constant if the system is isolated, that is, if the net external force acting on the system is zero. We have an analogous conservation law in rotational motion:

For rotational motion, we define torque.

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad \left[\equiv \frac{d\vec{p}}{dt} = \vec{F} \right]$$

So, if the net torque on a particle (or system) is zero $\frac{d\vec{L}}{dt} = 0$, i.e., $\vec{L} = \text{constant}$.

“The total angular momentum of a system is constant in both magnitude and direction if the net external torque acting on the system is zero, that is, if the system is isolated.”

This statement is often called the principle of **conservation of angular momentum** and is the basis of the **angular momentum version of the isolated system model**.

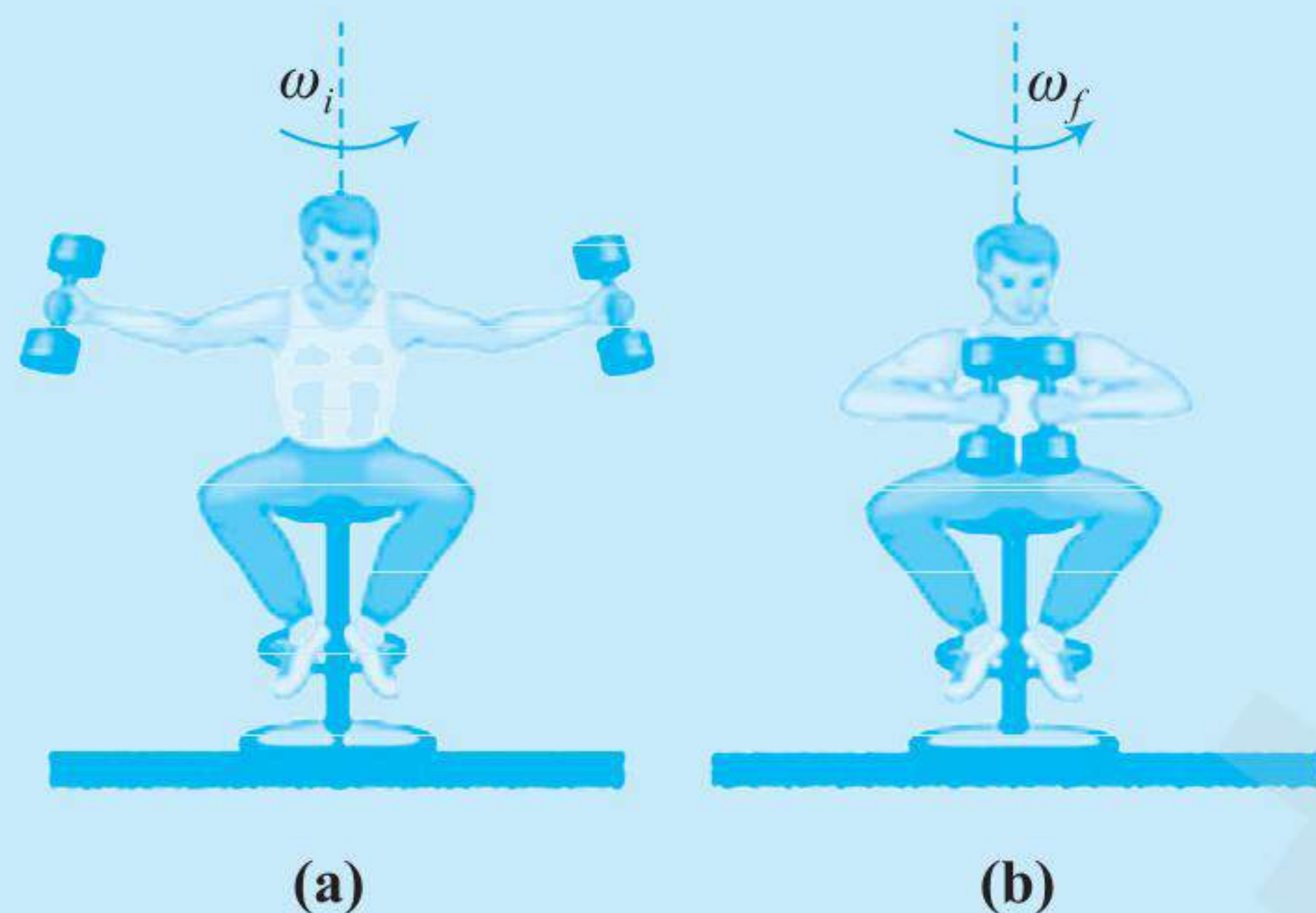
Hence, when the total angular momentum is conserved,

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \dots = \text{constant}$$

This shows that the angular momentum of an individual particle may change, but their sum remains constant in the absence of an external torque.

ILLUSTRATION 4.8

A man sits on a freely rotating stool holding two dumbbells, each of mass 2.0 kg. When his arms are extended horizontally [Fig.(a)], the dumbbells are 1.0 m from the axis of rotation and the man rotates with an angular speed of 0.50 rad/s. The moment of inertia of the man plus stool is 5.0 kgm² and is assumed to be constant. The man pulls the dumbbells inward horizontally to a position 0.50 m from the rotation axis (Fig. (b)). Find the new angular speed of the man.



Sol. A man is seated on a stool that can rotate freely about a vertical axis with his arms stretched horizontally with equal weights in each hand. As no net external torque acts on the system that consist of the ‘student, stool, and dumbbells’. It means, the angular momentum of that system about the rotation axis must remain constant. The total angular momentum of the system is conserved then,

$$\vec{L}_i = \vec{L}_f \quad \text{or} \quad I_i \omega_i = I_f \omega_f$$

Moment of inertia of the system,

$$I_{\text{total}} = I_{\text{student}} + I_{\text{weights}} = 5.0 + 2(mr^2)$$

Initially, $r_{\text{initial}} = 1.0 \text{ m}$

Thus, $I_i = 5.0 + 2 \times (2.0)(1.0)^2 = 9.0 \text{ kg m}^2$

Finally, $r_{\text{final}} = 0.50 \text{ m}$

Thus, $I_f = 5.0 + 2(2.0)(0.50)^2 = 6.0 \text{ kg m}^2$

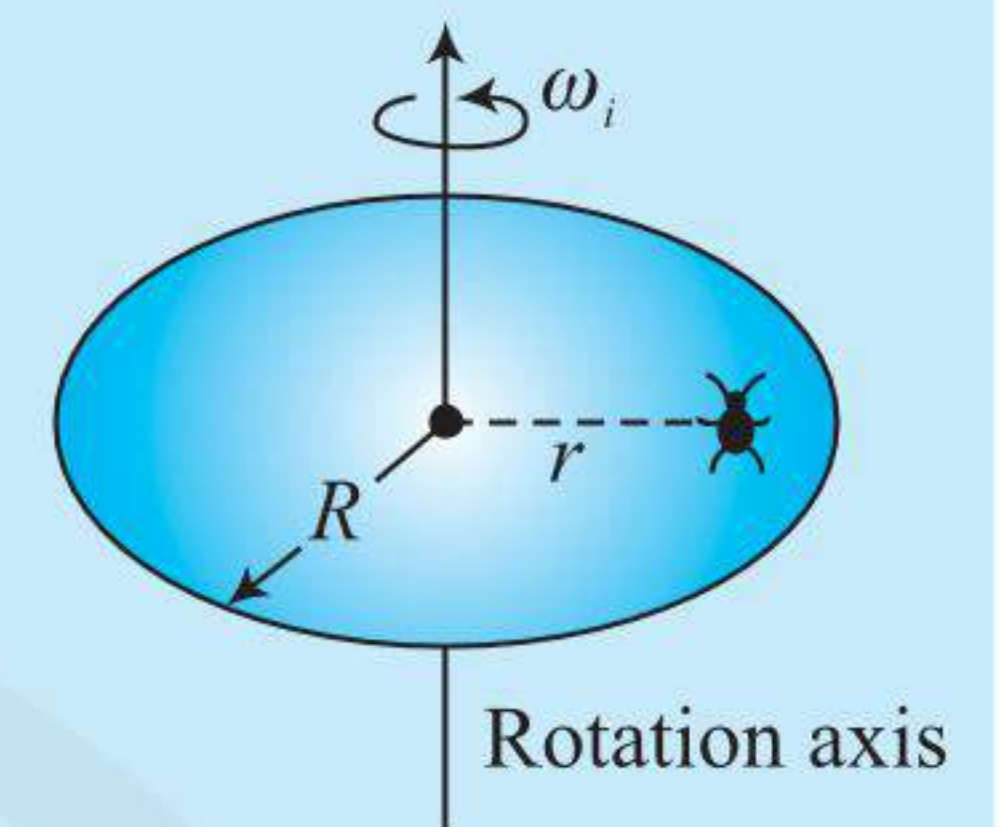
We now using conservation of angular momentum.

$$I_f \omega_f = I_i \omega_i$$

$$\text{or} \quad \omega_f = \left(\frac{I_i}{I_f} \right) \omega_i = \left(\frac{9.0}{6.0} \right) (0.50) = 0.75 \text{ rad/s}$$

ILLUSTRATION 4.9

In figure, a cockroach with mass m rides on a disk of mass $6.00m$ and radius R . The disk rotates like a merry-go-round around its central axis at angular speed $\omega_i = 1.50 \text{ rad/s}$. The cockroach is initially at radius $r = 0.800R$, but then it crawls out to the rim of the disk. Treat the cockroach as a particle. What then is the angular speed?



Sol. The cockroach’s crawl changes the mass distribution (and thus the rotational inertia) of the cockroach-disk system. The angular momentum of the system does not change because there is no external torque to change it. (The forces and torques due to the cockroach’s crawl are internal to the system.)

We want to find the final angular speed. Our key is to equate the final angular momentum L_f to the initial angular momentum L_i , because both involve angular speed. They also involve rotational inertia I . So, let’s start by finding the rotational inertia of the system of cockroach and disk before and after the crawl.

The rotational inertia of a disk rotating about its central axis is given by $I_d = \frac{1}{2} MR^2$. Substituting $6.00m$ for the mass M , our disk here has rotational inertia,

$$I_d = 3.00mR^2 \quad \dots(i)$$

We know that the rotational inertia of the cockroach (a particle) is equal to mr^2 . Substituting the cockroach’s initial radius ($r = 0.800R$) and final radius ($r = R$), we find that its initial rotational inertia about the rotation axis is,

$$I_{ci} = 0.64mR^2 \quad \dots(ii)$$

and its final rotational inertia about the rotation axis is

$$I_{cf} = mR^2 \quad \dots(iii)$$

So, the cockroach-disk system initially has the rotational inertia,

$$I_i = I_d + I_{ci} = 3.64mR^2 \quad \dots(iv)$$

and finally has the rotational inertia,

$$I_f = I_d + I_{cf} = 4.00mR^2 \quad \dots(v)$$

Next, we use equation ($L = I\omega$) to write the fact that the system’s final angular momentum L_f is equal to the system’s initial angular momentum L_i :

$$I_f \omega_f = I_i \omega_i \quad \text{or} \quad 4.00mR^2 \omega_f = 3.64mR^2 (1.50 \text{ rad/s})$$

After canceling the unknowns m and R , we come to,

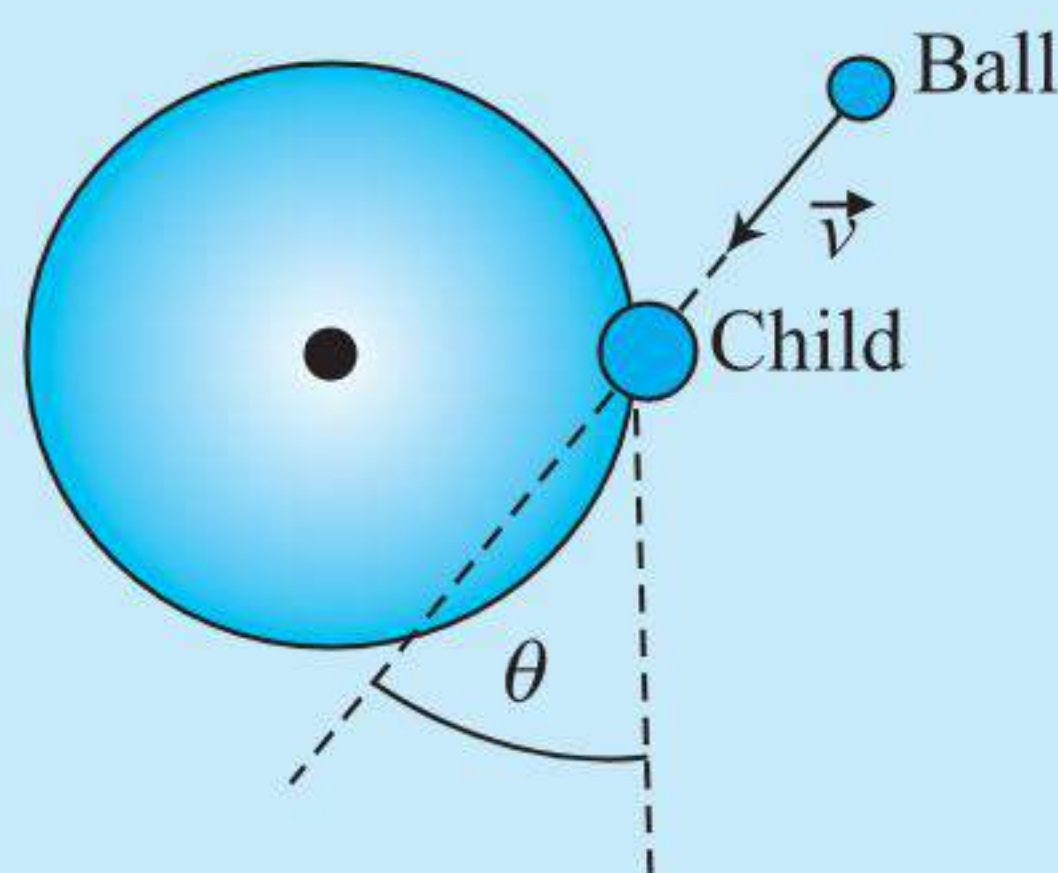
$$\omega_f = 1.37 \text{ rad/s}$$

Note that ω decreased because part of the mass moved outward, thus increasing that system’s rotational inertia.

ILLUSTRATION 4.10

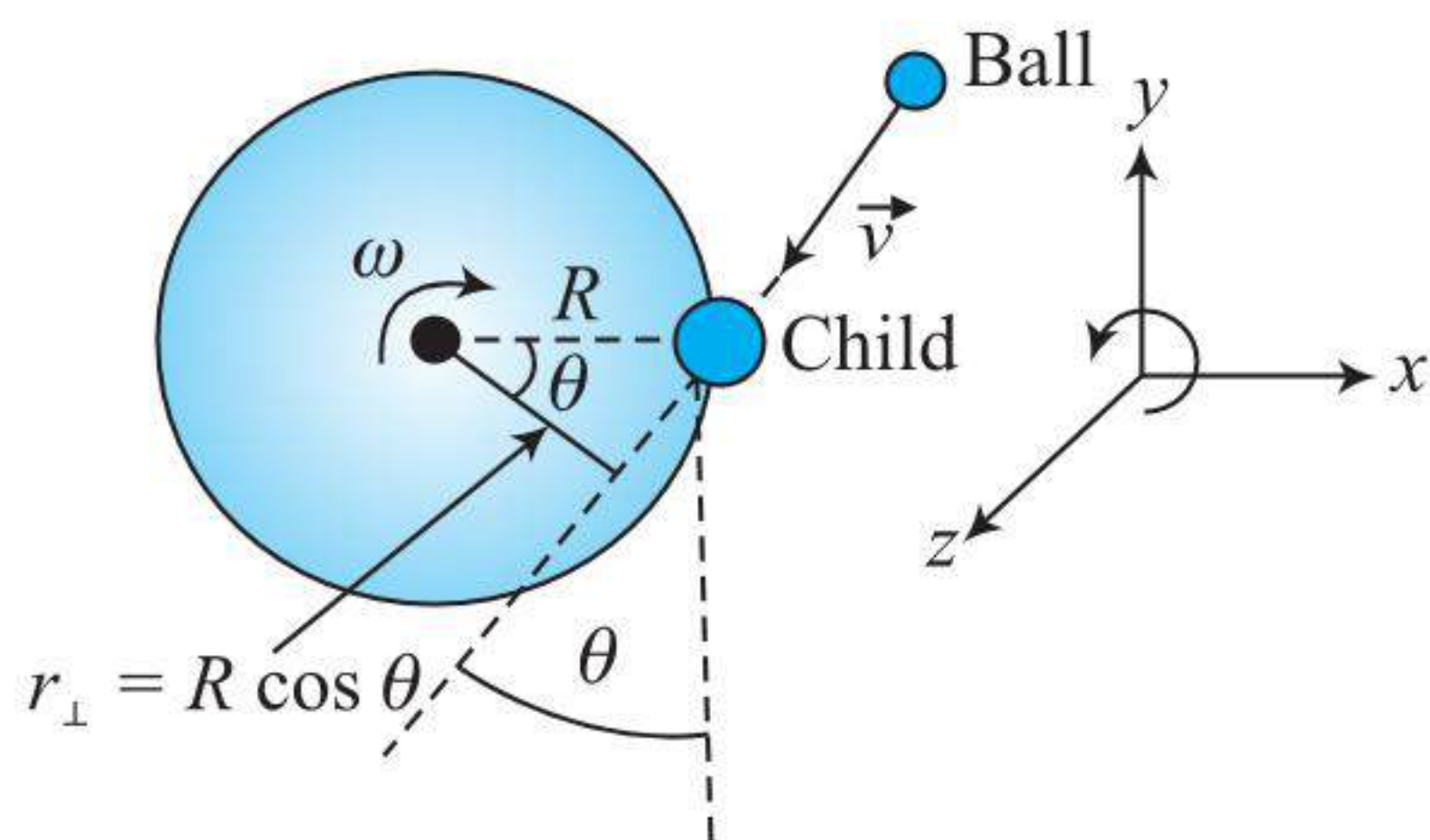
A child of mass 39 kg stands on the edge of a stationary merry-go-round of radius 2.5 m. The moment of inertia of the merry-go-round about its rotation axis is 150 kg m². The child catches a ball of mass 1.0 kg thrown by a friend. Just before the ball

is caught, it has a horizontal velocity of magnitude 50 m/s, at angle $\theta = 37^\circ$ with a line tangent to the outer edge of the merry-go-round, as shown. What is the angular speed of the merry-go-round just after the ball is caught?



Top view of child on merry-go-round

Sol. There is no net external torque acts on the system that consist of the ‘child, merry-go-round and ball’. It means, the angular momentum of that system about the rotation axis must remain constant.



We define the magnitude of angular momentum of a particle,

$$L = mvr_{\perp}$$

where $r_{\perp}$ is the perpendicular distance between the axis of rotation to line of action of $\vec{p}$.

Let m and v_0 be the mass and initial speed of the ball and R the radius of the merry-go-round. In this case,

$$r_{\perp} = R \cos 37^\circ = 2.5 \times \frac{4}{5} = 2 \text{ m} \quad \dots(i)$$

The initial angular momentum is

$$L_i = mv_0 r_{\perp} = 1 \times 50 \times 2 = 100 \text{ kg m}^2/\text{s}$$

in clockwise direction. In vector form, we can write

$$\vec{L}_i = 100(-\hat{k}) \text{ kg m}^2/\text{s}$$

After catching the ball, the system ‘child, merry-go-round and ball’ rotate with common angular velocity in clockwise direction to keep angular momentum conserve. Let this angular velocity be ω , then final angular momentum

$$L_f = I_{\text{system}} \omega \quad \dots(ii)$$

Now let us calculate moment of inertia of the system,

$$I_{\text{system}} = I_{\text{merry go round}} + I_{\text{child}} + I_{\text{ball}}$$

$$\text{or, } I_{\text{system}} = 150 + (39)R^2 + (1.0)R^2 = 150 + 40R^2 = 400 \text{ kg m}^2$$

Substituting the value of I_{system} in Eq. (i), we get

$$L_f = 400\omega$$

In vector form we can write,

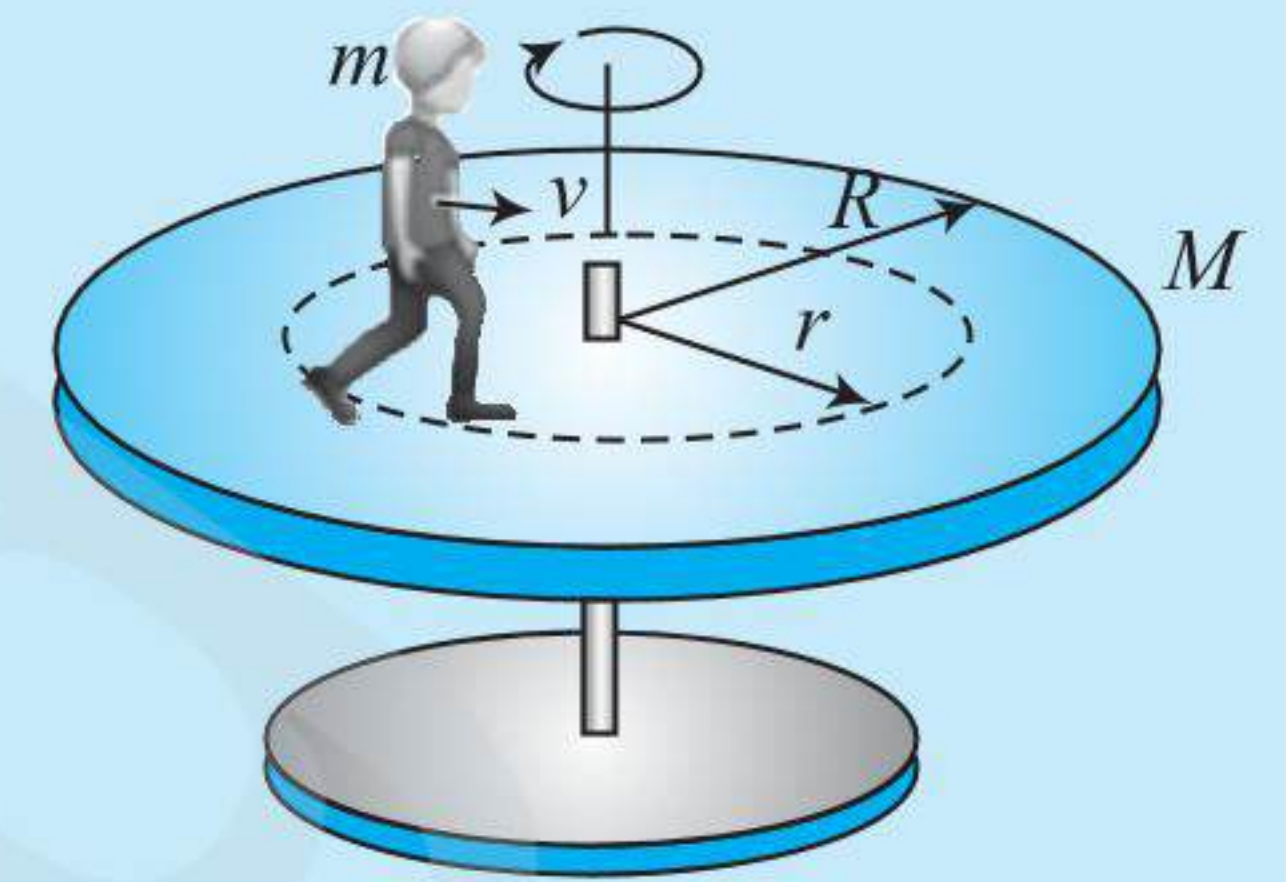
$$\vec{L}_f = 400\omega(-\hat{k}) \text{ kg m}^2/\text{s} \quad \dots(iii)$$

As angular momentum is conserved then, $\vec{L}_i = \vec{L}_f$

$$\text{or, } 100(-\hat{k}) = 400\omega(-\hat{k}), \text{ hence } \omega = 0.25 \text{ rad/s.}$$

ILLUSTRATION 4.11

A man of mass m stands on a horizontal platform in the shape of a disc of mass M and radius R , pivoted on a vertical axis through its centre about which it can freely rotate. The man starts to move around the centre of the disc in a circle of radius r with a velocity v relative to the disc. Calculate the angular velocity of the disc.



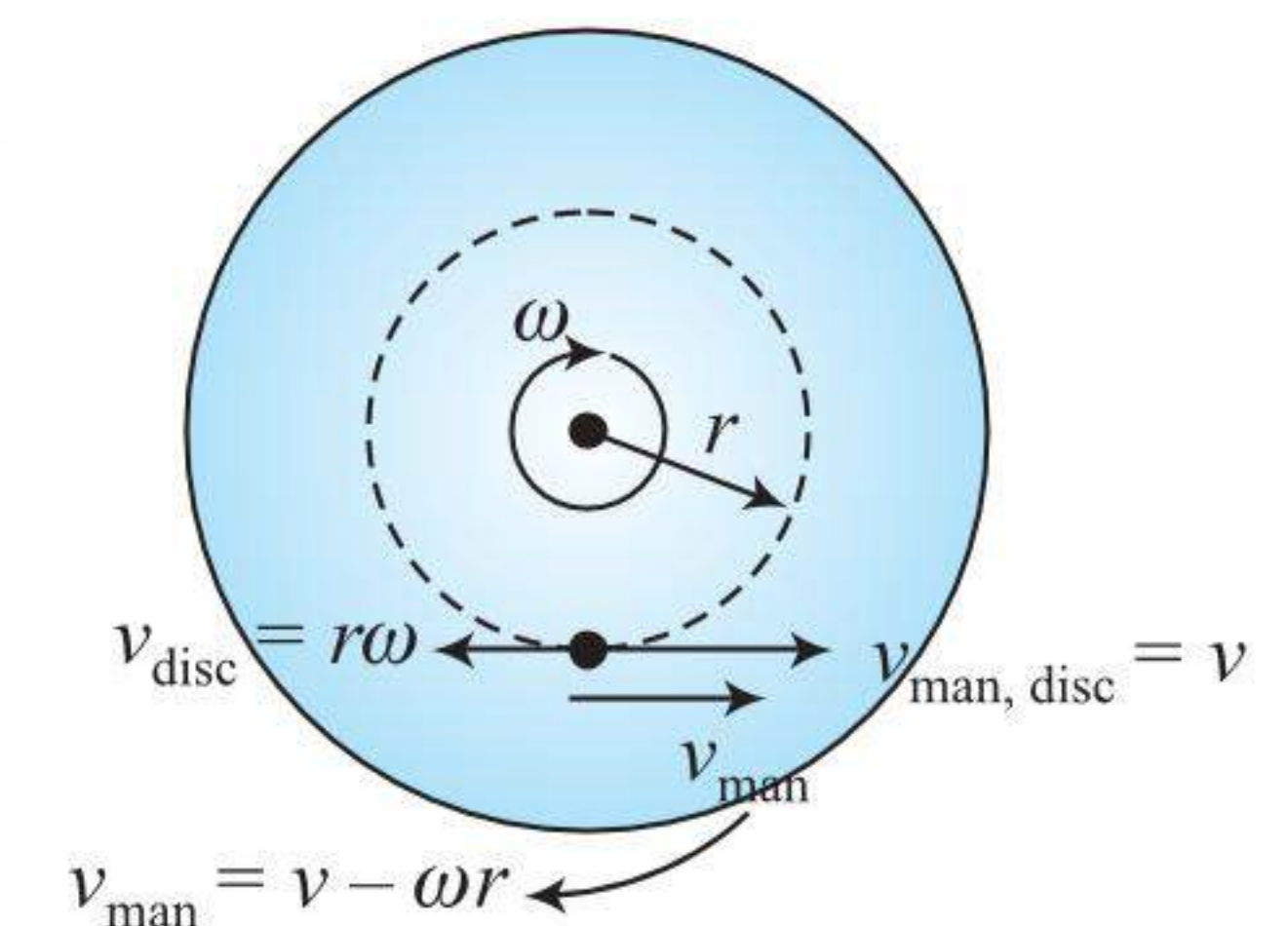
Sol. If we take ‘disc + man’ as a system, there is no torque acting about the axis of rotation of the disc, so the angular momentum of the system remains constant. Initially it is zero. When the man starts rotating about the axes of the disc, to maintain zero angular momentum the disc start rotating in opposite sense of rotation of man. Suppose ω is the angular velocity of the disc in clockwise direction.

Let us take counter clockwise rotation as positive. The velocity of the man with respect to the ground will be:

$$\vec{v}_{\text{man, disc}} = \vec{v}_{\text{man}} - \vec{v}_{\text{disc}}$$

$$\Rightarrow \vec{v}_{\text{man}} = \vec{v}_{\text{man, disc}} + \vec{v}_{\text{disc}}$$

$$\Rightarrow |\vec{v}_{\text{man}}| = v - \omega r$$



Thus, angular momentum of the man $\vec{L}_{\text{man}} = m(v - \omega r)r\hat{k}$

$$\text{and } \vec{L}_{\text{disc}} = I_{\text{disc}} \vec{\omega} = -\frac{MR^2}{2} \omega \hat{k}$$

(taking counter clockwise rotation as positive)

By conservation of angular momentum, we have

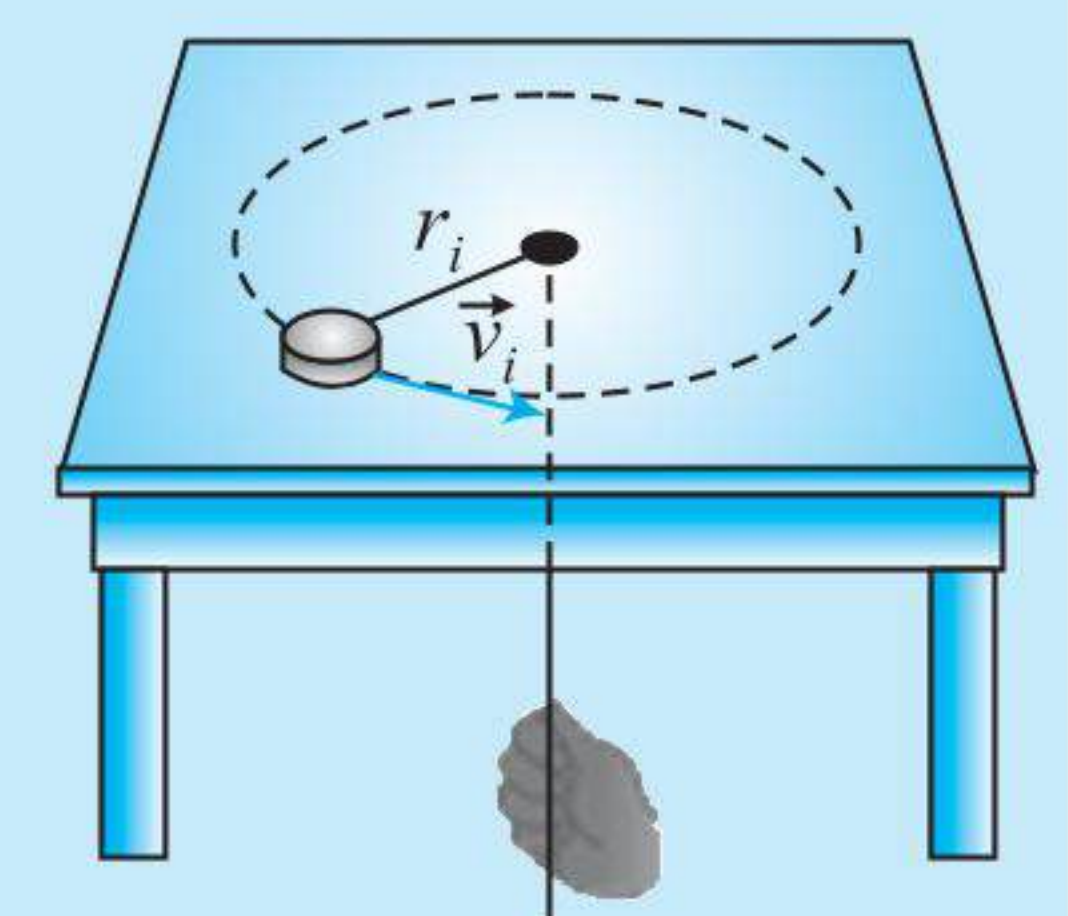
$$\vec{L}_{\text{initial}} = \vec{L}_{\text{final}} \Rightarrow 0 = m(v - \omega r)r - \frac{MR^2}{2} \omega$$

After solving, we get

$$\omega = \frac{mvr}{\left(mr^2 + \frac{MR^2}{2} \right)}$$

ILLUSTRATION 4.12

A puck of mass $m = 50.0 \text{ g}$ is attached to a taut cord passing through a small hole in a frictionless, horizontal surface (figure). The puck is initially orbiting with speed $v_i = 1.50 \text{ m/s}$ in a circle of radius $r_i = 0.300 \text{ m}$. The cord is then slowly pulled from below, decreasing the radius of the circle to $r = 0.100 \text{ m}$.



(a) What is the puck’s speed at the smaller radius?

(b) Find the tension in the cord at the smaller radius.

Sol.

(a) The puck’s linear momentum is always changing. Its mechanical energy changes as work is done on it. But its angular momentum stays constant because although an

external force (the tension of the rope) acts on the puck, no external torques act. Therefore, $L = \text{constant}$, and at any time, $mvr = mv_i r_i$ giving us,

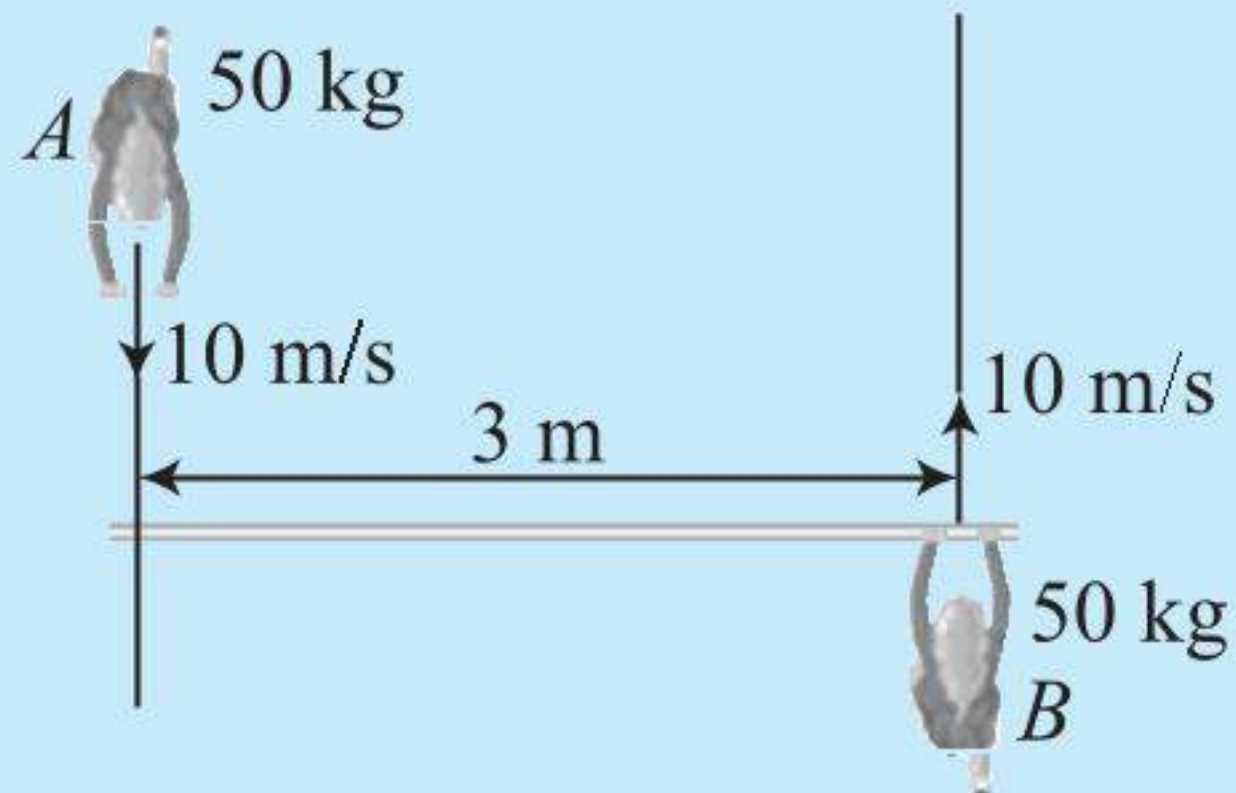
$$v = \frac{v_i r_i}{r} = \frac{(1.50 \text{ m/s})(0.300 \text{ m})}{0.100 \text{ m}} = 4.50 \text{ m/s}$$

(b) From Newton's second law, the tension is always

$$T = \frac{mv^2}{r} = \frac{(0.050 \text{ kg})(4.50 \text{ m/s})^2}{0.100 \text{ m}} = 10.1 \text{ N}$$

ILLUSTRATION 4.13

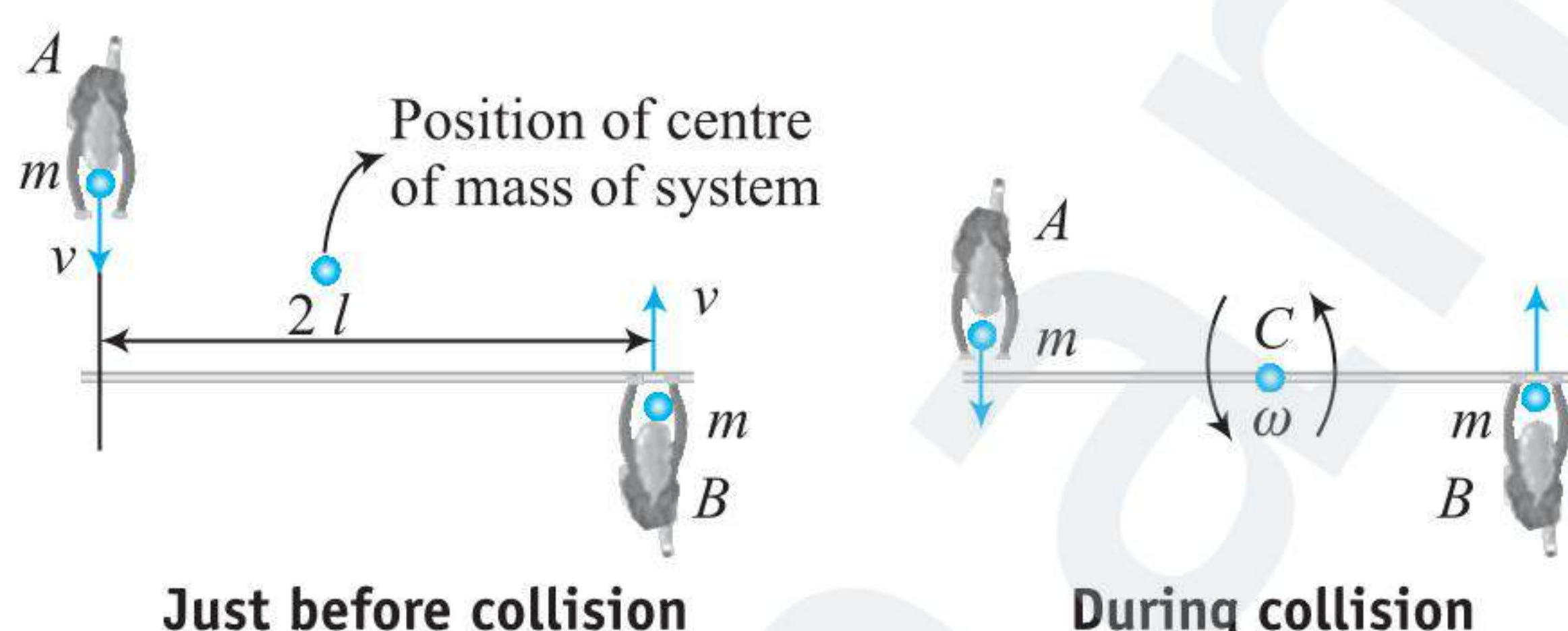
Two skaters, each of mass 50 kg, approach each other along parallel paths separated by 3 m. They have equal and opposite velocities of 10 m/s. The first skater carries a long light pole, 3 m long, and the second skater grabs the end of it as he passes (assume frictionless ice).



- (a) Describe quantitatively the motion of the skaters after they are connected by the pole.
 (b) By pulling on the pole the skaters reduce their distance to 1 m. What is their motion then?

Sol.

- (a) If we take skater 'A', skater 'B' and pole as system there is no net external force acting on system, the linear momentum of the system will remain conserved. As the initial linear momentum of the system (skaters + pole) is zero, the centre of mass will be at rest before and after the collision. After connecting with pole, the skaters and the pole will rotate around the centre of mass (at the midpoint of the pole).



Applying the conservation of angular momentum about an axis through C and perpendicular to the plane of the figure,

$$mvl + mvl = I\omega \text{ where } I = 2m(l)^2$$

$$\omega = \frac{2mvl}{I} = \frac{v}{l} = \frac{20}{3} \text{ rad/s}$$

- (b) Now, the separation reduces to $2l' = 1 \text{ m}$,
 Again, applying conservation of angular momentum, we get

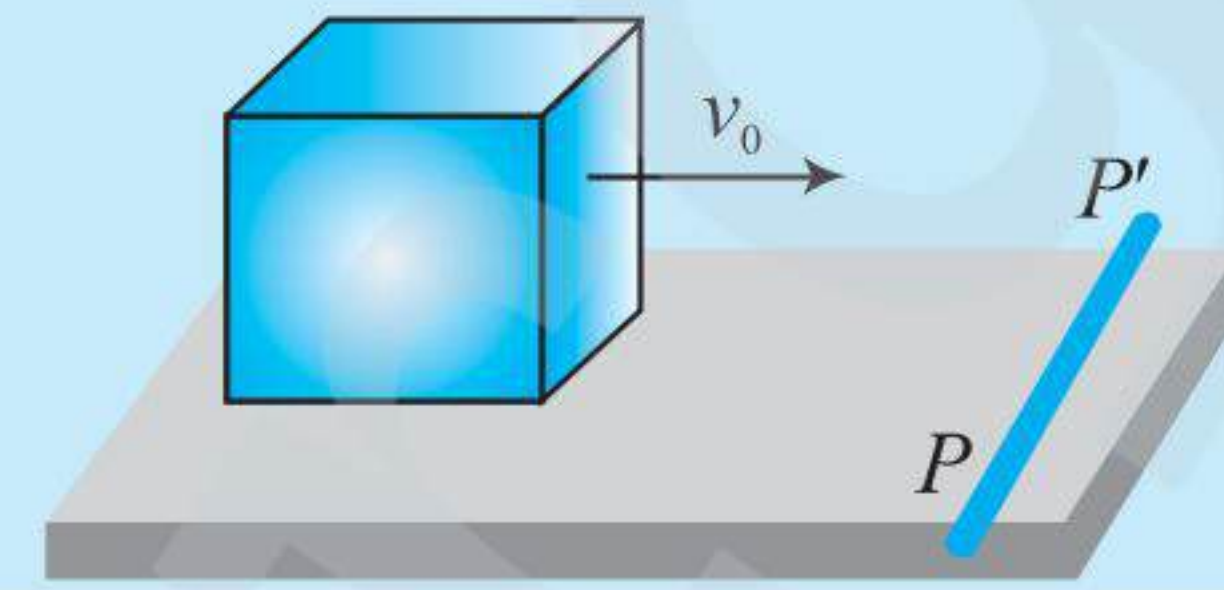
$$I\omega = I'\omega'$$

$$\omega' = \frac{I\omega}{I'} = \frac{2ml^2\omega}{2ml'^2} = \frac{l^2\omega}{l'^2} = 9\omega = 60 \text{ rad/s}$$

Therefore, angular velocity increases.

ILLUSTRATION 4.14

A cube of mass m and edge length l is sliding without friction on a smooth horizontal floor with velocity v_0 , it strikes a long obstruction PP' of small height. The obstruction is parallel to the leading bottom edge of the cube. The leading bottom edge of the cube gets pivoted with the obstruction and the cube starts rotating about the edge. Determine angular velocity of the cube immediately after the impact.



Sol. Before the impact, there is no external force in the horizontal direction and the cube slides with uniform velocity v_0 , and during the impact reaction forces of the obstruction stops its leading bottom edge and cause it to rotate about its leading bottom.

During the impact external forces acting on the cube are its weight, the normal reaction from the ground and reaction from the obstruction. The weight and the normal reaction from the ground both are finite in magnitude and the impact ends in infinitesimally small time interval so their impulses and angular impulses about any axes are negligible. It is the reaction from the obstruction which has finite impulse during the impact. Its horizontal component changes the momentum of the cube during the impact, but its angular impulse about the obstruction is zero, therefore the angular momentum of the cube about an axis coincident with the leading bottom edge remain conserved.

Let the velocity of the mass center and angular velocity of the cube immediately after the impact are v_0 and ω_0 . These velocities are shown in the adjacent figure.



We denote the angular momentum of the cube about axis coincident with the obstruction edge before and after the impact by L_{P1} and L_{P2} .

Applying principle of conservation of angular momentum about an axis coincident with the obstruction, we have

$$\vec{L}_{P1} = \vec{L}_{P2} \Rightarrow \vec{r}_{C/P} \times m\vec{v}_0 = I_P \vec{\omega}$$

Using theorem of parallel axes for moment of inertia I_P about the leading bottom edge, we get

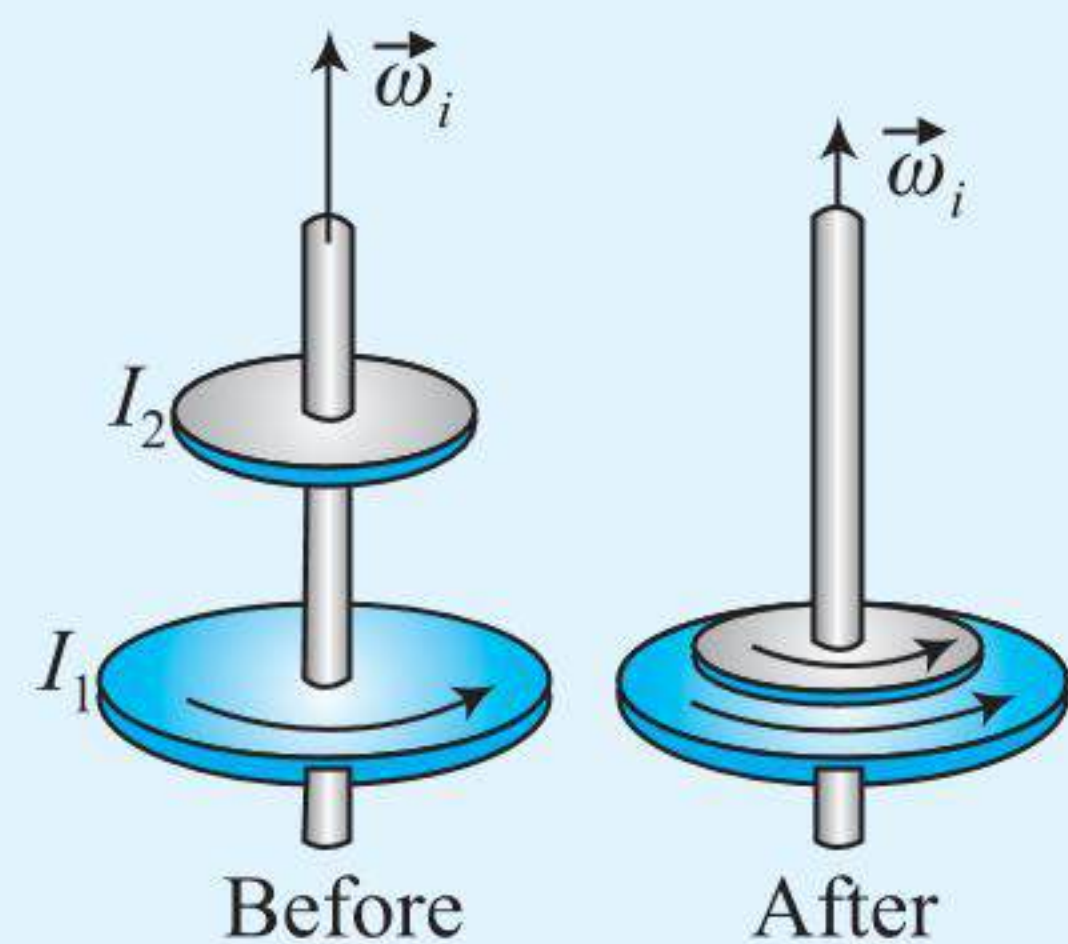
$$I_P = I_C + mr_{C/P}^2 = \frac{2}{3}ml^2$$

Substituting this in the above equation, we have

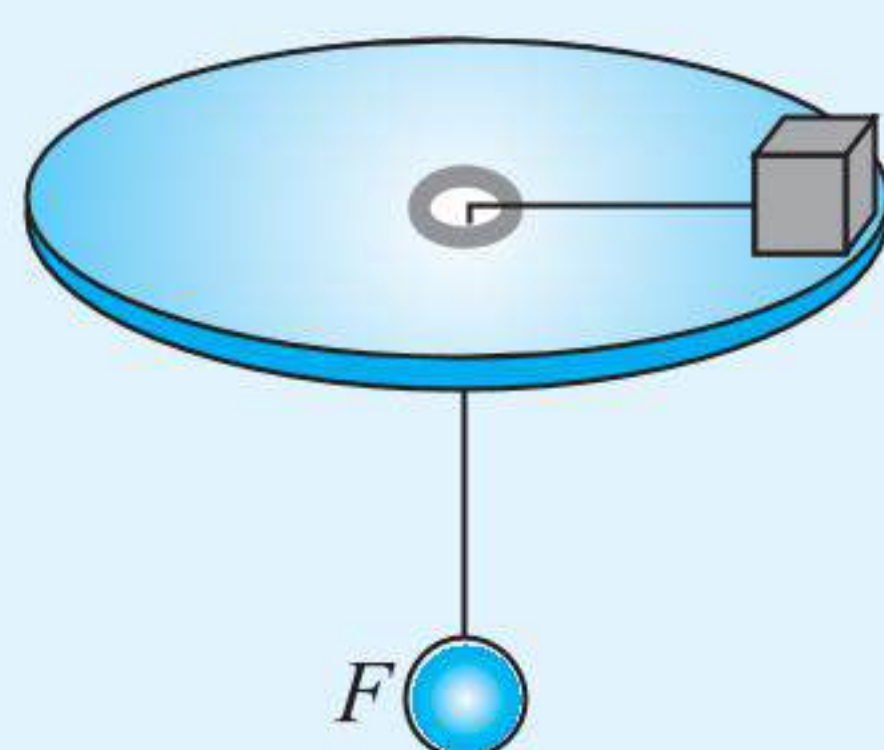
Angular velocity immediately after the impact, $\omega_0 = \frac{3v_0}{4l}$

CONCEPT APPLICATION EXERCISE 4.2

1. A disk with moment of inertia I_1 rotates about a frictionless, vertical axle with angular speed ω_i . A second disk, this one having moment of inertia I_2 and initially not rotating, drops onto the first disk (figure). Because of friction between the surfaces, the two eventually reach the same angular speed ω_f , calculate ω_f .



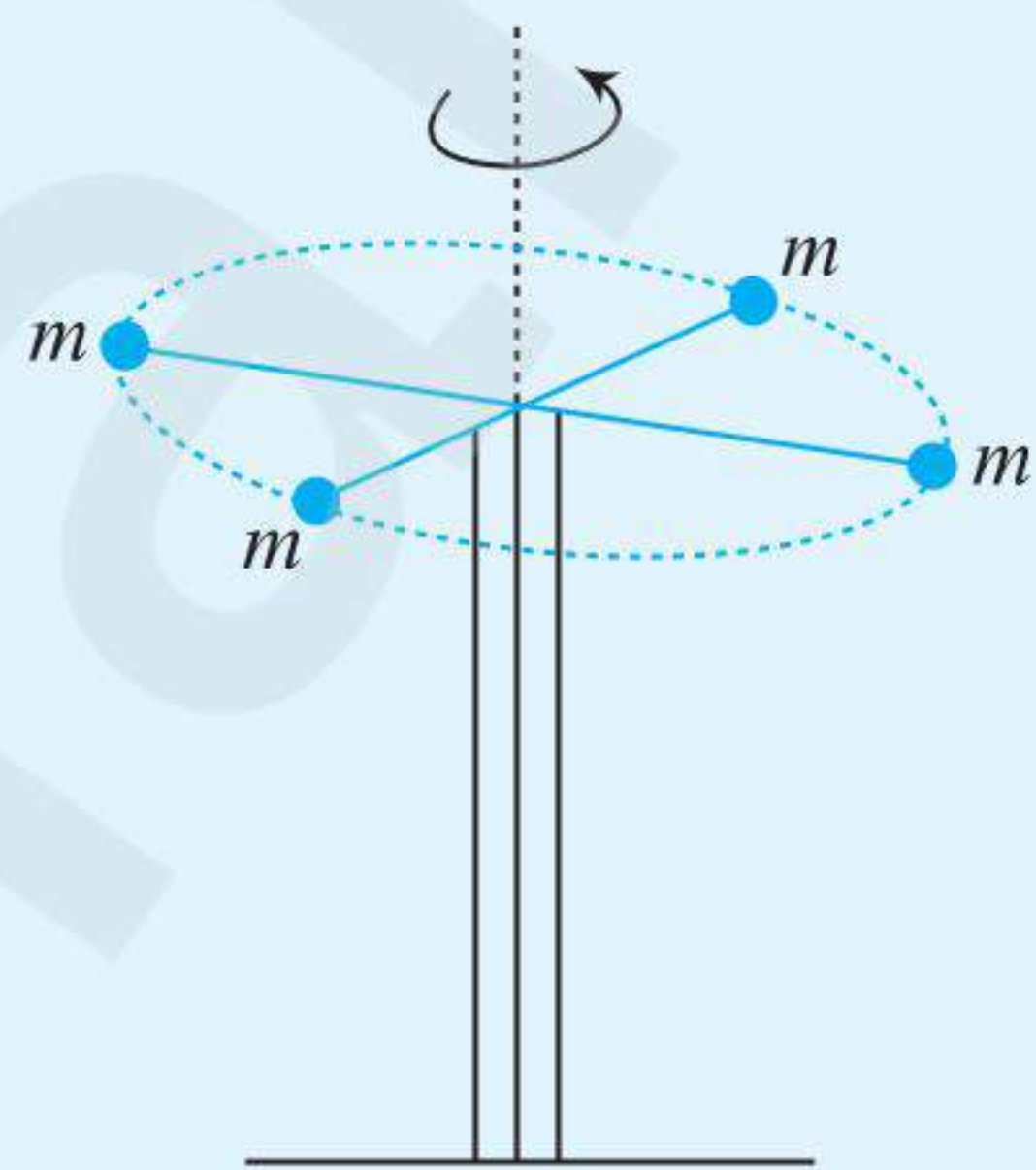
2. A small block of mass 4 kg is attached to a cord passing through a hole in a horizontal frictionless surface. The block is originally revolving in a circle of radius 0.5 m about the hole, with a tangential velocity of 4 m/s. The cord is then pulled slowly from below, shortening the radius of the circle in which the block revolves. The breaking strength of the cord is 600 N. What will be the radius of the circle when the cord breaks?



3. A man of mass 100 kg stands at the rim of a turntable of radius 2 m and moment of inertia 4000 kg m^2 mounted on a vertical frictionless shaft at its centre. The whole system is initially at rest. The man now walks along the outer edge of the turntable with a velocity of 1 m/s relative to the earth.

- (a) With what angular velocity and in what direction does the turntable rotate?
 (b) Through what angle will table have rotated when the man reaches his initial position on the turntable?
 (c) Through what angle will table have rotated when the man reaches his initial position relative to the earth?

4. The device shown in figure rotates on the vertical axle as shown. The frame has negligible mass as compared to the four masses each of mass m . Initial angular velocity of the system is ω_0 . Due to an internal mechanism the spokes in the frame lengthen so that the radii of the masses become $2a$. Initially, it was a . What will be the new angular velocity of the system?

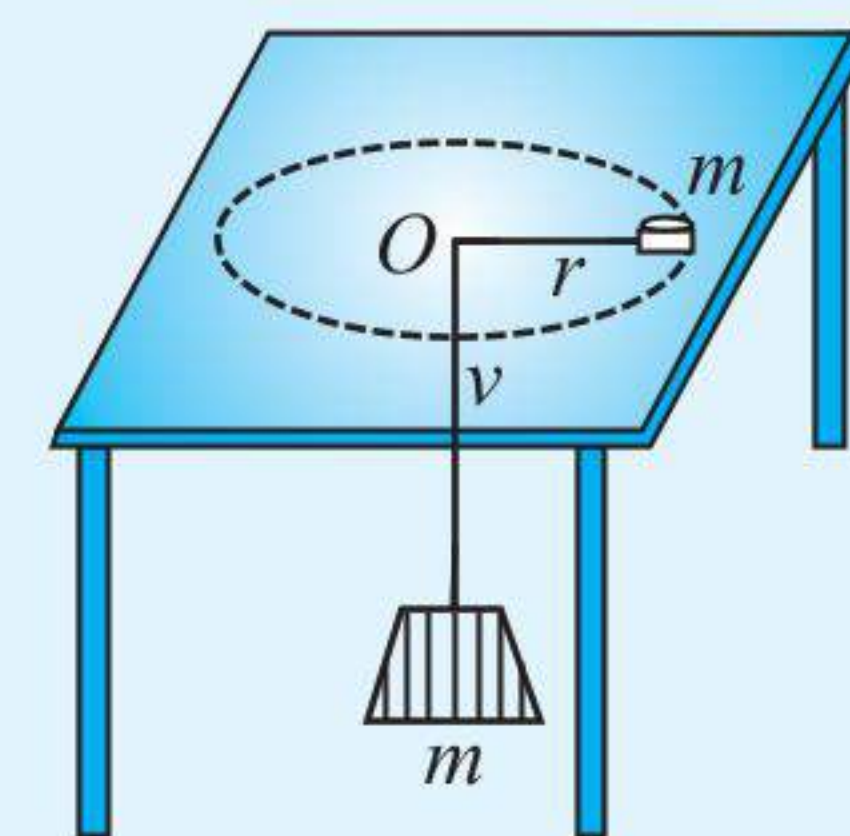


5. A girl jumps from a height h on the end of a see-saw. The see-saw consists of a uniform plank of length l pivoted at its centre. The plank is horizontal before the girl jumps. The mass of the see-saw is twice the mass of the girl. Find the angular velocity of the plank just after the girl jumps on the plank.
6. A child of mass m is standing on the periphery of a circular platform of radius R , which can rotate about its central axis. The moment of inertia of the platform is I . Child jumps off from the platform with a velocity u tangentially relative to the platform. Find the angular speed of the platform after the child jumps off.

7. Suppose in the previous problem, the child stays at rest on the platform and one of his friend throws a ball of mass m_1 towards him with a velocity of v from a direction tangential to the platform and the child on the platform catches the ball. Find the angular velocity of the platform after he catches the ball.

8. In the previous problem, initially the platform with the child is rotating with an angular speed ω_1 . If the child starts walking along its periphery in opposite direction with speed u relative to the platform, what will be the new angular speed of the platform.

9. A thread is passing through a hole at the centre of a frictionless table. At the upper end a block of mass 0.5 kg is tied and a block of mass 8.0 kg is tied at the lower end which is freely hanging. The smaller mass is rotated on the table with a constant angular velocity about the axis passing through the hole so as to balance the heavier mass. If the mass of the hanging block is changed from 8.0 kg to 1.0 kg, what is the fractional change in the radius and the angular velocity of the smaller mass so that it balances the hanging mass again?



ANSWERS

1. $\frac{I_1}{I_1 + I_2} \omega_i$ 2. 0.3 m
 3. (a) 0.05 rad/s (b) $(2\pi/11)$ rad (c) 36°
 4. $\frac{\omega_0}{4}$ 5. $\frac{6}{5} \sqrt{\frac{2gh}{l}}$ 6. $\frac{muR}{mR^2 + I}$ 7. $\frac{m_1 u R}{I + mR^2 + m_1 R^2}$
 8. $\frac{(I + mR^2)\omega_1 + muR}{I + mR^2}$ 9. 1, $\frac{3}{4}$

DYNAMICS OF COMBINATION OF TRANSLATION AND ROTATION

When the axis of rotation is not fixed (stationary in space), the motion of a rigid body is considered as combination of motion of centre of mass plus a rotation about centre of mass. We follow following steps for analyzing this type of motion

- For translational motion we write force equation:

$$\Sigma \vec{F}_{\text{ext}} = m \vec{a}_{\text{CM}}$$

- For rotational motion we write torque equation:

$$\Sigma \vec{\tau}_{\text{CM}} = I_{\text{CM}} \vec{\alpha}$$

Care should be taken for writing torque equation, as we can apply this equation either about center of mass or about a fixed axes or about instantaneous axis of rotation.

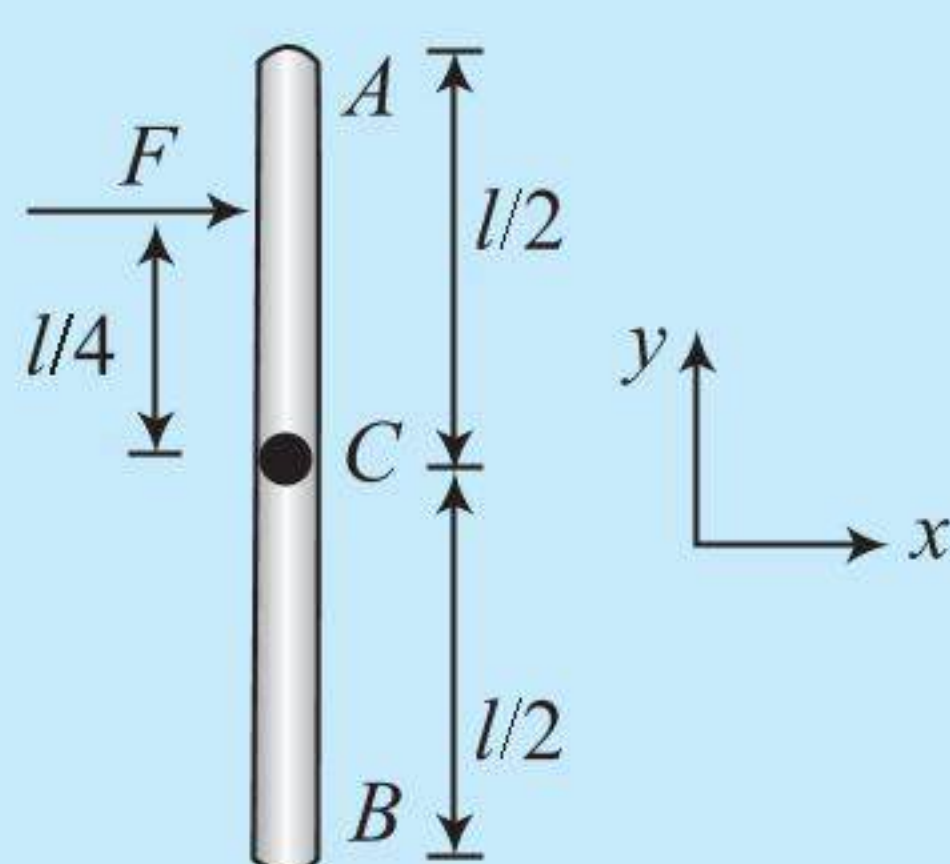
- When the number of variables exceed the number of equations (force and torque equations), then we write constraint equations.

Here we are going to learn method through illustrations.

ILLUSTRATION 4.15

A thin uniform rod of mass m and length l is placed on smooth horizontal surface. It is acted upon by a force F as shown in the figure. Find:

- the linear and angular acceleration of the rod.
- the location of a point on the rod which has zero acceleration.

**Sol.**

- The external force acting on the rod in x -direction, hence the centre of mass of the rod will accelerate along x -direction. Also because of this force there will be net torque acting on the rod in clockwise sense, hence the motion of the rod will be combined rotational and translational.

Equation of motion for rod:

$$F = ma \Rightarrow a = \frac{F}{m} \quad \dots(i)$$

$$\tau = I_c \alpha \Rightarrow F \frac{l}{4} = \frac{ml^2}{12} \alpha$$

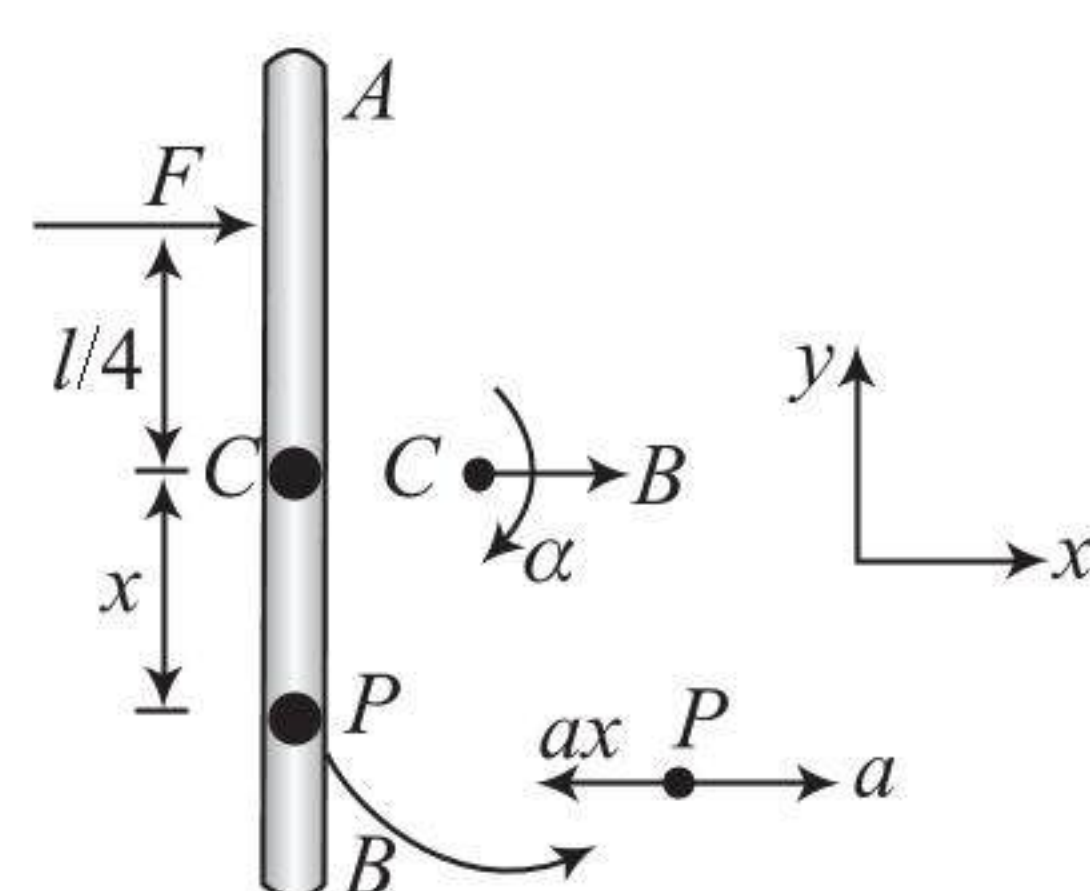
$$\text{which gives } \alpha = \frac{3F}{ml} \quad \dots(ii)$$

- Point of zero acceleration on the rod should be between C and B , as the vectors due to acceleration due to translation and acceleration due to rotation act in opposite directions as shown in figure.

$$\text{At point } P, a_p = a - \alpha x = \frac{F}{m} - \frac{3F}{ml} x$$

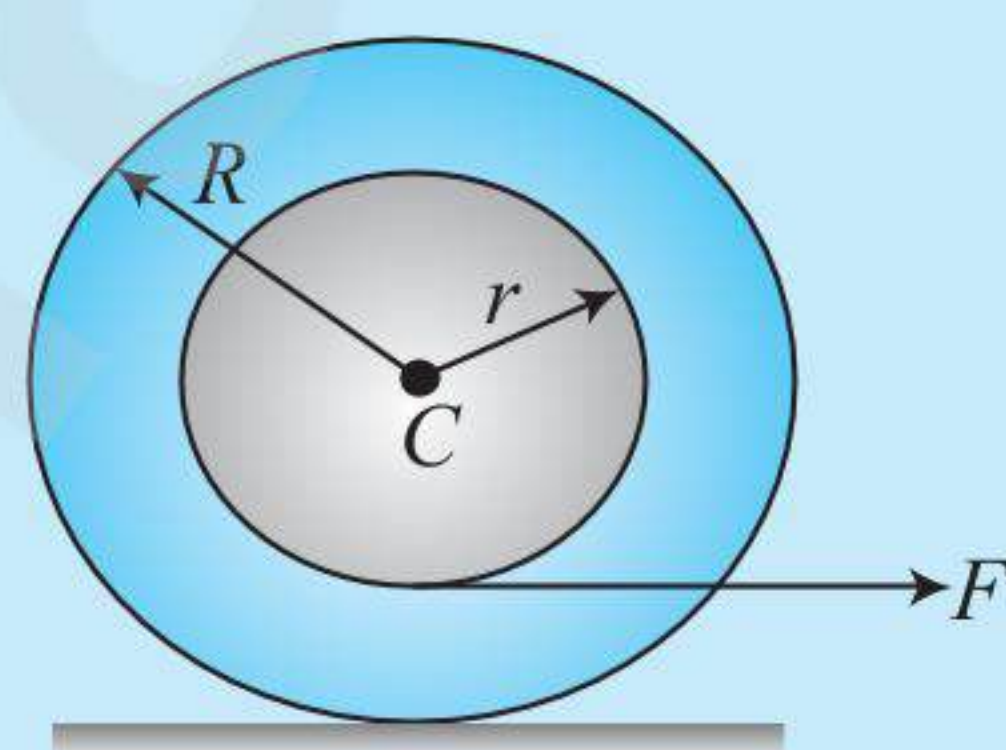
If the acceleration of this point is zero then

$$a_p = 0 = \frac{F}{m} - \frac{3F}{ml} x \text{ or } x = \frac{l}{3}$$

**ILLUSTRATION 4.16**

A stepped disc of mass m inner and outer radius r and R respectively is placed on a smooth horizontal surface. It is wrapped by a thread which is pulled by a horizontal force F . If the radius of gyration of the disc is k , find:

- the acceleration of CM of the disc
- the angular acceleration of the disc.

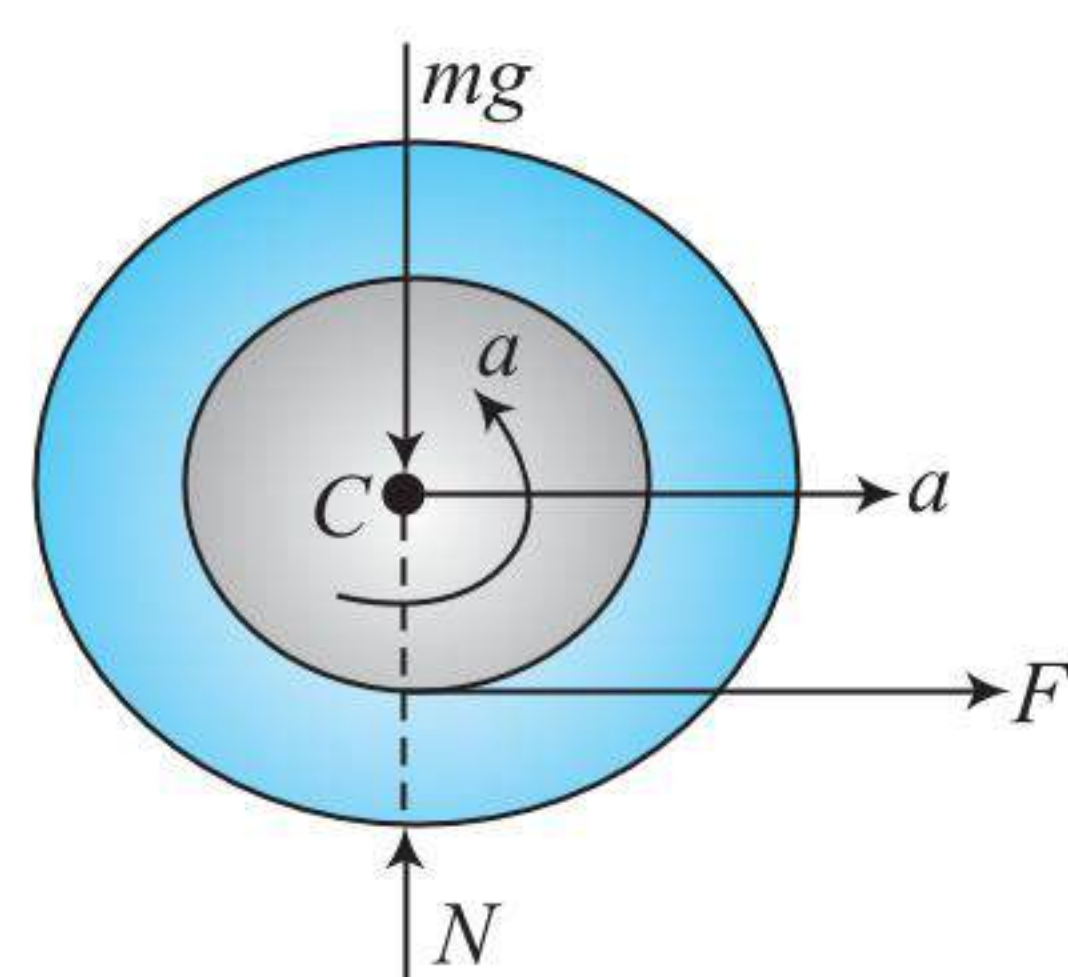


Sol. The motion of the disc is combined rotation and translation.

The disc will move in horizontal direction and it will rotate in counter clockwise sense.

- Force equation:

$$F = ma \Rightarrow a = \frac{F}{m}$$

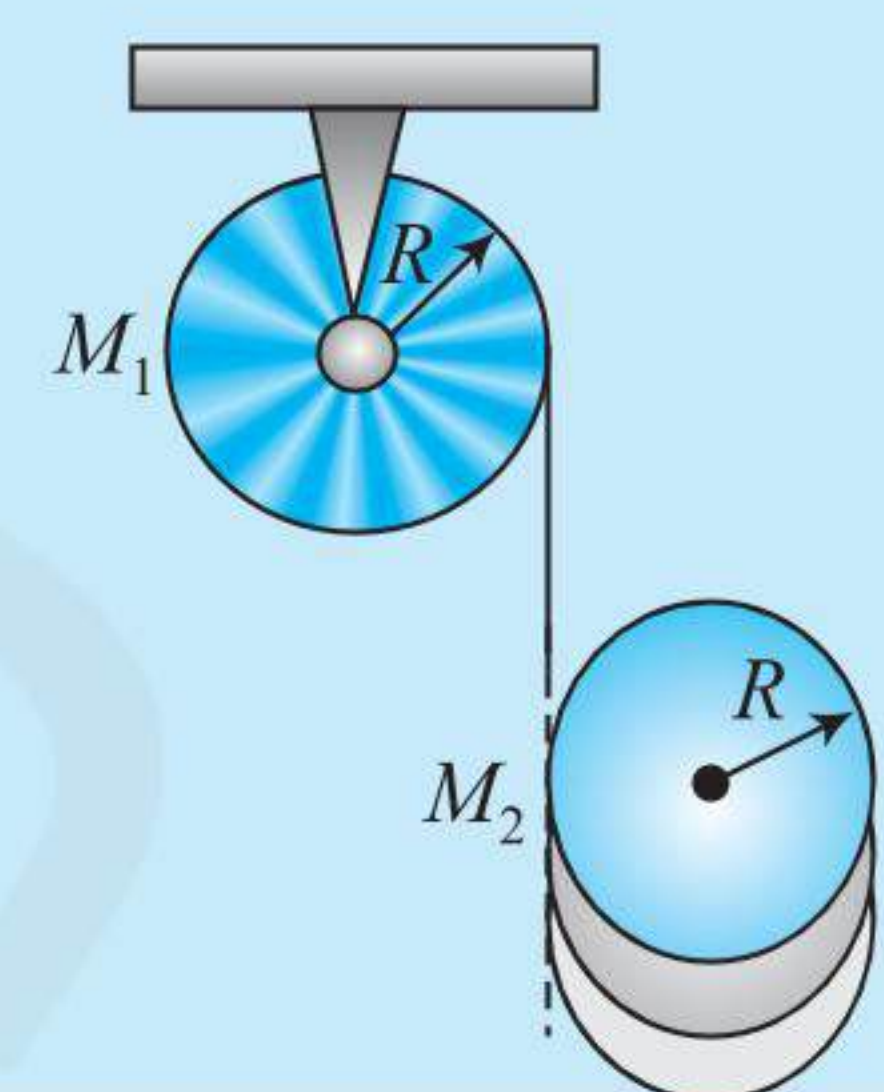


- Torque equation about C , $\tau = I_c \alpha$

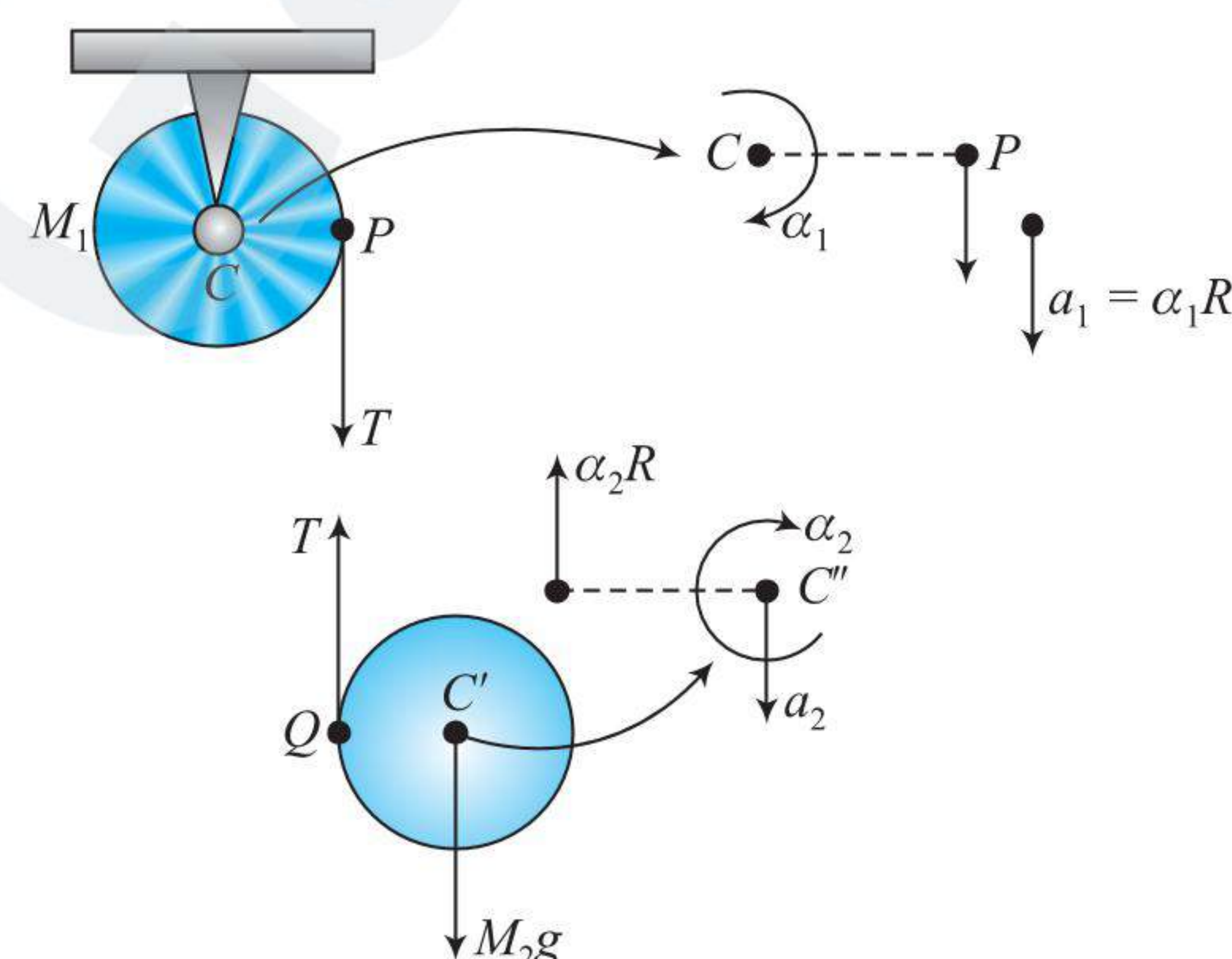
$$\Rightarrow F \cdot r = (mk^2) \alpha \Rightarrow \alpha = \frac{Fr}{mk^2}$$

ILLUSTRATION 4.17

An extensible string is wound over a rough pulley of mass M_1 and radius R and a cylinder of mass M_2 and radius R such that as the cylinder rolls down, (refer figure) the string unwound over the pulley as well the cylinder. Find the acceleration of cylinder.



Sol. Through this illustration we will learn the application of torque equation and constraint relation in more complex case here. Pulley M_1 will have only pure rotation while cylinder M_2 will have rotation and translation combined. Let us analyse step by step in the same way as the previous illustration.



Step I: Analyse the motions of the pulley and the cylinder.

Pulley: rotational acceleration α_1 (clockwise)

Cylinder: rotational acceleration α_2 (clockwise) and a linear acceleration a_2 (downward).

Step II: Equation of motion for M_2

$$M_2 g - T = M_2 a_2 \quad \dots(i)$$

Step III: Torque equation for pulley: $\tau_c = I_c \alpha$

$$TR = \left(\frac{M_1 R^2}{2} \right) \alpha_1 \Rightarrow \alpha_1 = \frac{2T}{M_1 R} \quad \dots(ii)$$

Torque equation for the cylinder (about centre of mass of cylinder):

$$\tau_c = I_c \alpha$$

$$TR = \left(\frac{M_2 R^2}{2} \right) \alpha_2 \Rightarrow \alpha_2 = \frac{2T}{M_2 R} \quad \dots(iii)$$

Step IV: The acceleration of P and Q should be equal as both are connected with the same inextensible string.

Acceleration of P , $a_p = \alpha_1 R$ (downwards) Acceleration of Q ,

$$a_Q = a_2 - \alpha_2 R \text{ (downwards)}$$

Hence, constraint relation: $\alpha_1 R = a_2 - \alpha_2 R \quad \dots(iv)$

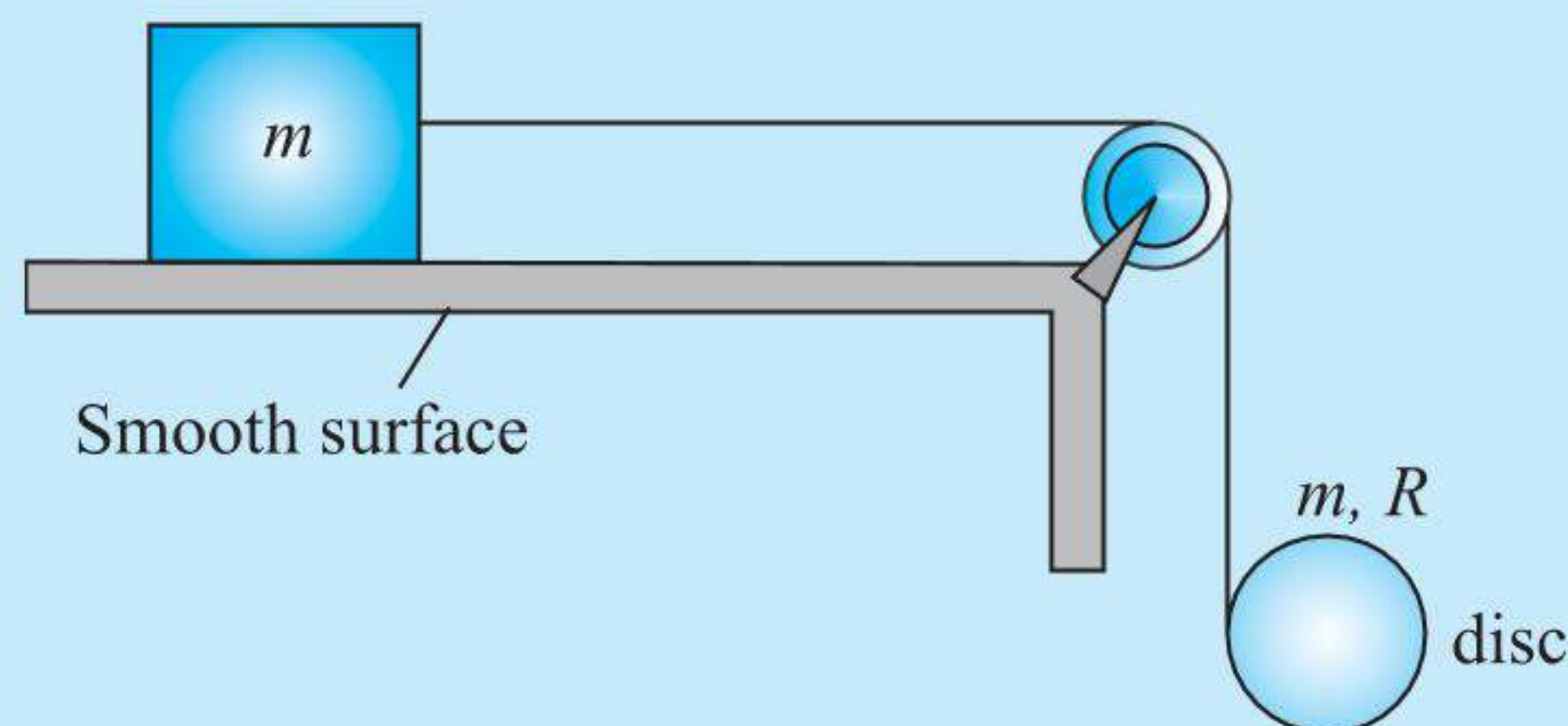
Step V: Solving equations:

After solving Eqs. (i) (ii), (iii) and (iv), we get

$$a_2 = \left[\frac{2(M_1 + M_2)}{3M_1 + 2M_2} \right] g$$

ILLUSTRATION 4.18

A block of mass m slides on a smooth horizontal surface. It is connected with a disc of same mass and radius R , through a light string which passes through light pulley as shown in figure. Assuming pulley to be massless and frictionless and thread inextensible, calculate acceleration of centre of the disc.



Sol. The motion of the block is pure translational and that of disc is translational and rotational. Let acceleration of block and disc be a and b respectively and angular acceleration of disc be α (clockwise). The acceleration of point P on the rim should be equal to the magnitude of the acceleration of the block.

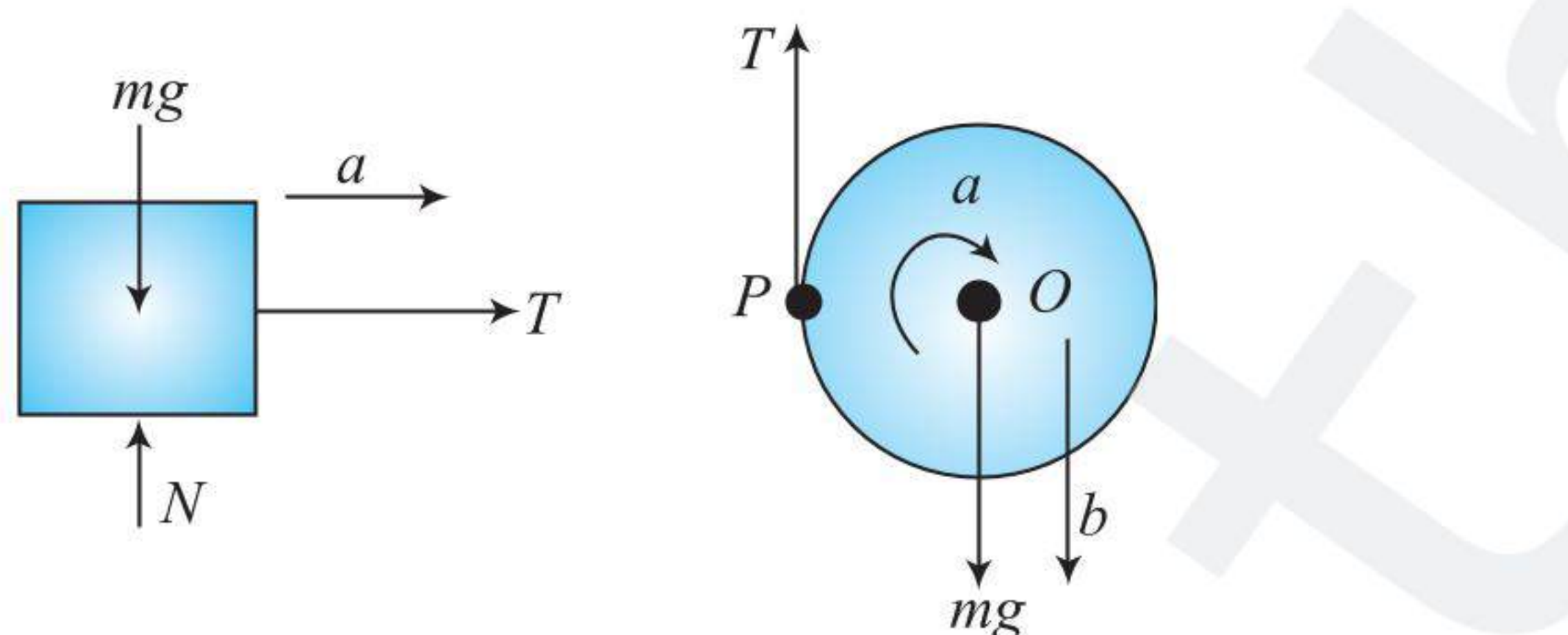
$$\vec{a}_P = \vec{a}_{P,O} + \vec{a}_O$$

$$\Rightarrow a = (-R\alpha) + b$$

Hence the net downward acceleration of centre of disc will be equal to,

$$b = (a + R\alpha) \quad \dots(i)$$

Now consider FBD of block and disc



For vertical equilibrium of the block, $N = mg$

For horizontal forces, $T = ma$...(ii)

Here the axis of rotation of moving, hence we can apply torque equation about centroidal axis of the disc,

$$T \cdot R = I_O \alpha$$

$$T \cdot R = \frac{mR^2}{2} \alpha \quad \text{or} \quad T = \frac{1}{2} mR\alpha \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$R\alpha = 2a \quad \text{or} \quad b = 3a \quad \dots(iv)$$

Now analyzing vertical forces, acting on the disc,

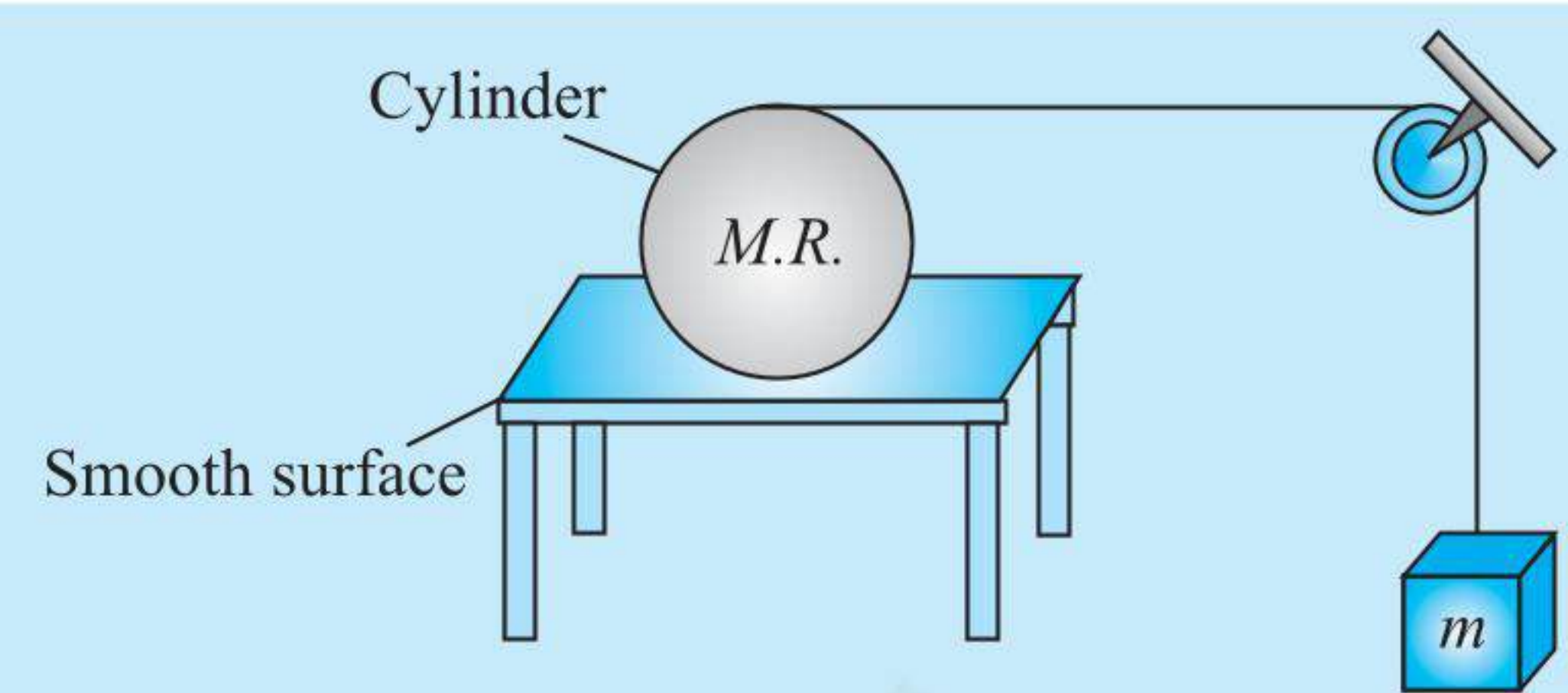
$$mg - T = mb \quad \dots(v)$$

From (ii), (iv) and (v) we get

$$a = \frac{g}{4} \quad \text{and} \quad b = \frac{3g}{4}$$

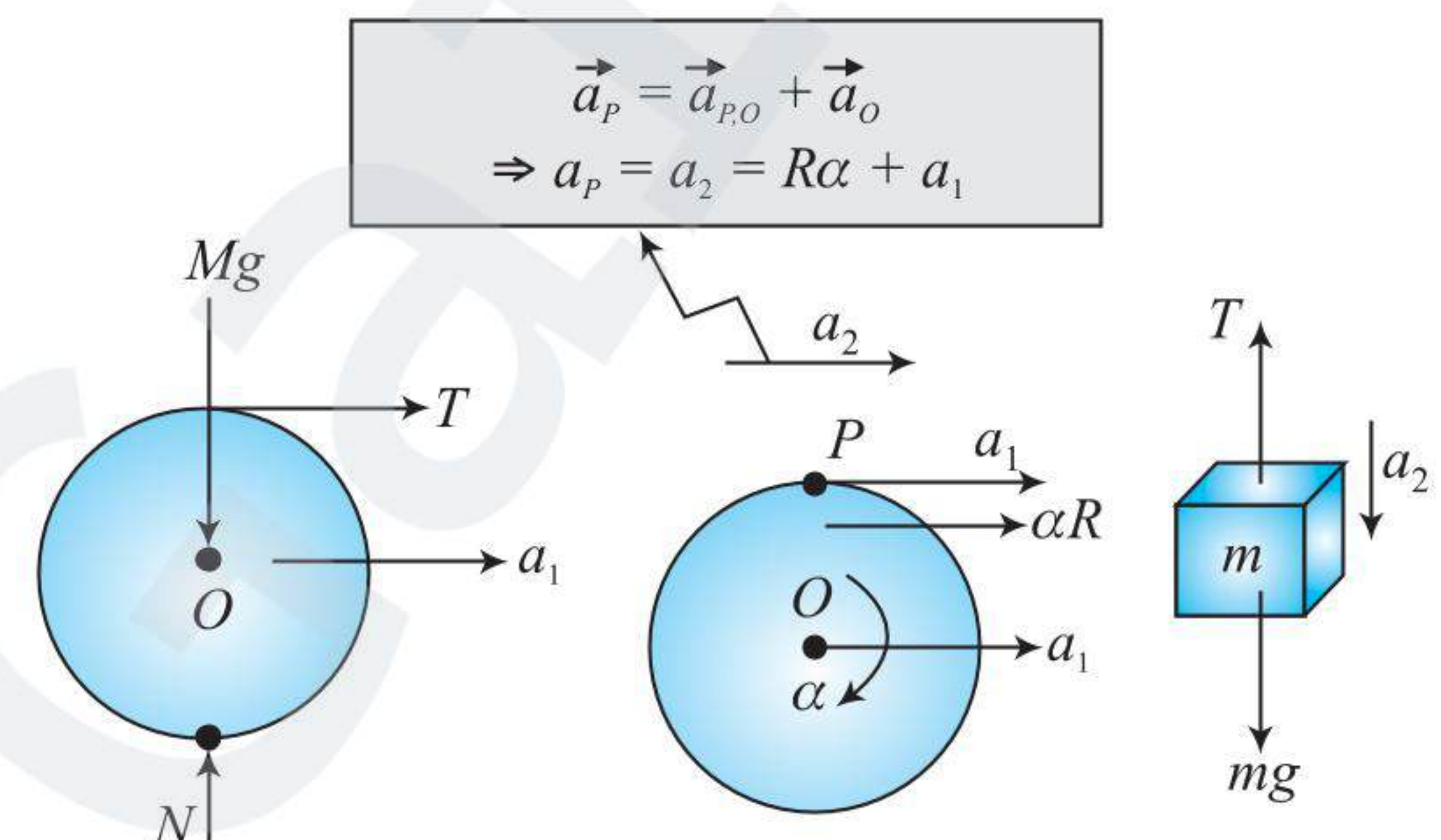
ILLUSTRATION 4.19

An inextensible thread is wound round a cylinder of mass $M = 7.0$ kg and radius $R = 10$ cm. The cylinder is placed over a smooth horizontal surface and thread is passed over a massless and frictionless pulley such that the part of the thread between cylinder and pulley is horizontal as shown in figure.



A block of mass $m = 1.0$ kg is attached with free end of thread. If the system is released from rest, calculate acceleration of the block. ($g = 10 \text{ m s}^{-2}$).

Sol. As horizontal surface is smooth, and force is acting on the cylinder, therefore the cylinder will slip over the surface.



FBD of cylinder and block

Let acceleration of axis of the cylinder be a_1 (rightward) and its angular acceleration be α (clockwise) then downward acceleration of the block will be equal to,

$$a_2 = a_1 + R\alpha$$

Taking moment of forces acting on the cylinder about O,

$$T \cdot R = I\alpha \quad \text{where} \quad I = \frac{MR^2}{2}$$

$$T = \frac{1}{2} MR\alpha \quad \dots(i)$$

For horizontal forces on the cylinder,

$$T = Ma_1 \quad \dots(ii)$$

From Eqs. (i) and (ii)

$$R\alpha = 2a_1$$

hence $a_2 = 3a_1$

Now considering forces acting on the block,

$$mg - T = ma_2$$

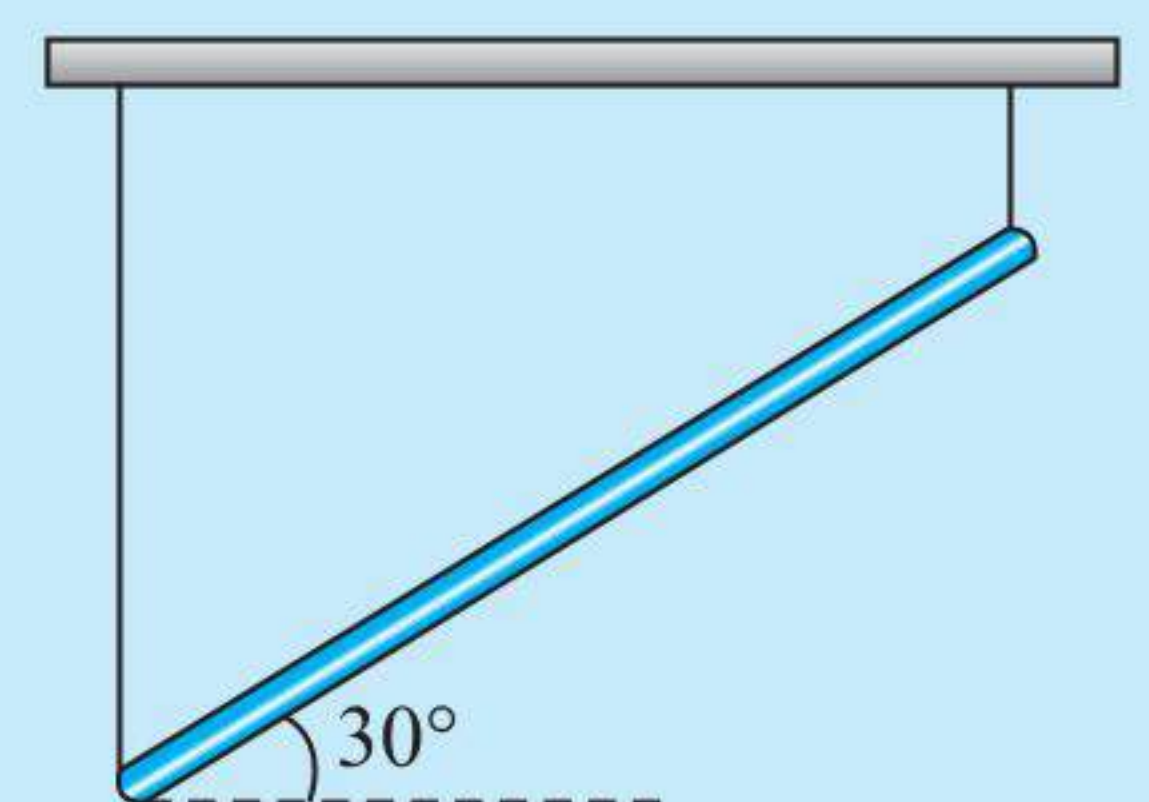
Substituting $T = Ma_1$ and $a_2 = 3a_1$

$$a_1 = \frac{mg}{(M + 3m)} \quad \text{or} \quad a_1 = 1 \text{ m s}^{-2}$$

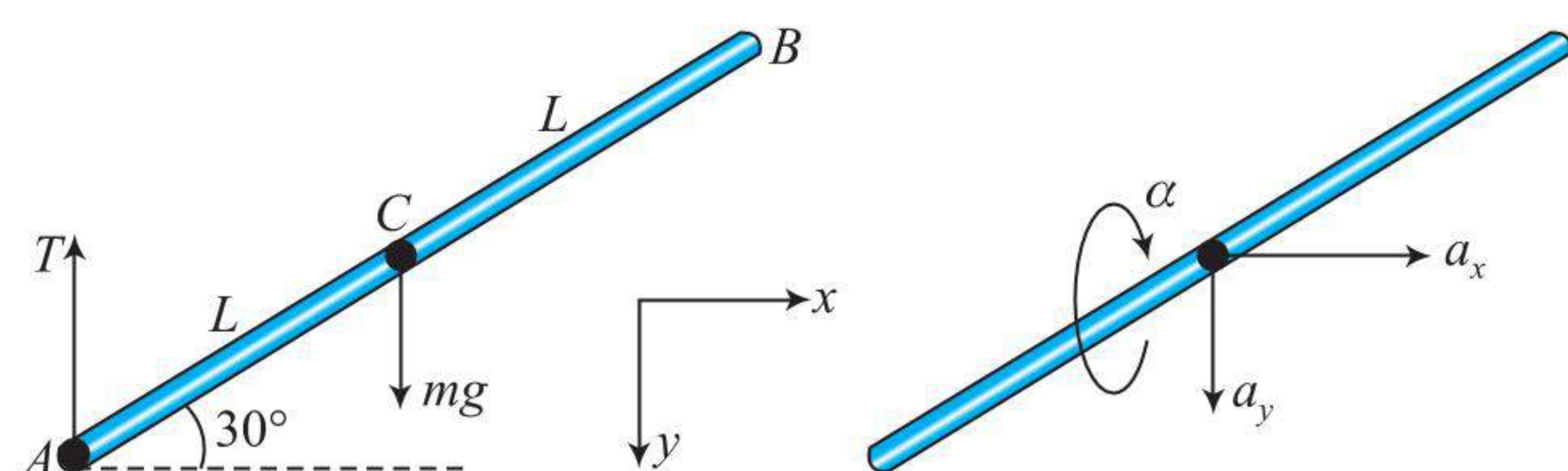
Hence, acceleration of the block, $a_2 = 3 \text{ m s}^{-2}$

ILLUSTRATION 4.20

A thin uniform bar of mass m and length $2L$ is held at an angle 30° with the horizontal by means of two vertical inextensible strings, at each end as shown in figure. If the string at the right end breaks, leaving the bar to swing, determine the tension in the string at the left end and the angular acceleration of the bar immediately after string breaks.



Sol. Just after cutting the string the tension of right string will disappear and tension in left string will change and motion of the rod will be combined rotational and translational. Let us draw free body diagram of rod immediately after the string breaks.



Let a_x and a_y be the linear acceleration of centre of mass and α the angular acceleration of the rod.

As no force in horizontal direction. Then

$$\sum F_x = 0$$

$$\therefore a_x = 0$$

$$\therefore a_y = \frac{\sum F_y}{m} = \frac{mg - T}{m} \quad \dots(i)$$

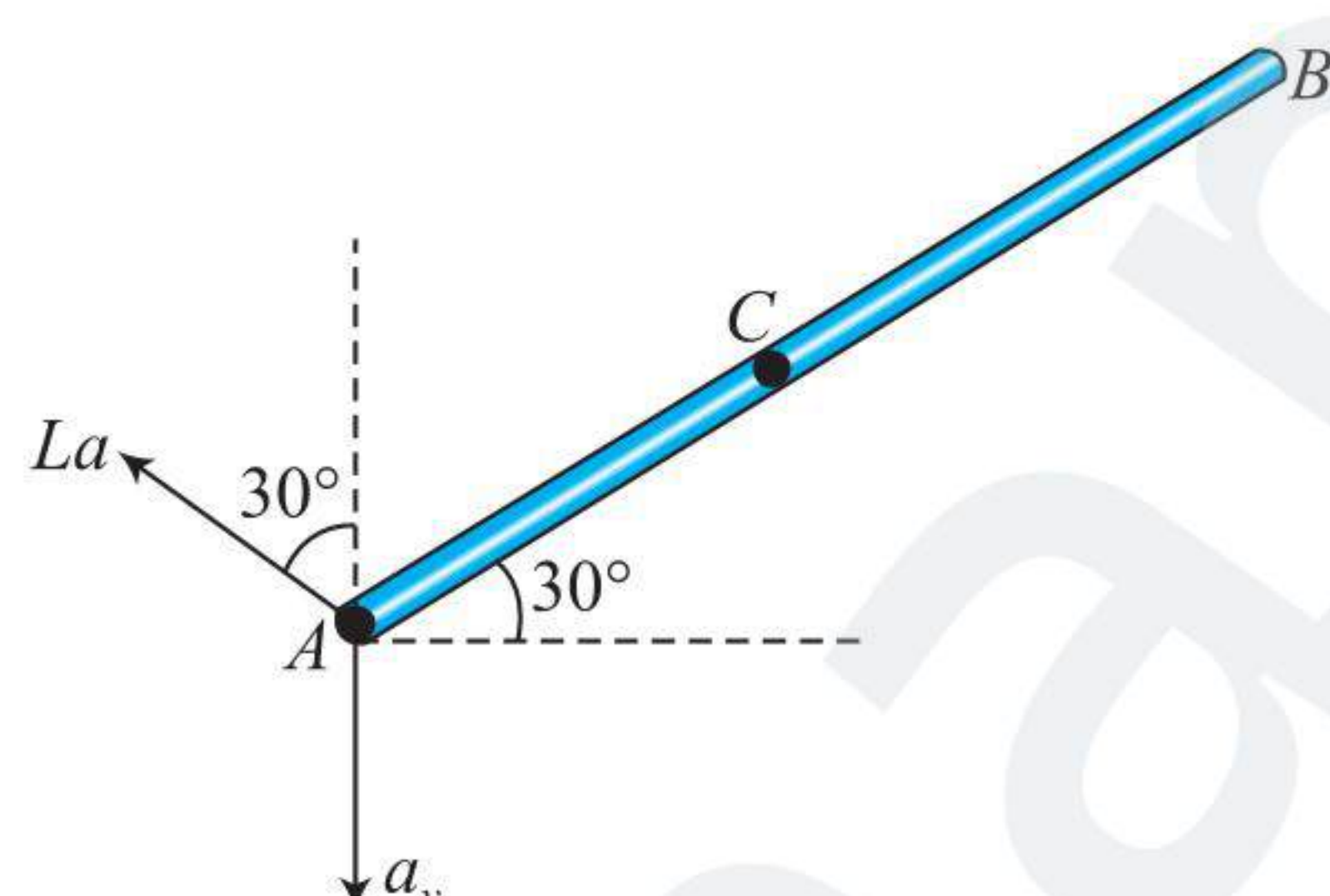
The torque equation is applicable about a fixed point or about centre of mass. In this case we have no fixed point, hence applying torque equation about center of mass

$$\tau = I_{cm} \alpha \Rightarrow T \cdot (L \cos 30^\circ) = \frac{m(2L)^2}{12} \cdot \alpha$$

$$\Rightarrow \alpha = \frac{\tau}{I_{cm}} = \frac{TL \cos 30^\circ}{m(2L)^2/12} = \frac{3\sqrt{3}T}{2mL} \quad \dots(ii)$$

Now just after the string breaks, acceleration of point A in the vertical direction should be zero as the string is inextensible.

$$\text{i.e., } a_y = L\alpha \cos 30^\circ \quad \dots(iii)$$



Solving Eqs. (i), (ii), and (iii), we get

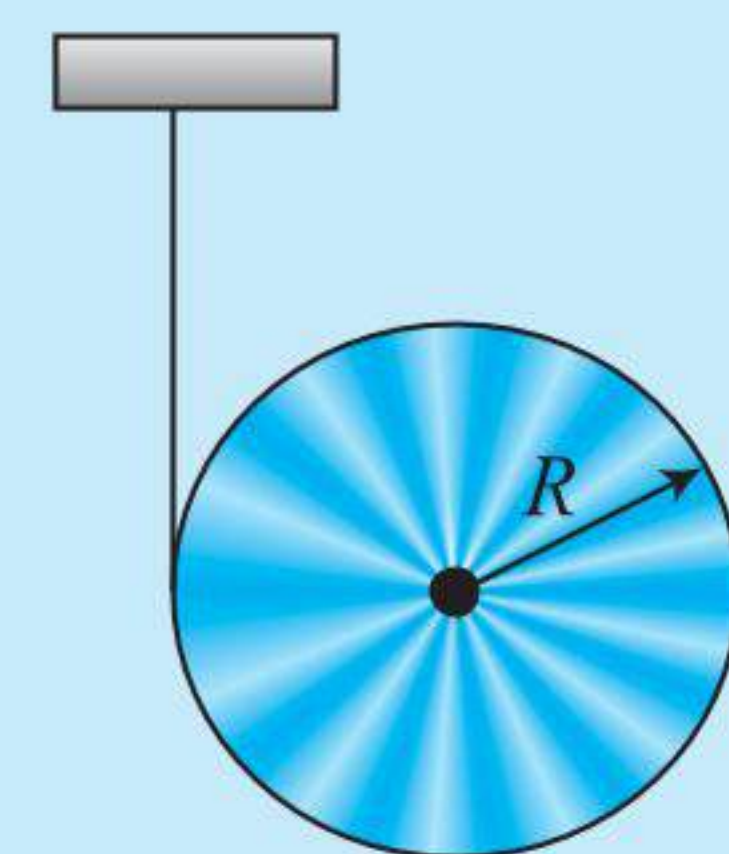
$$T = \frac{4mg}{13} \text{ and } a = \frac{9g}{13}$$

APPLYING TORQUE EQUATION ABOUT INSTANTANEOUS CENTRE OF ROTATION (ICR)

The torque equation is applicable about centroidal axes or about a fixed axis. In case of combined rotation and translation motion we apply torque equation about centroidal axes. We know instantaneous centre of rotation (ICR) is a point which can be considered stationary at certain instant. It means we can consider the axes passing through ICR as a fixed axis and apply torque equation. In this way we can solve the problems of combined rotational and translational motion in lesser steps. Let us learn this through following illustrations.

ILLUSTRATION 4.21

A uniform disc of mass M has a thin string wrapped several times around its circumference. The string is fixed at one end and the disc is released. Determine the magnitude of the downward acceleration of the mass as it falls.

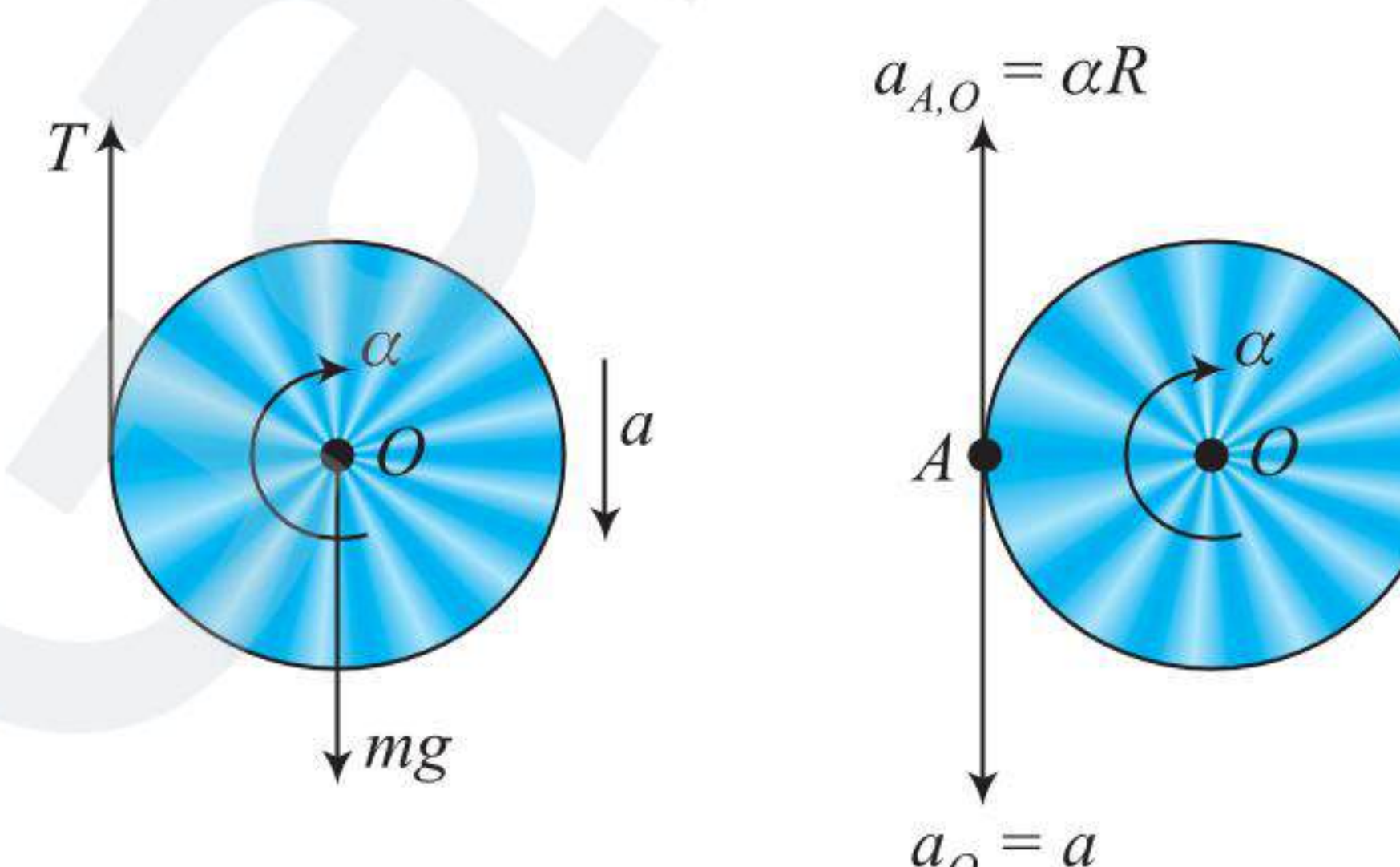


Sol. Method 1: The motion of the disc will be combined rotational and translational. Let a be the linear acceleration of centre of mass and α the angular acceleration of the disc.

The equation of motion of disc is

$$mg - T = ma \quad \dots(i)$$

The torque equation is applicable about a fixed axis or about centroidal axes.



Now writing torque equation for the disc (about centroidal axes of disc):

$$\tau_c = I_c \alpha \Rightarrow TR = \frac{mR^2}{2} \alpha$$

$$\text{or } T = \frac{1}{2} mR\alpha \quad \dots(ii)$$

Let us now write constraint relation. As one end of the string is fixed, hence along the length of the string, acceleration must be zero. It means the acceleration of a point A situated on the disc should be zero i.e., the resultant of a_{CM} and $R\alpha$ is zero, i.e.,

$$a = R\alpha \quad \dots(iii)$$

From (ii) and (iii),

$$T = \frac{1}{2} ma \quad \dots(iv)$$

$$\text{From (i) and (iv), } a = \frac{2}{3} g$$

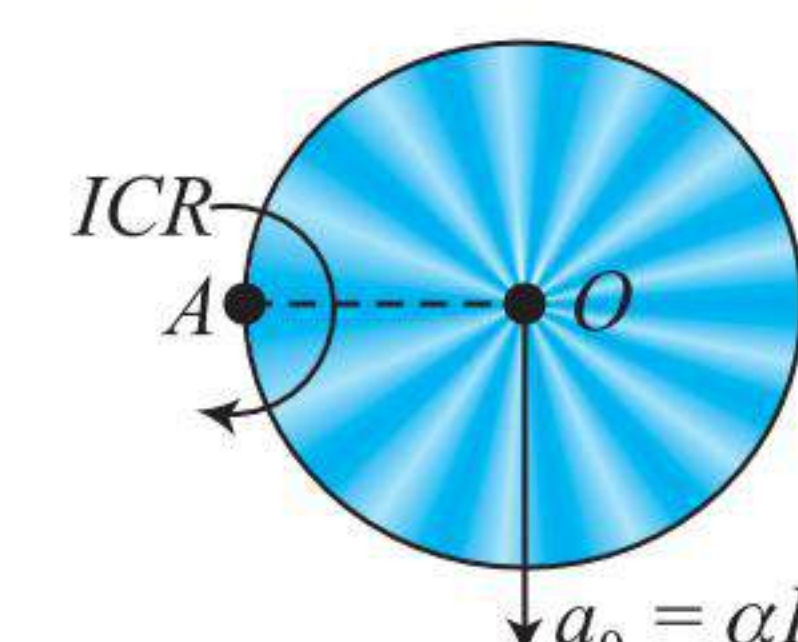
Method 2: The string is fixed at one end it means point A, the contact point of string with disc as shown in figure, should not move. It means point A is instantaneous centre of rotation (ICR) of the disc. We can apply the torque equation about ICR. Now writing torque equation for the disc (about point A):

$$\tau_A = I_A \alpha \Rightarrow mgR = I_A \alpha \quad \dots(i)$$

In this case moment of inertia is obtained from the parallel axis theorem.

$$I_A = I_{CM} + mR^2 = \frac{mR^2}{2} + mR^2 = \frac{3}{2} mR^2 \quad \dots(ii)$$

$$\Rightarrow mgR = \frac{3}{2} mR^2 \alpha \text{ or } \alpha = \frac{2g}{3R} \quad \dots(iii)$$

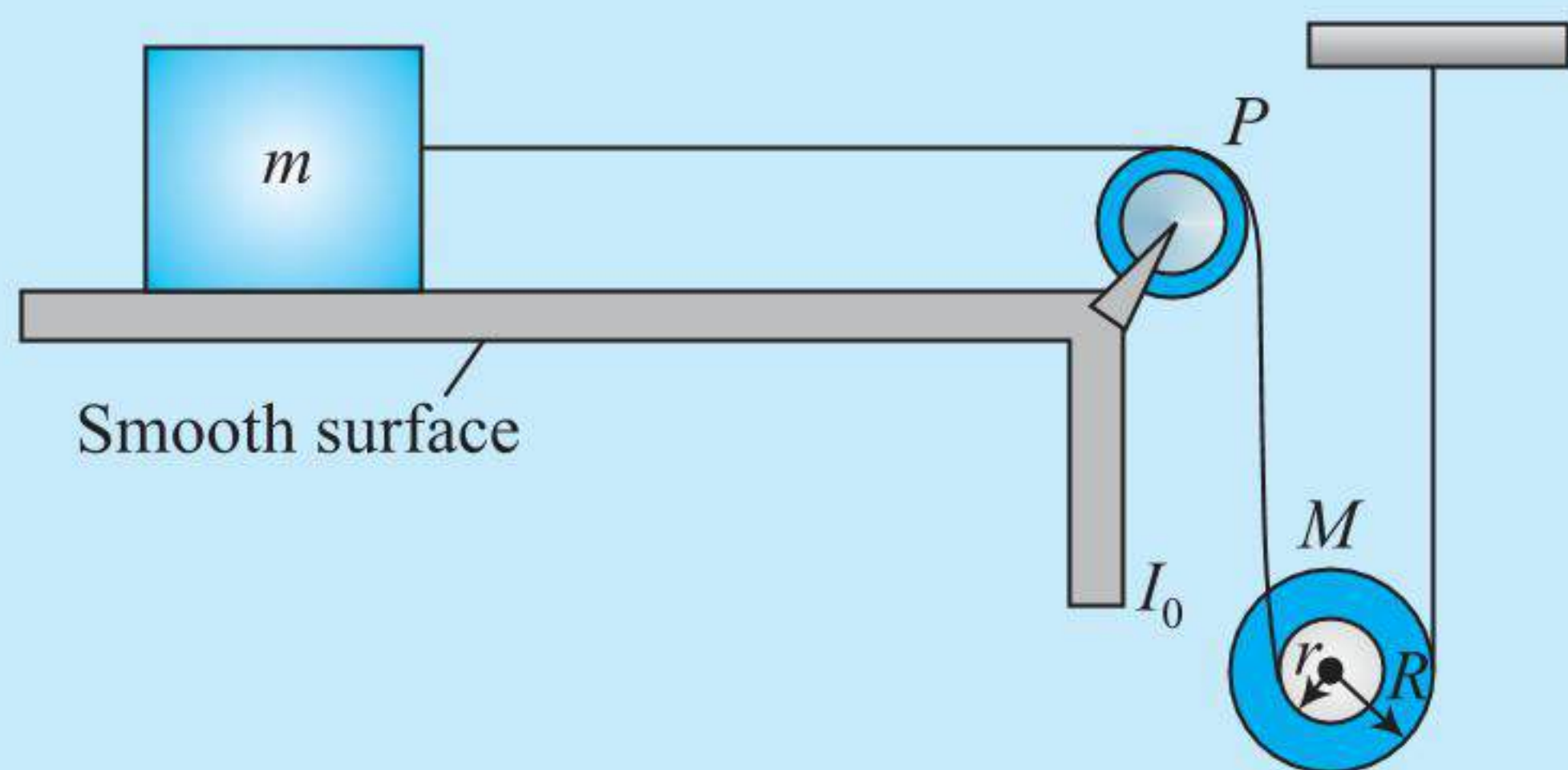


Hence the acceleration of CM,

$$a_c = R\alpha = R\left(\frac{2g}{3R}\right) = \frac{2g}{3}$$

ILLUSTRATION 4.22

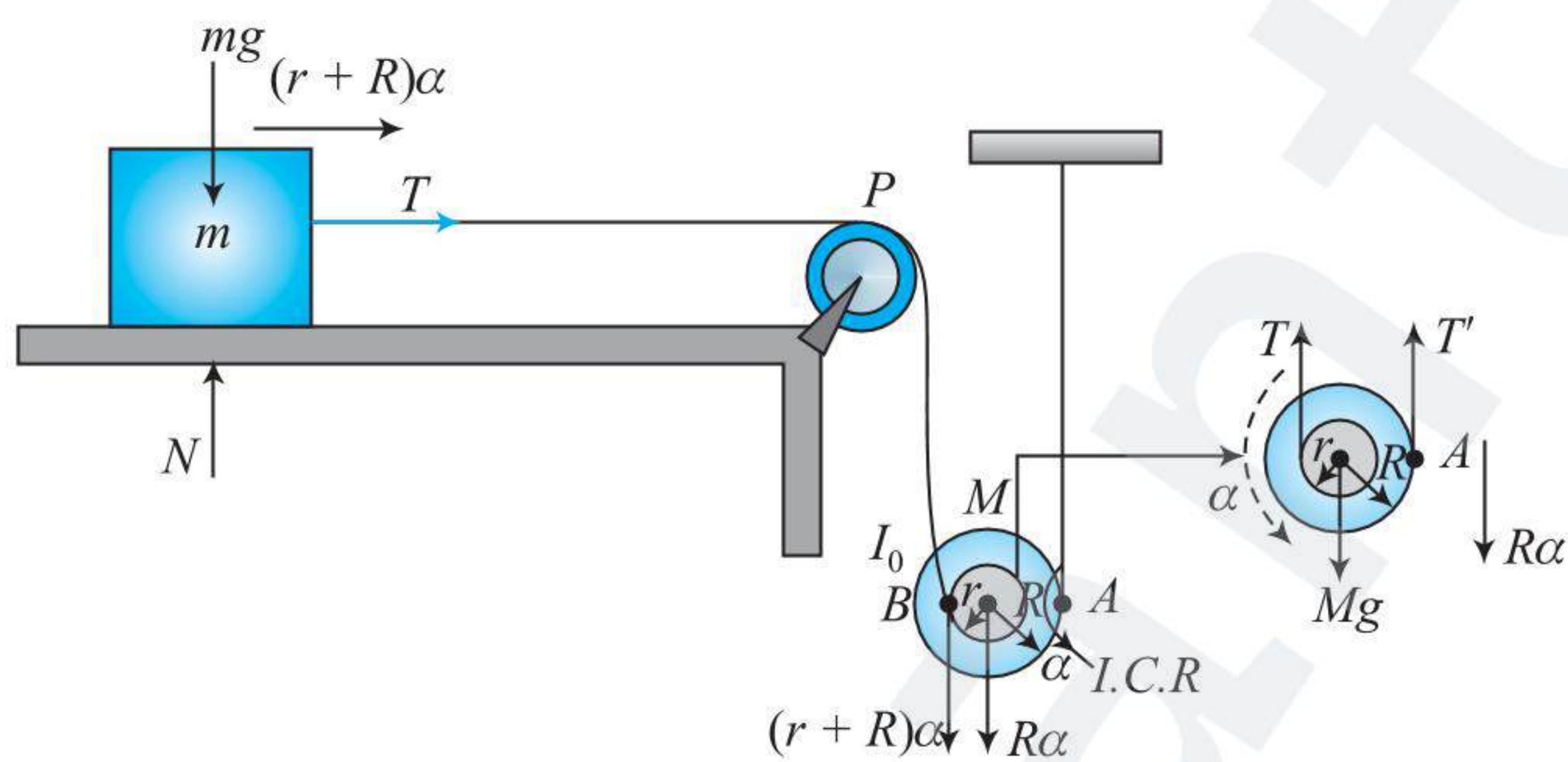
A block of mass m placed over a smooth horizontal surface is connected with a stepped pulley through a light string which passes through a massless and frictionless pulley 'P' as shown in figure. Inner and outer radii of the stepped pulley are r and R respectively. Mass of the pulley is M and its moment of inertia about its own axis is I_0 . Calculate the acceleration of the block.



Sol. Since upper end of the right thread is fixed it means point A, the contact point of string with disc as shown in figure, should not move. It means point A is instantaneous centre of rotation (ICR) of the disc. Therefore, instantaneous axis of rotation will pass through A. We can apply the torque equation about ICR.

Let angular acceleration of the pulley be α (anticlockwise), then downward acceleration of its centre will be equal to $R\alpha$ and rightward acceleration of the block will be equal to $(r + R)\alpha$.

FBD of pulley and block will be as shown in figure.



For the block, $N = mg$

$$\text{and } T = ma_{\text{block}} = m(r + R)\alpha \quad \dots(i)$$

Now writing torque equation for the disc (about point A):

$$Mg \cdot R - T(r + R) = I_A \alpha$$

where, $I_A = (I_0 + MR^2)$

$$MgR - T(r + R) = (I_0 + MR^2) \cdot \alpha \quad \dots(ii)$$

$$\text{From Eqs. (i) and (ii), } \alpha = \frac{MgR}{I_0 + MR^2 + m(r + R)^2}$$

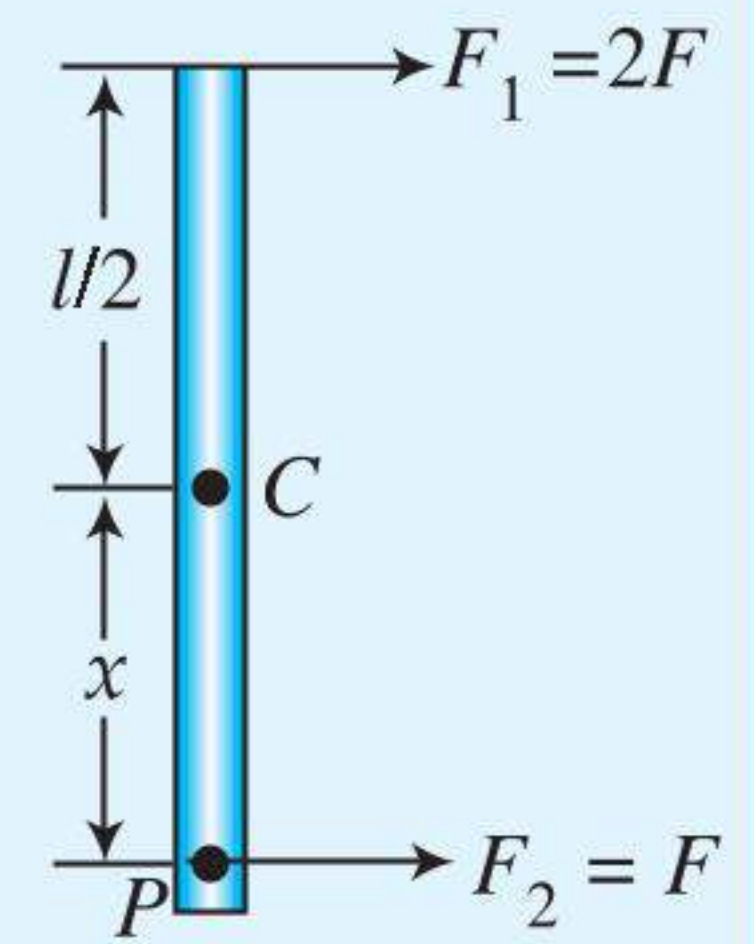
Since, acceleration of the block is equal to $(r + R)\alpha$,

$$\text{Therefore, it is equal to, } a_{\text{block}} = \frac{MgR(r + R)}{I_0 + MR^2 + m(r + R)^2}$$

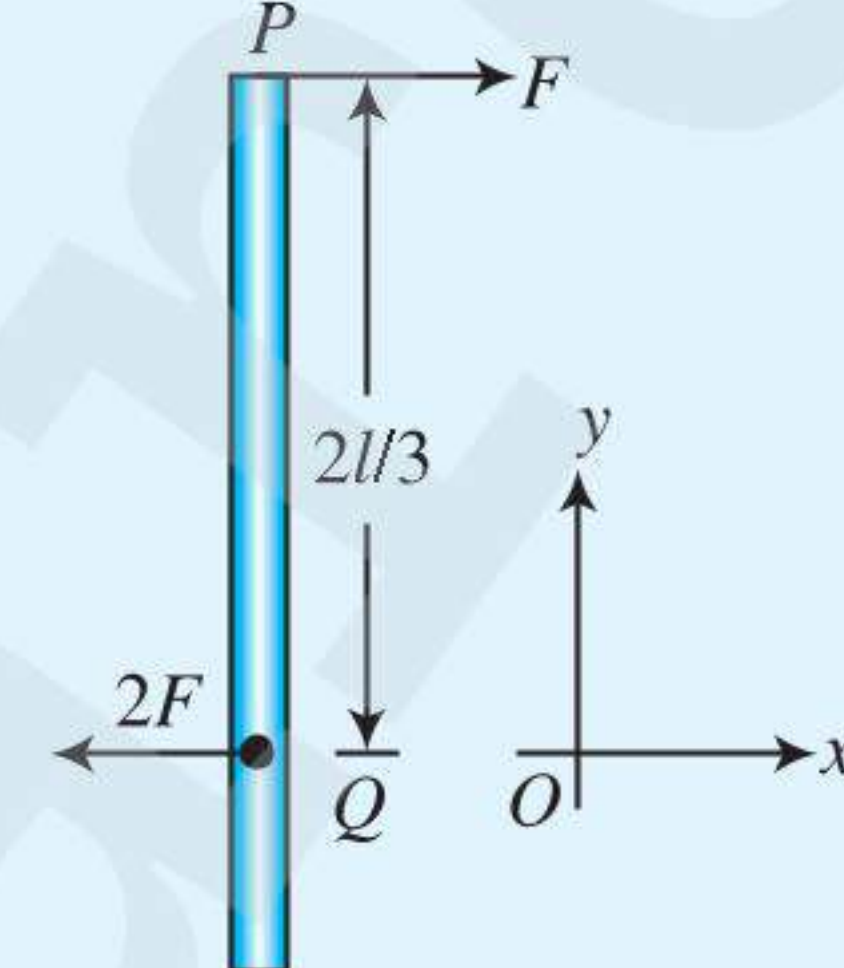
CONCEPT APPLICATION EXERCISE 4.3

1. A uniform rod of mass m and length l is acted upon by the forces F_1 and F_2 . Find that:

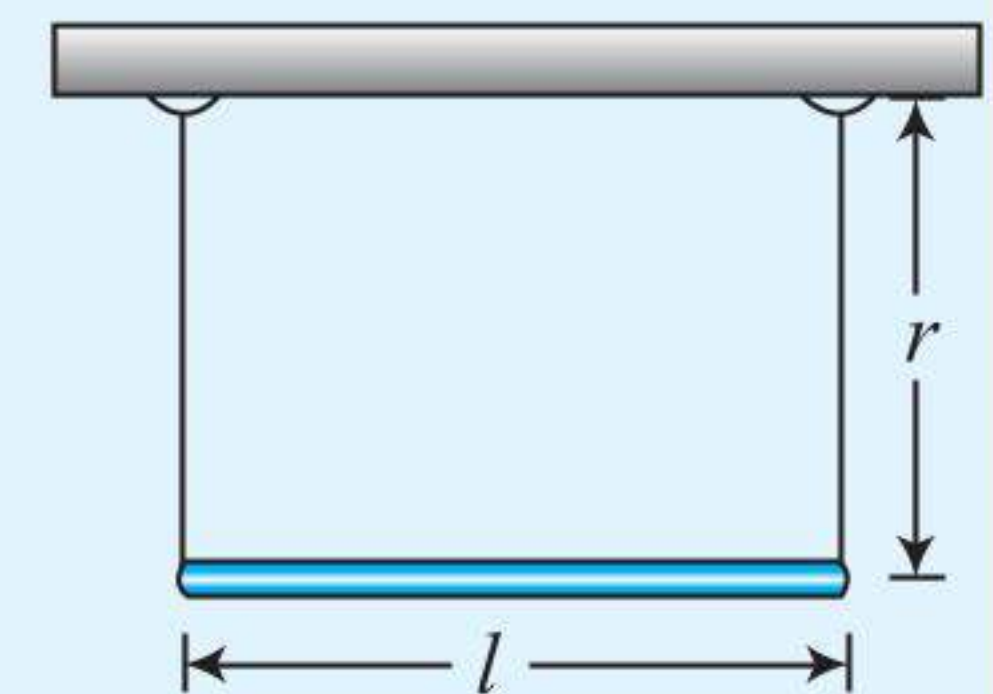
- (a) linear and angular acceleration of the rod.
- (b) value of x for which the point P does not accelerate.



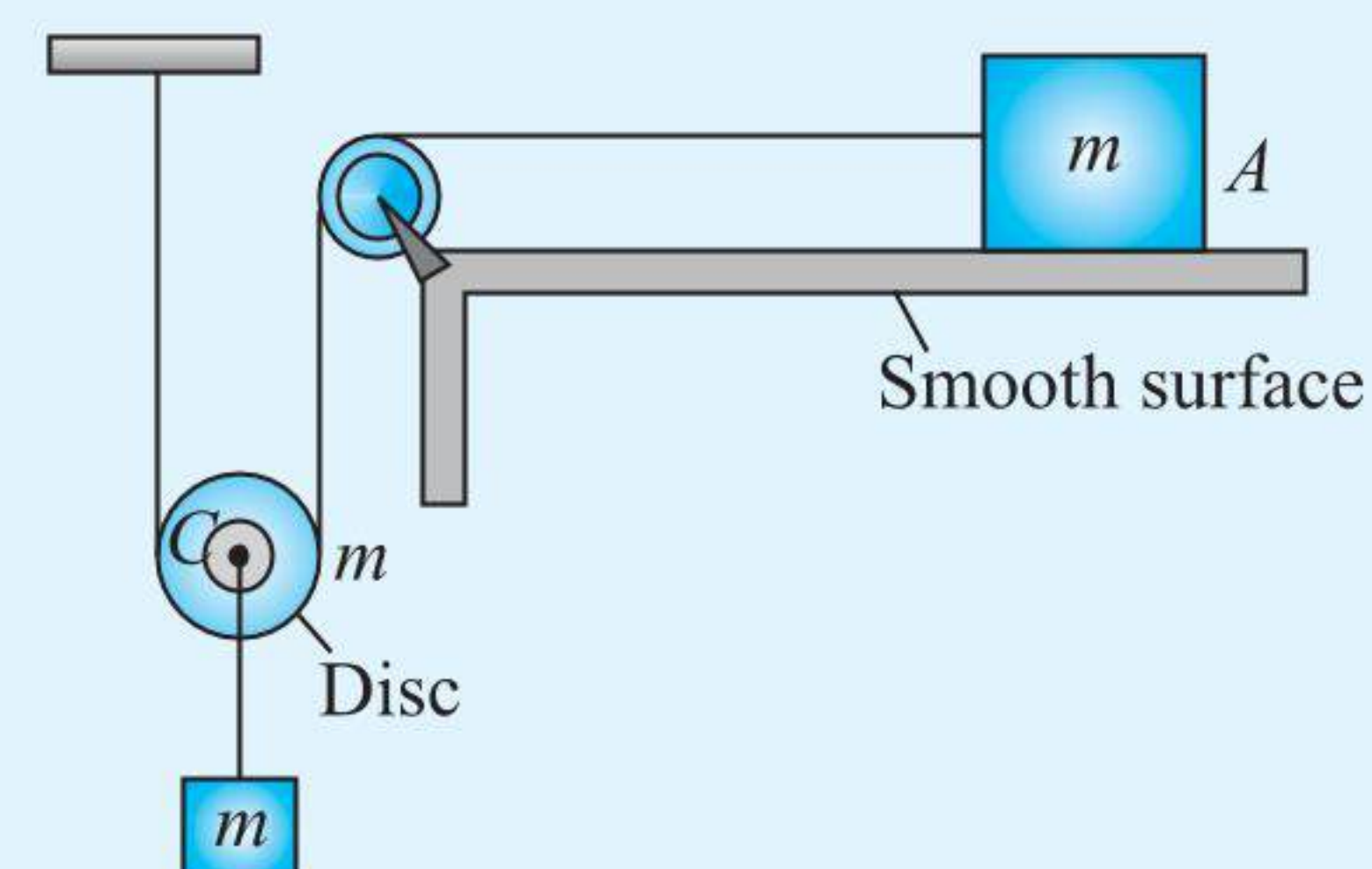
2. Find a_c and α of the smooth rod of mass m and length l .



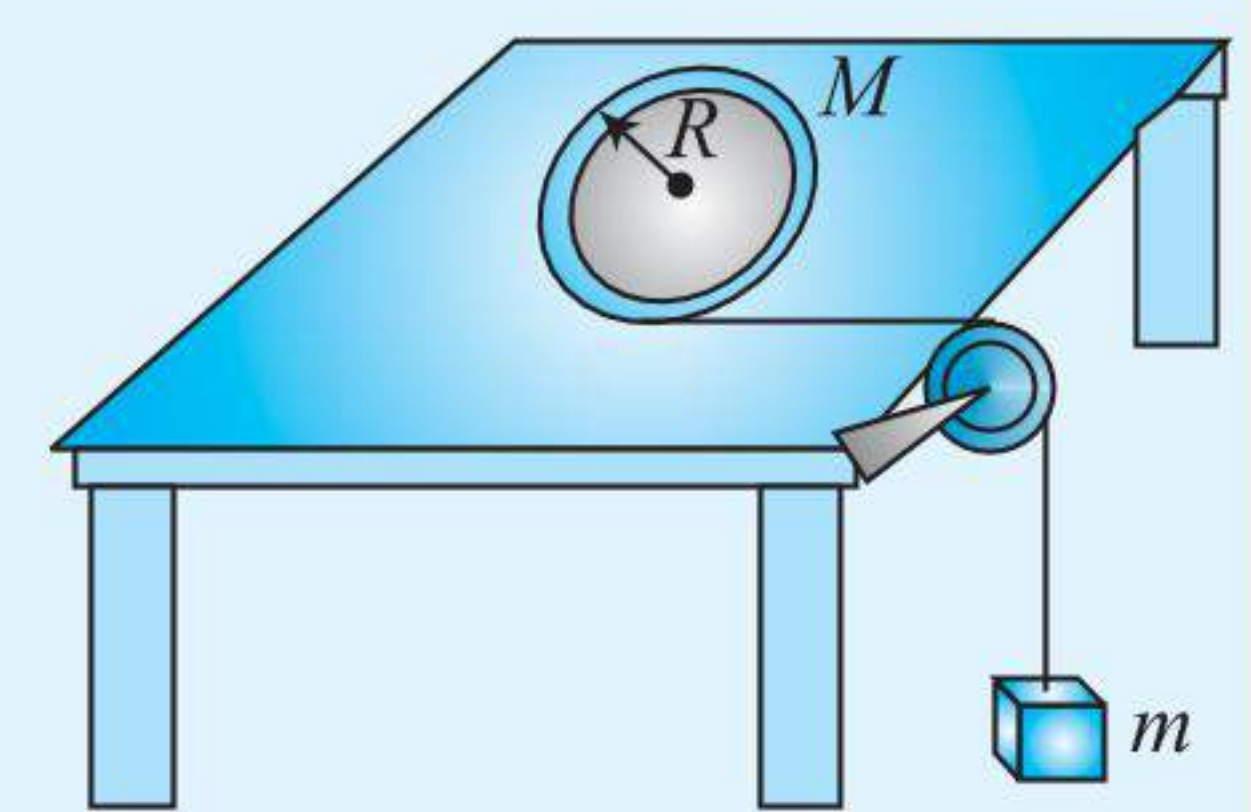
3. A uniform rod of mass M and length l is suspended by two vertical inextensible strings as shown in figure. Calculate tension in left string at the instant, when right string snaps.



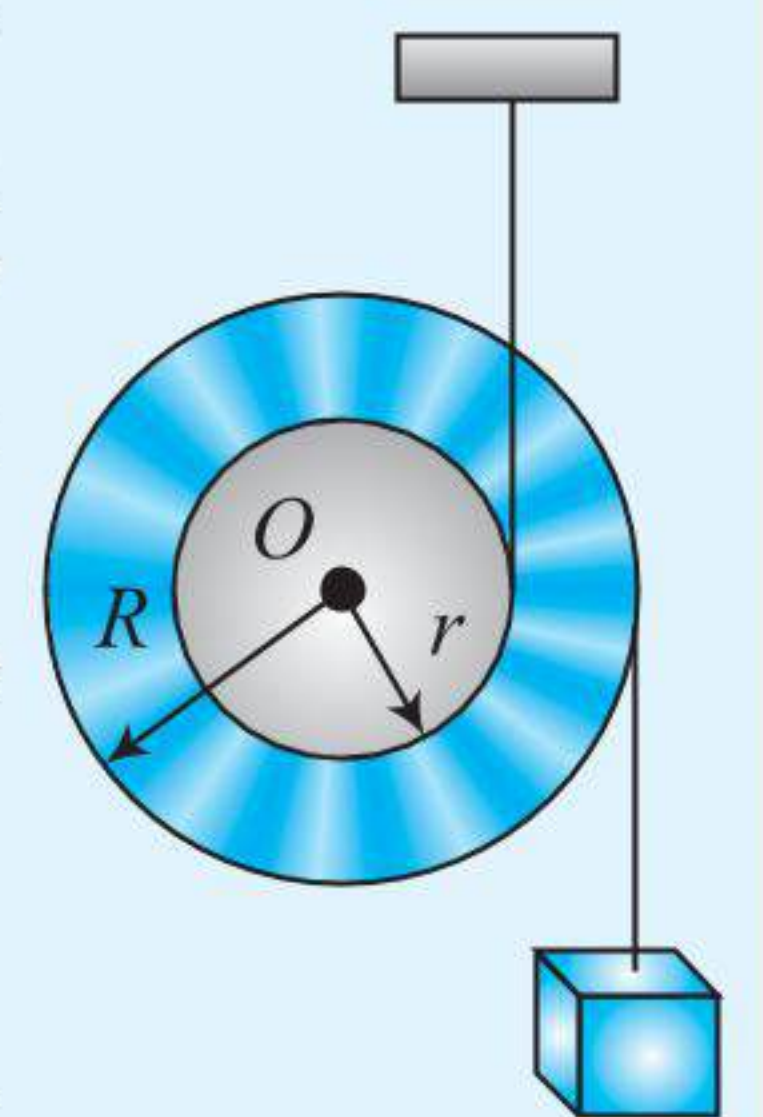
4. In the figure A and B are identical blocks, each of mass m and C is a circular disc of equal mass m . Pulley is massless and frictionless and thread is inextensible. Neglecting friction, find the acceleration of block A and B.



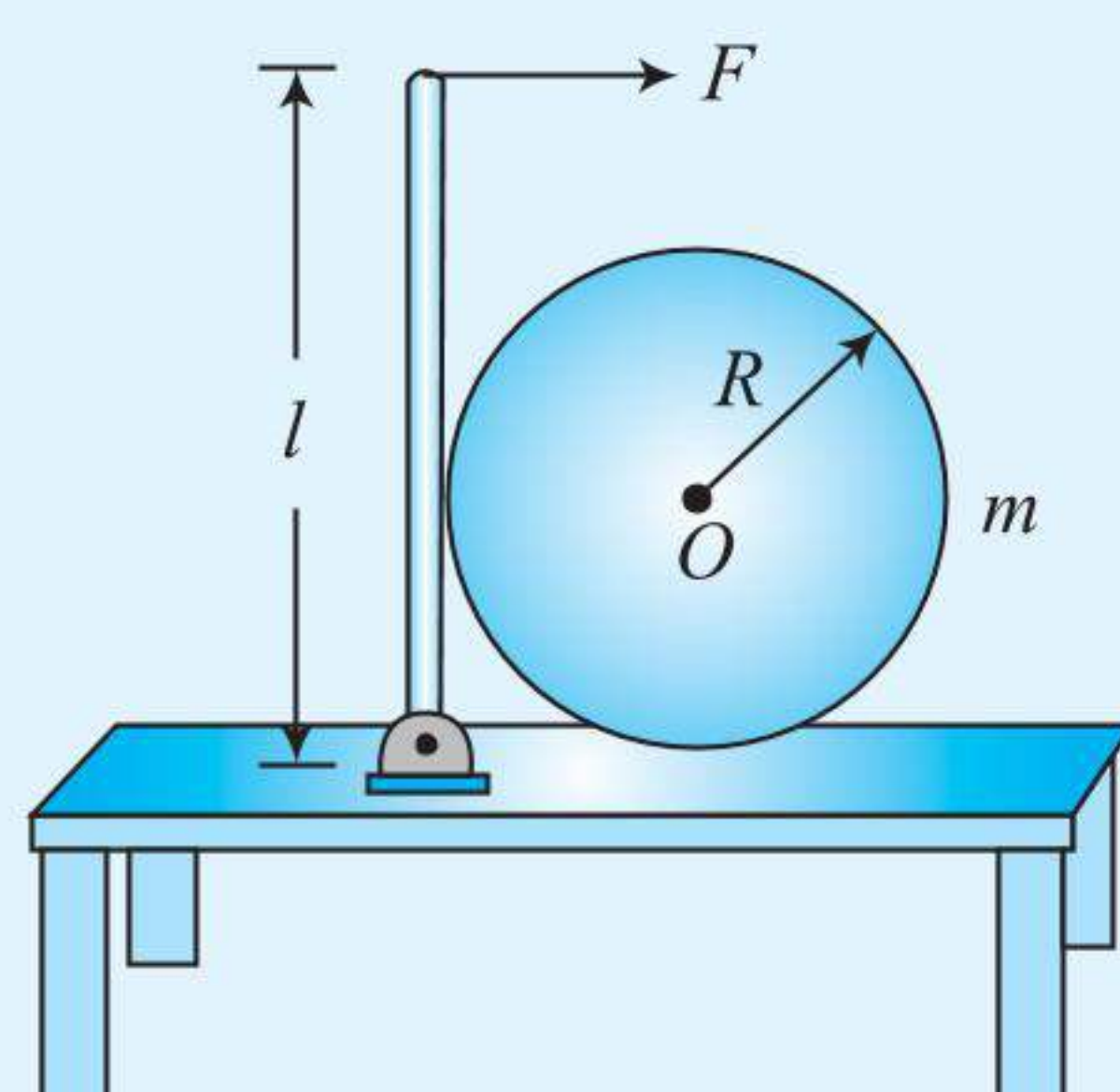
5. A light thread has been tightly wrapped around a disc of mass M and radius R . The disc has been placed on a smooth table, lying flat as shown. The other end of the string has been attached to a mass m as shown. The system is released from rest. Find the acceleration of the disc and block.



6. In the arrangement shown, the double pulley has a mass M and the two massless threads have been tightly wound on the inner (radius = r) and outer circumference (radius $R = 2r$). The block shown has a mass $4M$. The moment of inertia of the double pulley system about a horizontal axis passing through its centre and perpendicular to the plane of the figure is $I = \frac{Mr^2}{2}$. Find the acceleration of the center of the pulley after the system is released.



7. A uniform rod of mass m and length l which can rotate freely in vertical plane without friction, is hinged at its lower end on a table. A sphere of mass m and radius $R = l/3$ is placed in contact with the vertical rod and a horizontal force F is applied at the upper end of the rod.



- (a) Find the acceleration of the sphere just after the force starts acting.
 (b) Find the horizontal component of hinge reaction acting on the rod just after force F starts acting.

ANSWERS

1. (a) $\frac{3F}{m}, \frac{12F(l-x)}{ml^2}$ (b) $\frac{l}{2}$ 2. $\frac{F}{m}, \frac{8F}{ml}$ 3. $Mg/4$
 4. $8g/13$ 5. $a_{\text{block}} = \frac{3mg}{3m+M}$ and $a_{\text{disc}} = \frac{mg}{3m+M}$
 6. $6g/11$ 7. (a) $3F/4m$ (b) $7F/8$

DYNAMICS OF ROLLING MOTION

Before going to learn dynamics of rolling motion let us revise, what is rolling? Rolling can be viewed as a combination, or superposition, of purely translational motion (moving a wheel from one place to another with no rotation) and purely rotational motion (only rotation with no movement of the center of the wheel). The point of contact of the rolling object does not slide with respect to the surface on which it is moving. This is also called pure rolling. The situations of combined rotation and translation where point of contact slide on the surface is called rolling with slipping. There is a special connection between the translational component of the motion and the rotational component. Here we will learn the cases of dynamics of rolling into two categories

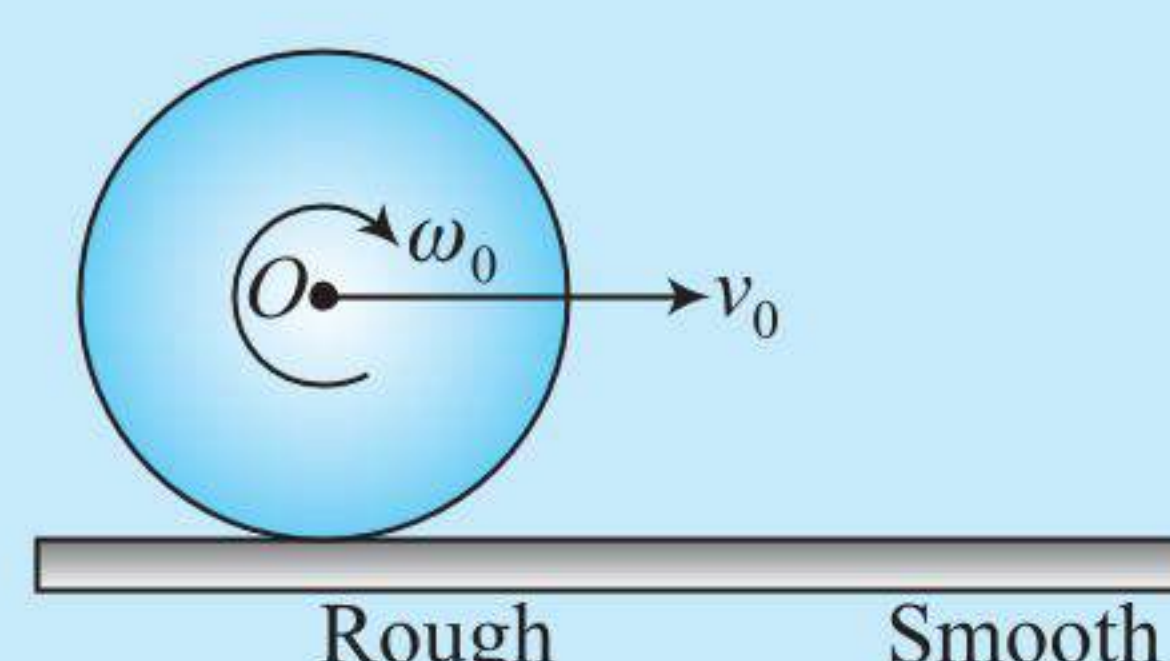
- (i) Rolling of an object without net external force
 (ii) Rolling of an object under net external force

ROLLING OF AN OBJECT WITHOUT NET EXTERNAL FORCE

Let us first take rolling of an object without net external force, for understanding the concept, let us take some illustrations.

ILLUSTRATION 4.23

A rolling body of mass m and radius R is rolling on a rough surface without any pulling force as shown in figure. What happens when it enters on a smooth surface?



Sol. The point of contact of the rolling object does not slide with respect to the surface on which it is moving. It means when body is rolling on the rough surface no friction force acts on it.

When the body enters the smooth surface. In this case, the linear speed v_0 also remains constant. The body keeps on rolling on the smooth horizontal surface and the point of contact is stationary with respect to the surface.

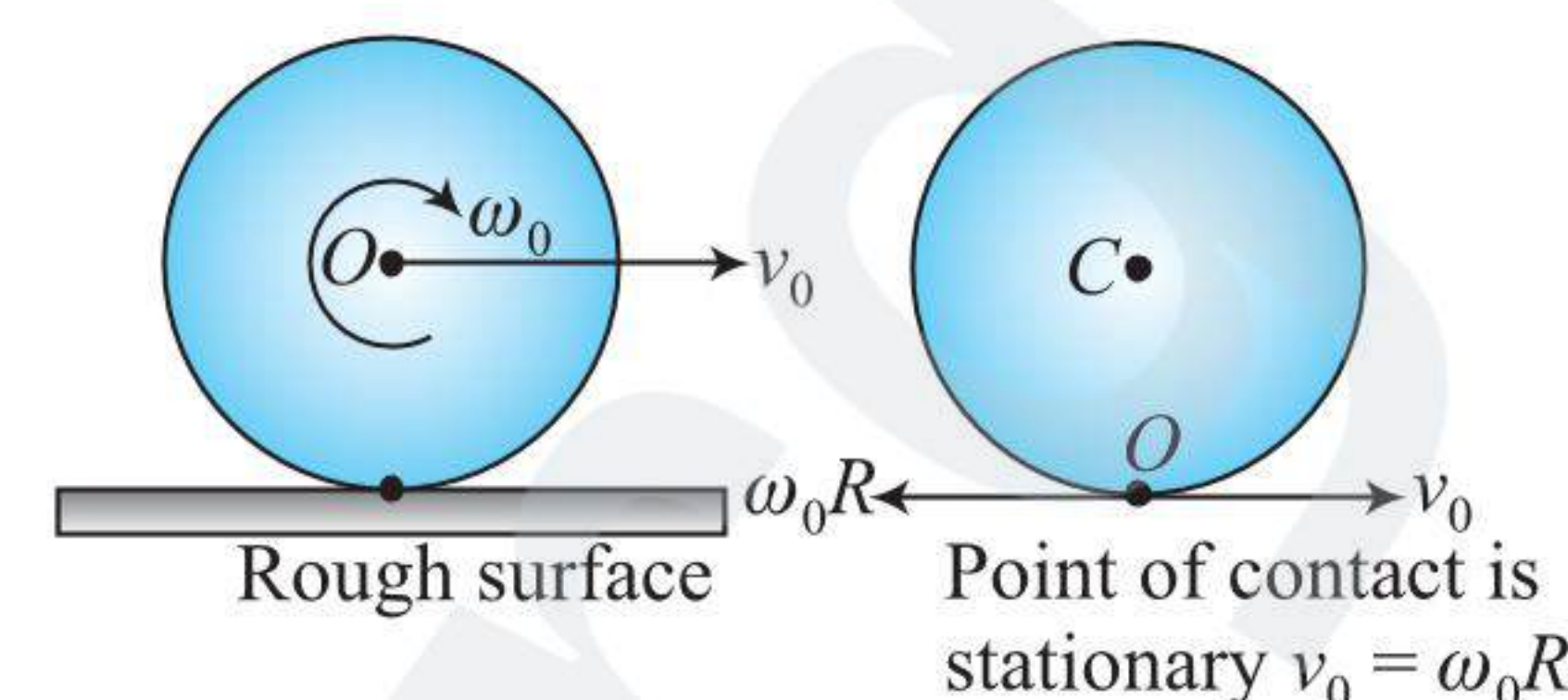
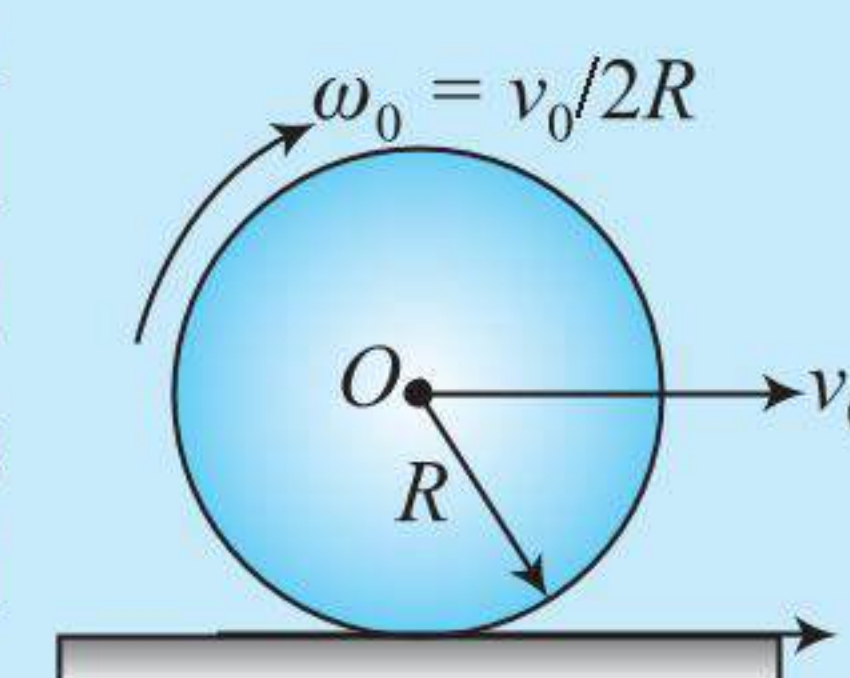
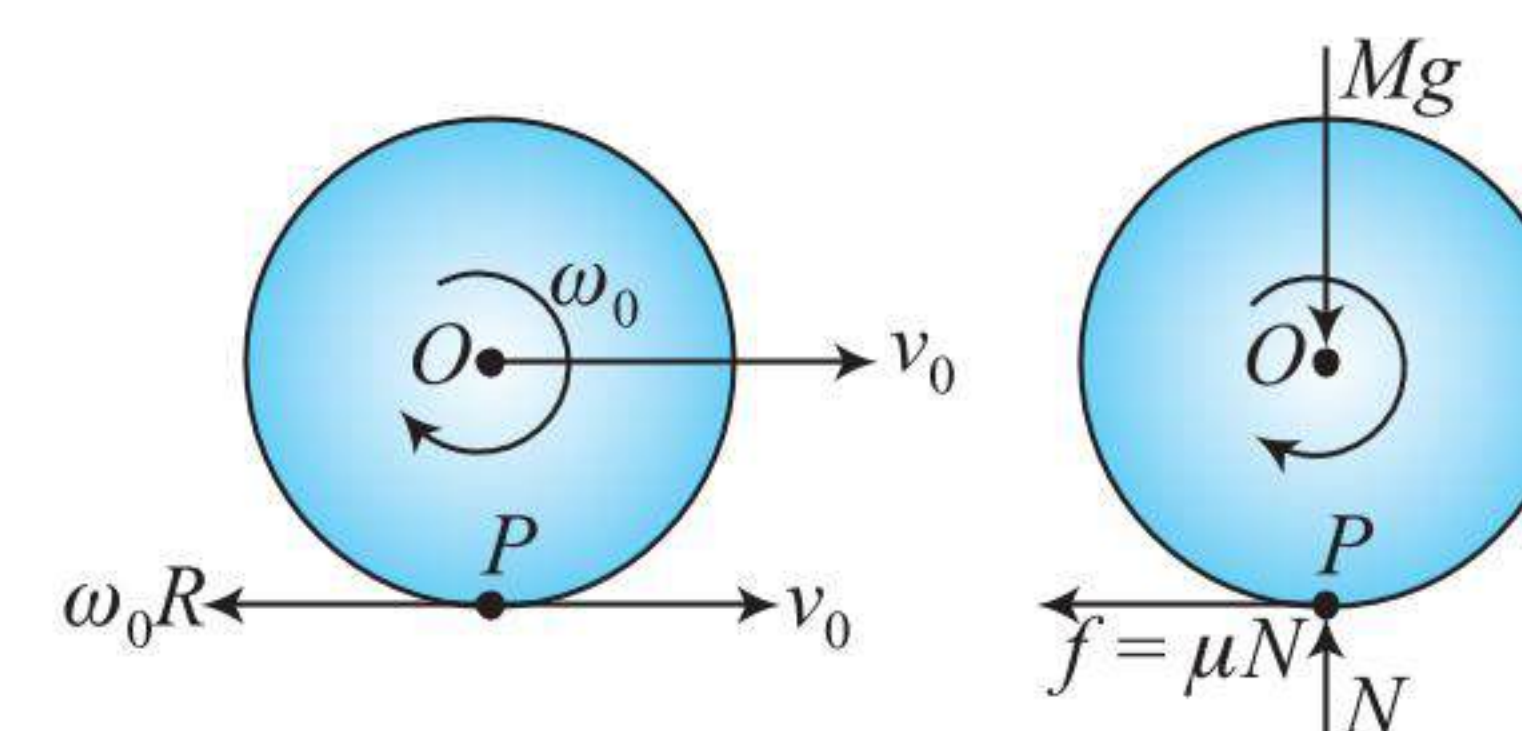


ILLUSTRATION 4.24

A sphere of mass M and radius R shown in the figure slips on a rough horizontal plane. At some instant it has translational velocity v_0 and rotational velocity about the centre $v_0/2R$. Find the translational velocity after the sphere starts pure rolling.



Sol. Method 1. Force/torque method: It is given at certain instant, the velocity of the centre of mass of the disc is v_0 and the angular velocity about the centroidal axes is $v_0/2R$.



As, $v_0 > \omega_0 R$, the point of contact 'P' of sphere with ground will slip forward and thus the friction by the plane will act backward on the sphere. This friction will be kinetic in nature and the sphere will be decelerated by

$$a_{\text{cm}} = \frac{f}{M} = \frac{\mu Mg}{M} = \mu g \quad \dots(i)$$

Hence velocity of centre of mass of the sphere at any time t is

$$v(t) = v_0 - \mu g t \quad \dots(ii)$$

This friction will also produce a torque about the centroidal axes. This torque is clockwise and in the direction of ω_0 . Hence the angular velocity of the sphere will increase. Here we can apply torque equation about centroidal axes

$$\tau_o = I_o \alpha \Rightarrow \mu Mg \times R = \frac{2}{5} MR^2 \alpha \quad \dots(iii)$$

From Eq. (iii), the angular acceleration of the sphere, $\alpha = \frac{5\mu g}{2R}$

and the angular velocity at time t will be

$$\omega(t) = \omega_0 + \alpha t = \frac{v_0}{2R} + \frac{5\mu g}{2R} t$$

The pure rolling starts when $v(t) = R\omega(t)$

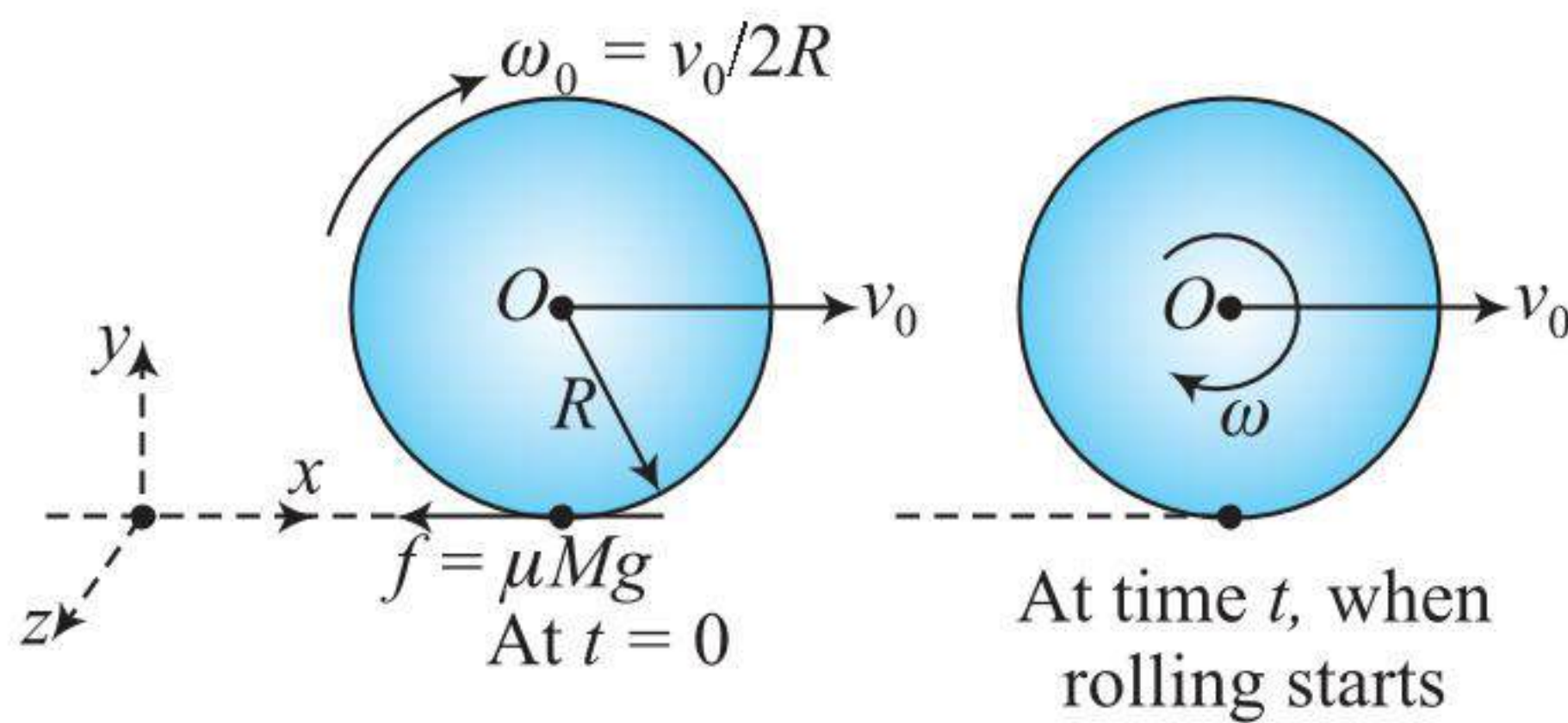
$$\text{i.e., } v_0 - \mu g t = R \left(\frac{v_0}{2R} + \frac{5\mu g}{2R} t \right) \Rightarrow t = \frac{v_0}{7\mu g}$$

On substituting the value of t in Eq. (ii), we get

$$v(t) = v_0 - \mu g \left(\frac{v_0}{7\mu g} \right) = \frac{6v_0}{7}$$

Thus, the sphere rolls with translational velocity $6v_0/7$ in the forward direction.

Method 2. By conservation of angular momentum: We can solve this problem by using conservation of angular momentum. We can conserve the angular momentum about any point situated on the ground. (As torque of friction force about any point on the ground is zero.)



Angular momentum about any point O situated on ground at $t = 0$

$$\begin{aligned} \vec{L}_{\text{initial}} &= \vec{L}_{\text{CM}} + \vec{r} \times m\vec{v} = \frac{2}{5}MR^2\omega_0(-\hat{k}) + Mv_0R(-\hat{k}) \\ &= \left[\frac{2}{5}MR^2 \left(\frac{v_0}{2R} \right) + Mv_0R \right] (-\hat{k}) \\ \vec{L}_{\text{initial}} &= \frac{6}{5}Mv_0R(-\hat{k}) \end{aligned} \quad \dots(i)$$

Angular momentum at time t when rolling starts

$$\vec{L}_{\text{final}} = \vec{L}_{\text{CM}} + \vec{r} \times m\vec{v} = \frac{2}{5}MR^2\omega(-\hat{k}) + MvR(-\hat{k})$$

When rolling starts $v = \omega R$

$$\begin{aligned} \text{Hence, } \vec{L}_{\text{final}} &= \left[\frac{2}{5}MR^2 \left(\frac{v}{R} \right) + MvR \right] (-\hat{k}) \\ \Rightarrow \vec{L}_{\text{final}} &= \frac{7}{5}MvR(-\hat{k}) \end{aligned} \quad \dots(ii)$$

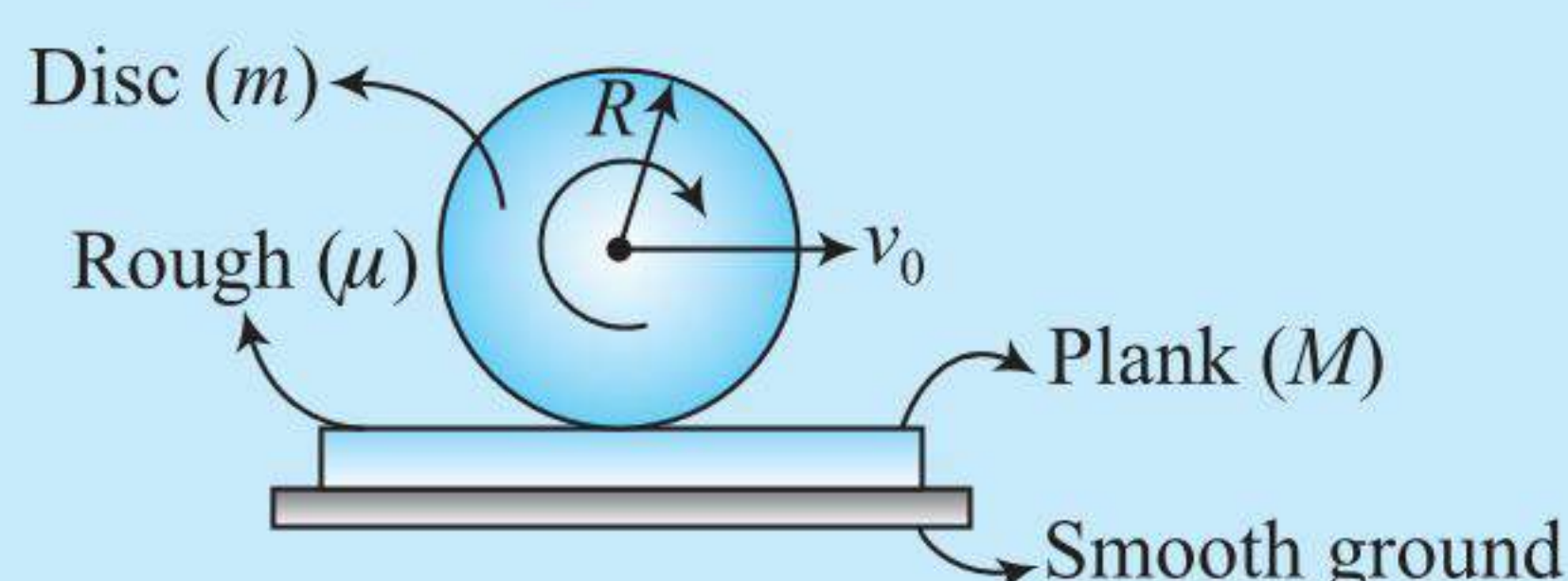
Conserving angular momentum, we get

$$\begin{aligned} \vec{L}_{\text{initial}} &= \vec{L}_{\text{final}} \\ \frac{6}{5}Mv_0R(-\hat{k}) &= \frac{7}{5}MvR(-\hat{k}) \end{aligned}$$

which gives $v = \frac{6}{7}v_0$

ILLUSTRATION 4.25

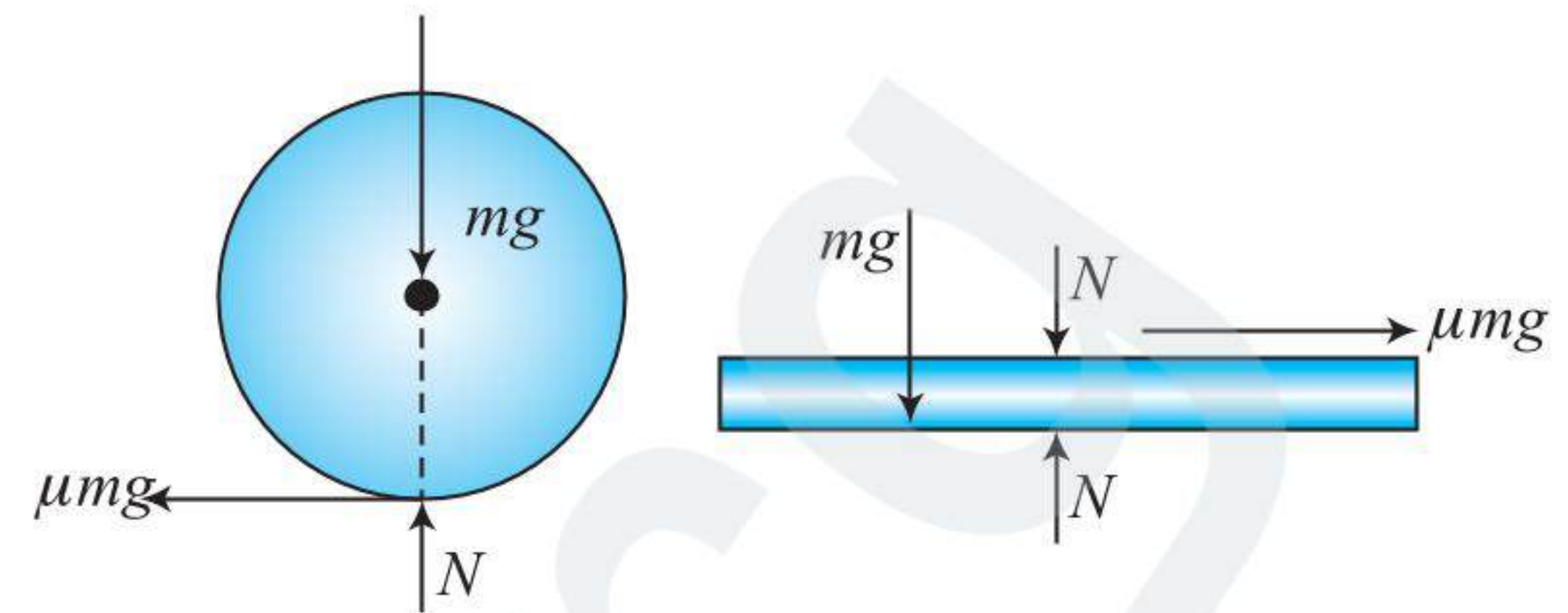
A plank of mass M , whose top surface is rough with coefficient of friction μ is placed on a smooth ground (figure). Now a disc of mass $m = M/2$ and radius r is placed on the plank. The disc is now given a velocity v_0 in the horizontal direction, at $t = 0$.



- Find the time when the disc starts rolling.
- Find the velocity of the plank and the disc up to that time.
- Find the distance travelled by the plank up to this instant.

Sol.

- (a) Initially the point of contact of the disc will slide on the plank, hence friction will be of kinetic nature and act on the disk in backward direction. It means the friction on the plank will act in forward direction.



Equation of motion

For the plank: $\mu mg = MA$

$$\Rightarrow A = \frac{\mu mg}{M} \quad (\text{towards right}) \quad \dots(i)$$

$$\Rightarrow A = \frac{\mu g}{2} \quad (\text{as } m = M/2)$$

For the disc: $\mu mg = ma$

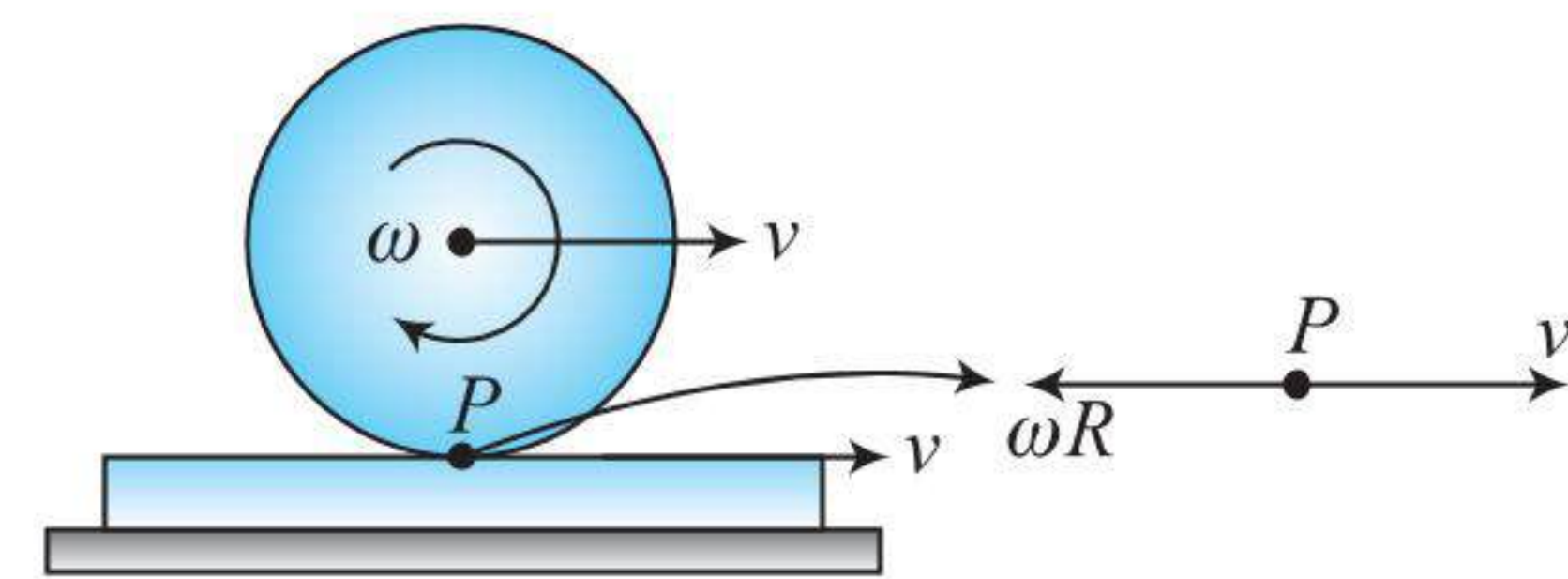
$a = \mu g$ (backwards, towards left)

Torque equation about centre of mass of the disc

$$\mu mgR = I_c \alpha = \frac{mR^2}{2} \alpha$$

$$\Rightarrow \alpha = \frac{2\mu g}{R} \quad (\text{clockwise, will be positive})$$

Now let us write constraint relations. At the time of rolling, point P should not slide with respect to the plank.



$$V = v - \omega R$$

$$(0 + At) = (v_0 - at) - (0 + \alpha t)R$$

Now substituting the values of A and a , we can calculate ' t '.

$$\frac{\mu mg}{M}t = (v_0 - \mu g t) - \left(\frac{2\mu g}{R}t \right)R$$

$$\left(\frac{\mu mg}{M} + \mu g + 2\mu g \right)t = v_0$$

which gives

$$t = \frac{v_0}{\mu g \left(\frac{m}{M} + 3 \right)} = \frac{2v_0}{7\mu g} \quad (\text{as } m = M/2)$$

- (b) Velocity of the plank when rolling starts,

$$V = At = \left(\frac{\mu g}{2} \right) \left(\frac{2v_0}{7\mu g} \right) = \frac{v_0}{7}$$

Velocity of the disc when rolling starts,

$$v = v_0 - at = v_0 - \mu g \left(\frac{2v_0}{7\mu g} \right) = \frac{5}{7}v_0$$

Angular velocity at the time of rolling,

$$\omega = \alpha t = \left(\frac{2\mu g}{R} \right) \left(\frac{2v_0}{7\mu g} \right) = \frac{4}{7} \frac{v_0}{R}$$

(c) Distance travelled by the plank,

$$S_{\text{plank}} = \frac{1}{2} A t^2 = \frac{1}{2} \left(\frac{\mu g}{2} \right) \left(\frac{2v_0}{7\mu g} \right)^2 = \frac{v_0^2}{49\mu g}$$

ROLLING OF AN OBJECT UNDER NET EXTERNAL FORCE

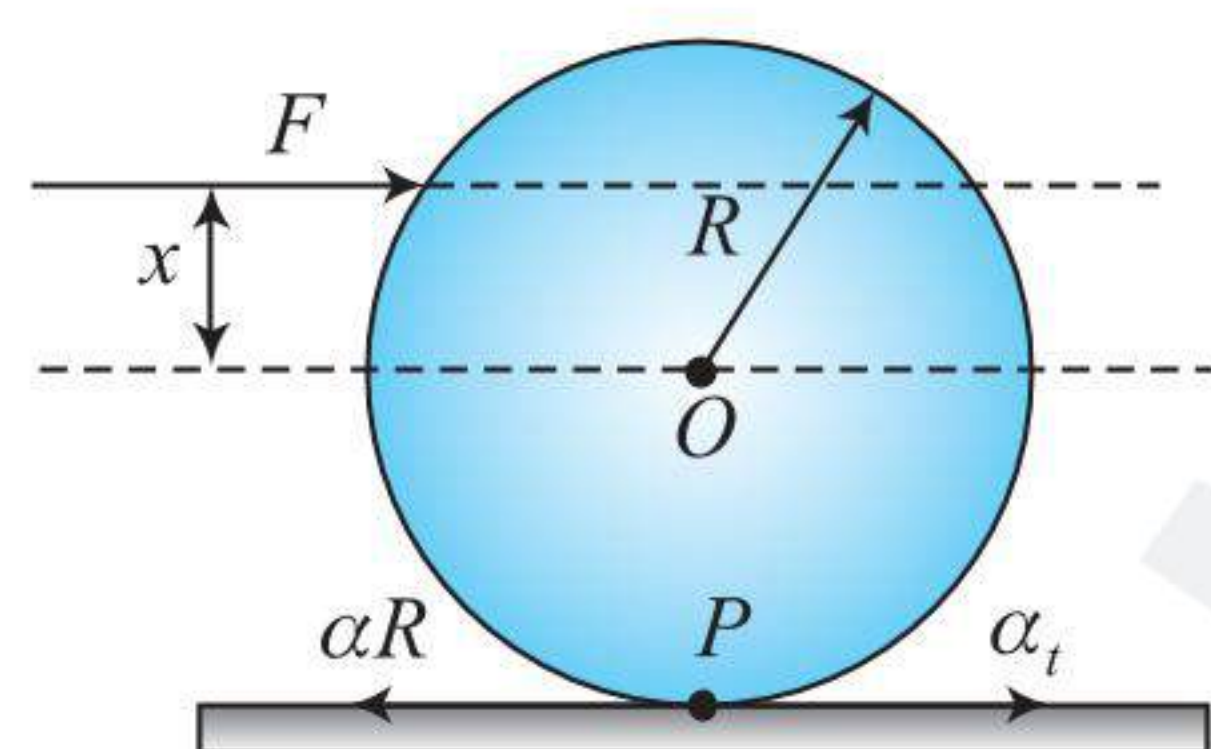
When a body rolls on a surface under external force, the frictional force on the body (if any) will be static in nature, less than its limiting value. But if the object rolls with sliding the nature of friction should be kinetic nature. Let us first learn how to find the direction of friction force in case of combined translational and rotational motion.

Direction of Friction in Case of Translation and Rotation Combined

In case of translation motion, we can decide the direction of friction force by direct observation. But the direction of friction cannot be found by direct observation in case of rotational motion, as the body is translating as well as rotating. The direction of friction force is determined after deciding the motion tendency of the point of contact of the body under consideration with the ground.

Let us take an example of analysis of direction of friction in this case.

In the following diagram (figure), a rolling object of mass M and radius R is placed on a rough horizontal surface. A force F is applied as shown.



To find the direction of friction at P , let us consider the surface to be frictionless.

Acceleration of point P due to translation only,

$$a_t = \frac{F}{M} (\rightarrow)$$

Acceleration of point P due to rotation only,

$$a_r = \alpha R = \frac{\tau}{I} R \Rightarrow \frac{FR}{I} x (\leftarrow)$$

Net acceleration of point P is

$$\vec{a}_p = \vec{a}_t + \vec{a}_r$$

$$a_p = \frac{F}{M} - \frac{FRx}{I} (\rightarrow) \quad \dots(i)$$

From Eq. (ii), it is clear that motion tendency at point P depends upon both x and I .

Equation (i) can be written as

$$a_p = \frac{F}{M} \left(1 - \frac{Rx}{K^2} \right) \quad \dots(ii)$$

The motion tendency of point P will be in forward direction if

$$1 - \frac{Rx}{K^2} > 0 \Rightarrow 1 > \frac{Rx}{K^2} \Rightarrow K^2 > Rx \quad \dots(iii)$$

If this condition is satisfied, friction will act in backward direction.

Note: We can summarize the situation as

If $K^2 > Rx$: friction will act in backward direction.

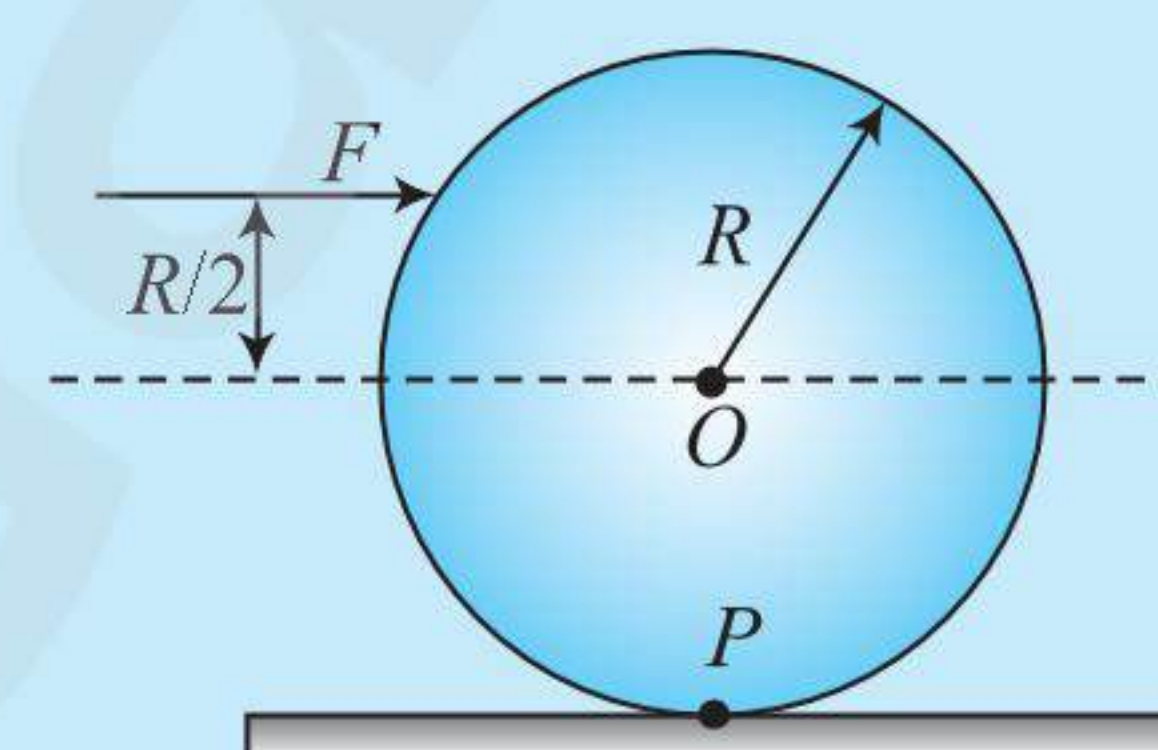
If $K^2 = Rx$: no friction will act.

If $K^2 < Rx$: friction will act in forward direction.

If force acts in lower diametric plane, friction will act in backward direction only.

ILLUSTRATION 4.26

Find the direction of friction force in the following cases. If rolling object is



- (a) a ring (b) a disc
(c) a solid sphere (d) a hollow sphere

Sol.

(a) For ring: $K^2 = R^2$ and $Rx = R \frac{R}{2} = \frac{R^2}{2}$

As $K^2 > Rx$

The friction will act in the backward direction.

(b) For disc: $K^2 = \frac{R^2}{2}$ and $x = R \frac{R}{2} = \frac{R^2}{2}$

$\Rightarrow K^2 = Rx$

No friction will act.

(c) For sphere: $K^2 = \frac{2}{5} R^2$ and $Rx = R \frac{R}{2} = \frac{R^2}{2}$

$\Rightarrow K^2 < Rx$

The friction will act in the forward direction.

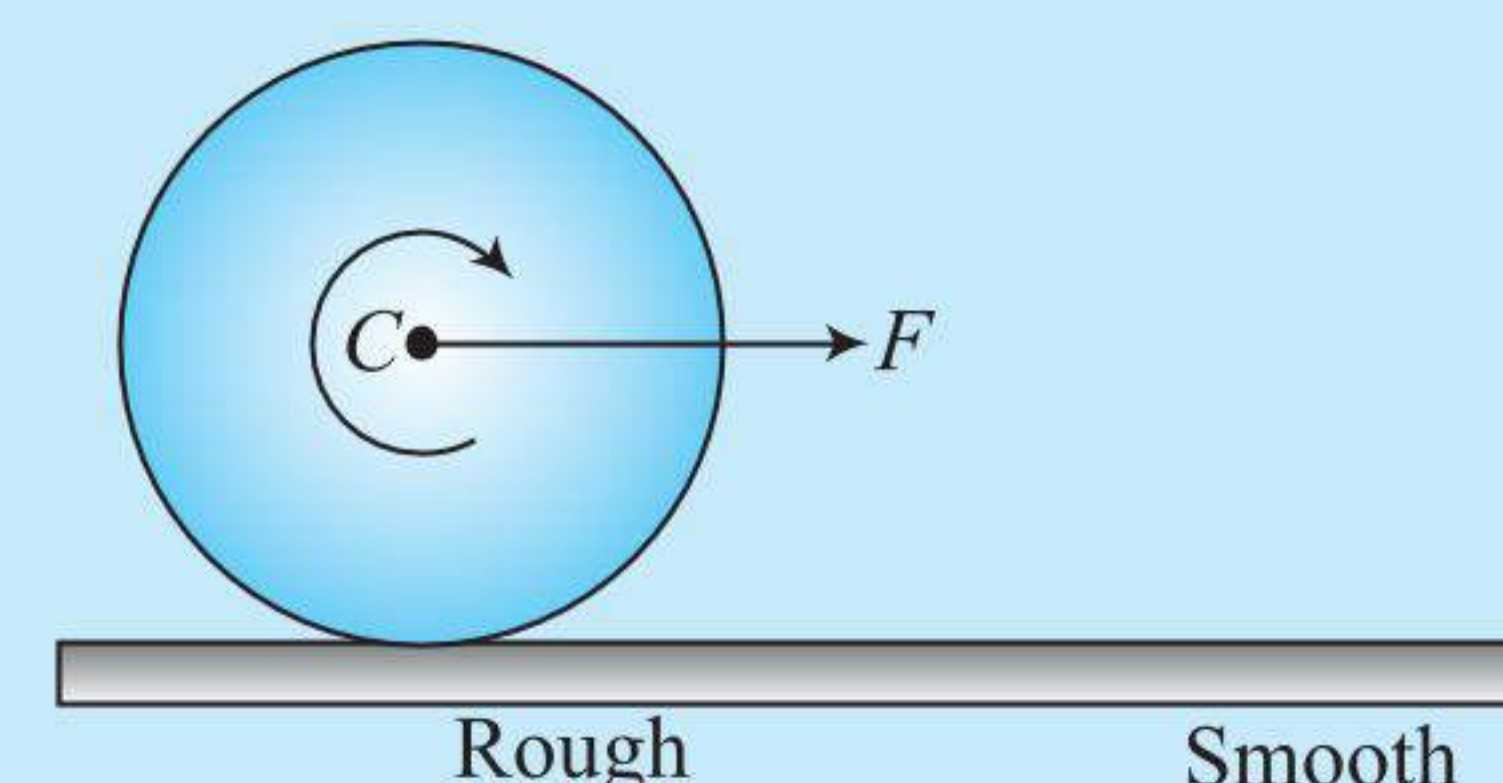
(d) For hollow sphere: $K^2 = \frac{2}{3} R^2$ and $Rx = R \frac{R}{2} = \frac{R^2}{2}$

$\Rightarrow K^2 > Rx$

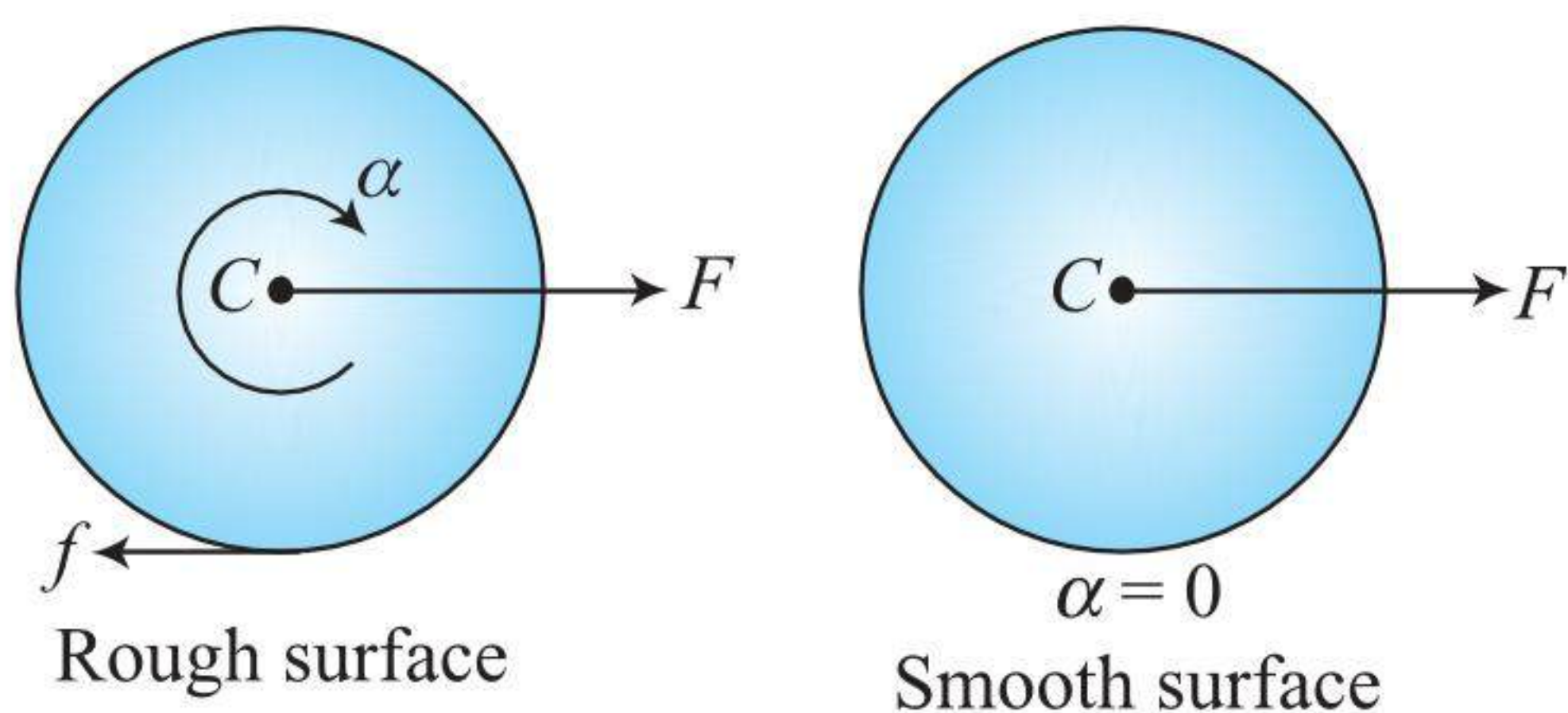
The friction will act in the backward direction.

ILLUSTRATION 4.27

A force of magnitude F is acting on a rolling body of mass m and radius R as shown in figure. What happens when it enters on a smooth surface?



Sol. When the body is rolling on the rough surface, the friction force acting on the surface acts in the backward direction. The force F increases velocity of centre of mass while friction force provides torque for providing angular acceleration. The velocity of centre of mass is always such that $v_c = \omega R$, i.e., rolling is present. When the body enters on the smooth surface, the pulling force F is still acting, i.e., $F \neq 0$.



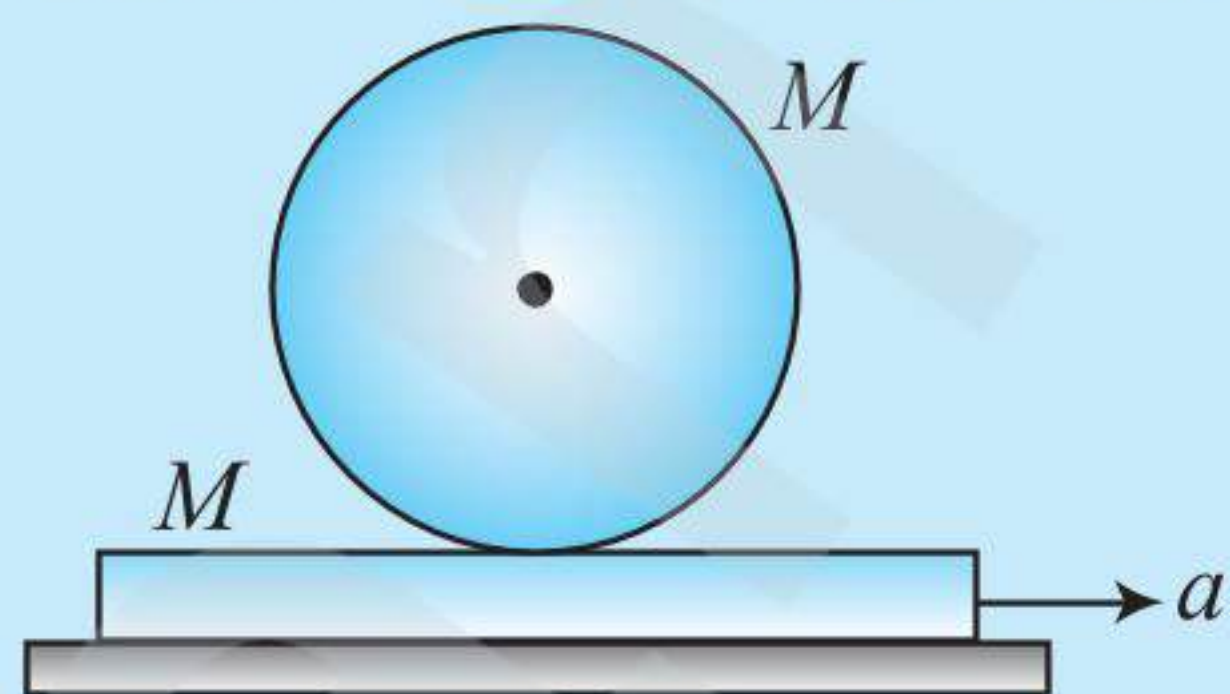
When the rolling body enters on a smooth surface, the force of friction disappears, i.e., $f=0$. As a result, the angular acceleration of the body about its centre of mass becomes zero.

Whatever is the angular speed attained by the body, it continues to rotate with that. On the contrary, the linear acceleration of the centre of mass suddenly increases to F/m .

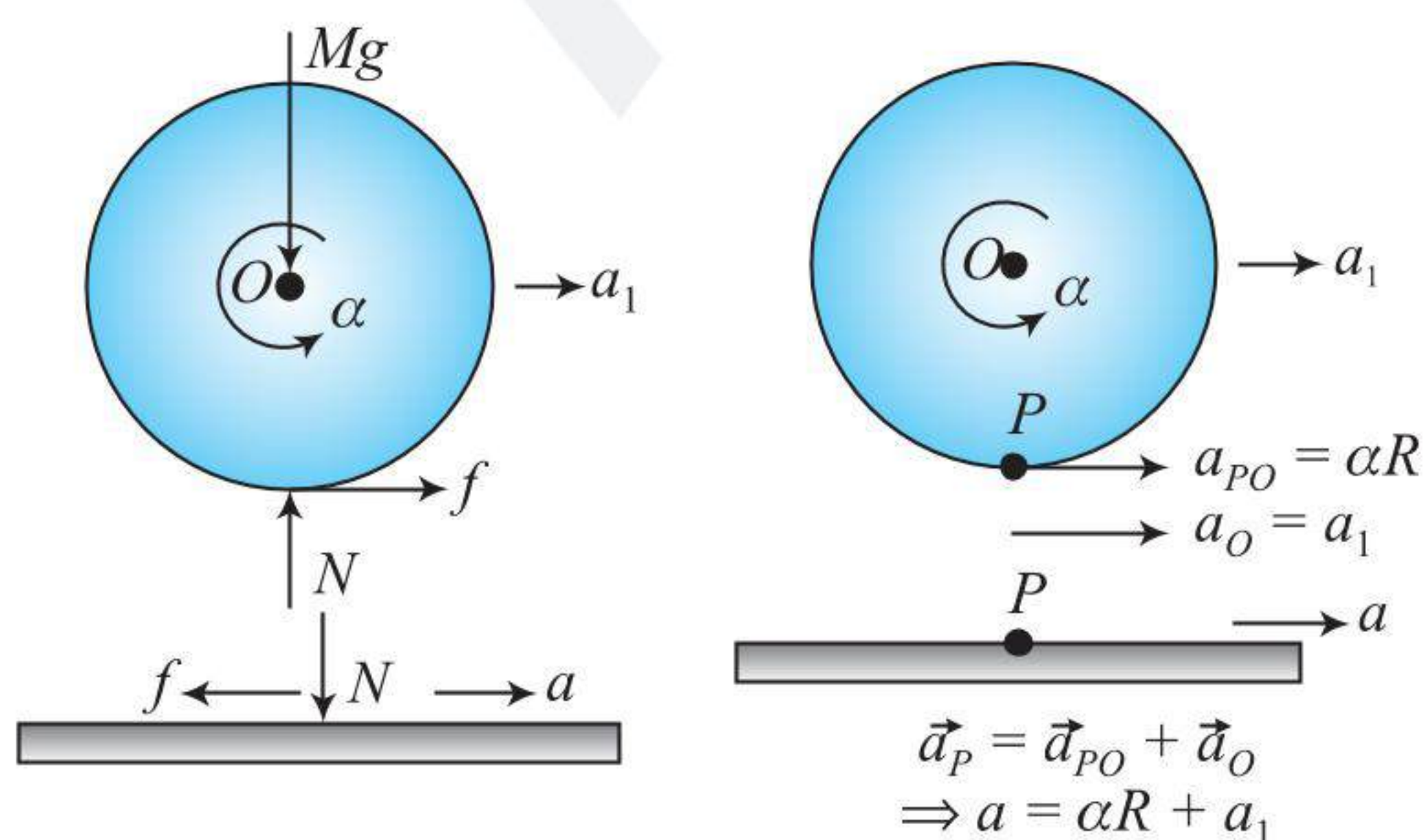
As a result, its linear speed increases rapidly and the condition $v_c = \omega R$ does not hold true and the body will not roll on the smooth surface. The point of contact slips forward ($v > \omega_0 R$).

ILLUSTRATION 4.28

A uniform cylinder (mass M) of radius R is kept on an accelerating platform (mass M) as shown in figure. If the cylinder rolls without slipping on the platform, determine the magnitude of acceleration of the centre of mass of the cylinder. Assuming the coefficient of friction μ , determine the maximum acceleration the platform may have without slip between the cylinder and the platform.



Sol. As there is no sliding between the cylinder and plank, the friction will be static in nature. For finding the direction of friction assume, there is no friction anywhere, it suggests sliding tendency of the cylinder w.r.t. plank will be in backward direction. It means friction force on the cylinder will act in forward direction. The friction acting on the cylinder will be responsible for angular acceleration of the cylinder.



From FBD of cylinder:

$$\text{Force equation of cylinder, } f = Ma_1 \quad \dots(i)$$

As axes of rotation is moving so we can apply torque equation about centripetal axis, $\tau_0 = I_0 \alpha$

$$\Rightarrow f \cdot R = \frac{MR^2}{2} \alpha \text{ or } R\alpha = \frac{2f}{M} \quad \dots(ii)$$

As the cylinder is rolling, it means point of contact of cylinder with plank will not slide. It means, acceleration of the plank = acceleration of point P

$$\text{or } a = a_1 + \alpha R \quad \dots(iii)$$

From (i), (ii) and (iii), we get

$$f = \frac{Ma}{3} \quad \dots(iv)$$

As there is no sliding, it means

$$f \leq \mu N$$

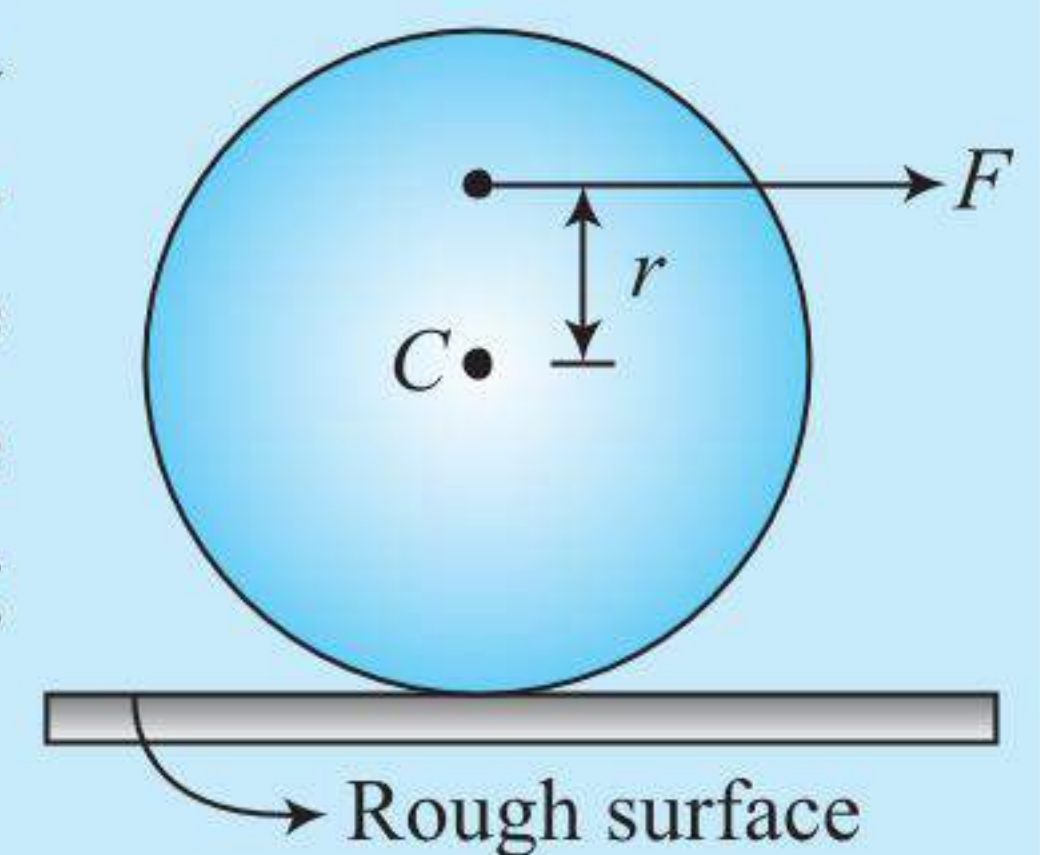
$$\frac{Ma}{3} \leq \mu Mg$$

$$\Rightarrow a_{\max} = 3\mu g$$

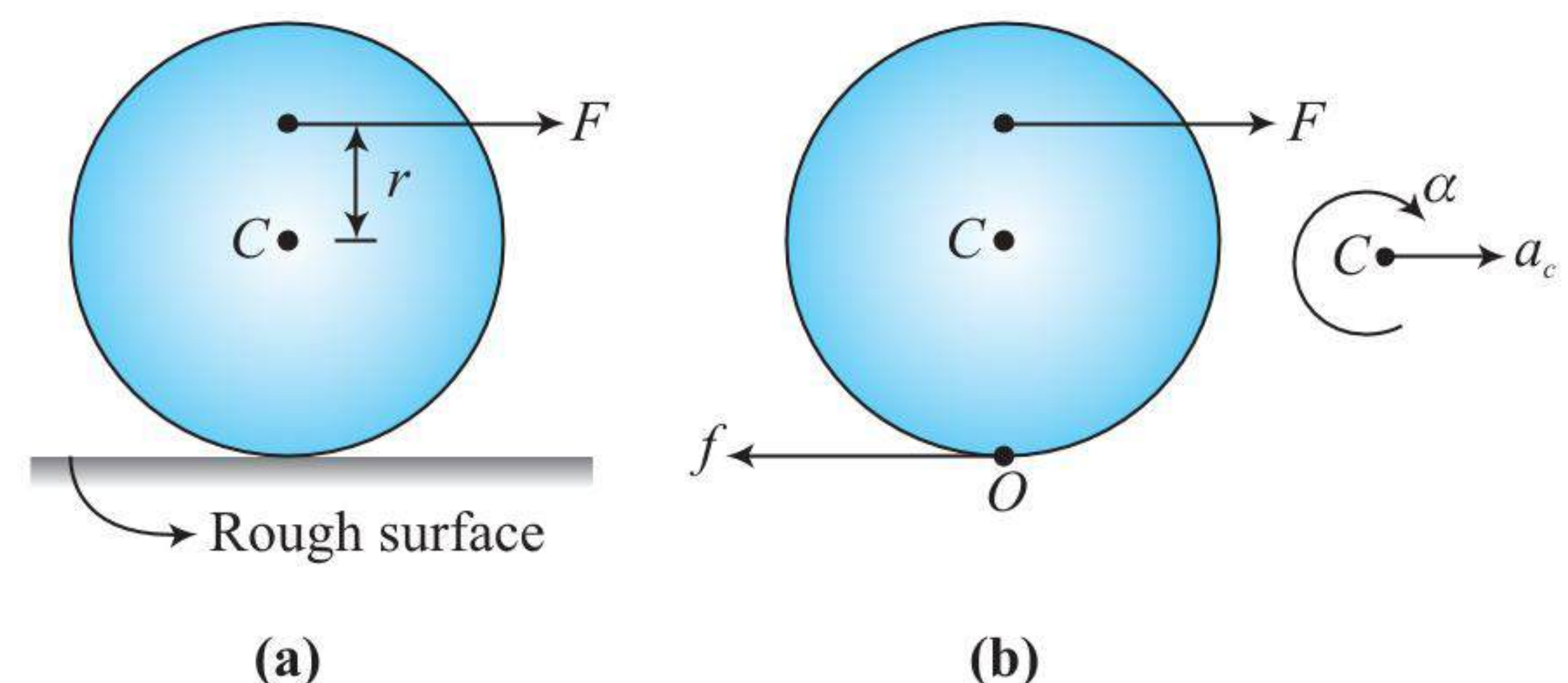
ILLUSTRATION 4.29

A force of magnitude F is acting on a rolling body of mass M and radius R as shown in figure. What happens if the pulling force F is removed? Discuss the different possibilities when the pulling force acts

- (a) at centre (b) above centre
(c) below centre



Sol. If the pulling force F is removed from a body rolling on a rough surface, the friction force also disappears, because the point of contact has no more tendency to slip with respect to the surface. The linear and angular speeds of the body remain constant and the body keeps on rolling.



As body is rolling, the friction will be of static nature and unknown (both in magnitude and direction). Let friction act in the backward direction.

Equation of motion:

$$F - f = Ma_c \quad \dots(i)$$

Here we can apply torque equation about centre of mass as well as about point of contact with ground (instantaneous centre of rotation)

$$\text{About centre of mass: } Fr + fR = I_c \alpha \quad \dots(ii)$$

About point of contact O : $F(R+r) = I_O \alpha$

$$F(R+r) = (I_c + MR^2) \alpha \quad \dots(iii)$$

Taking torque equation about O , we can directly calculate the value of α .

$$\text{Hence, } \alpha = \frac{F(R+r)}{(I_c + MR^2)} \quad \dots(iv)$$

As the body is rolling, $a_c = \alpha R = \frac{F(R+r)R}{(I_c + MR^2)}$

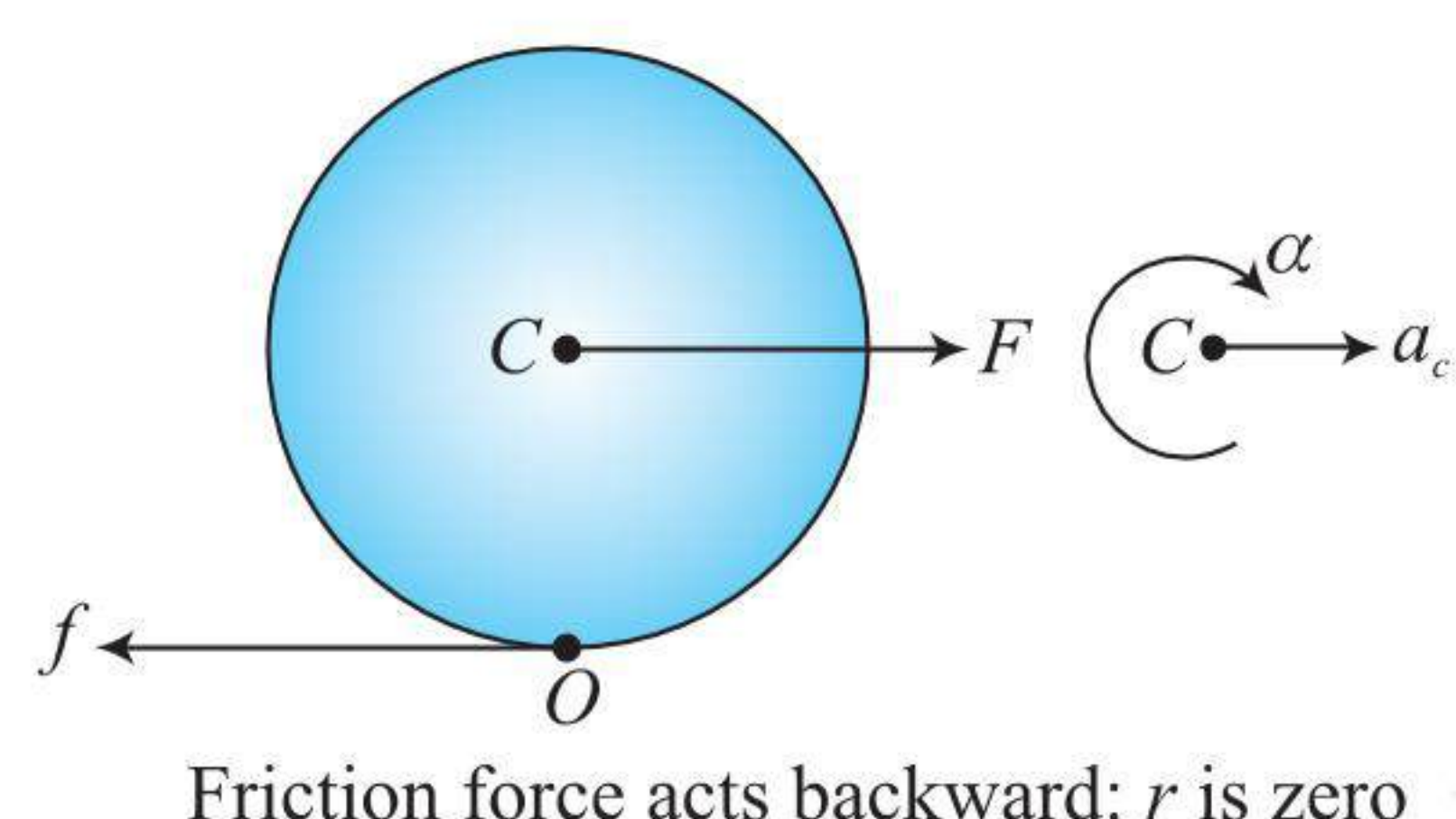
$$\text{or } a_c = \frac{F}{M} \left[\frac{\left(1 + \frac{r}{R}\right)}{\left(1 + \frac{I_c}{MR^2}\right)} \right] \quad \dots(v)$$

Substituting the value of a_c in Eq. (i), we get force of friction,

$$f = F \left[\frac{I_c - MrR}{I_c + MR^2} \right] \quad \dots(vi)$$

Note: We can find the values of a_c and f by using Eqs. (i) and (ii). But by taking torque equation about point of contact, the calculation steps are reduced.

(a) If force acts through C , i.e., $r = 0$:



$$\text{From Eq. (v), we get } a_c = \frac{F}{M} \left[\frac{1}{1 + \frac{I_c}{MR^2}} \right]$$

$$\text{From Eq. (vi), we get } f = F \left[\frac{1}{1 + \frac{MR^2}{I_c}} \right]$$

From the values of a_c and f , it is clear that

- body moves forward
- friction acts backward

(b) Force acts above C , i.e., r is positive:

From Eqs. (v) and (vi), it is clear that

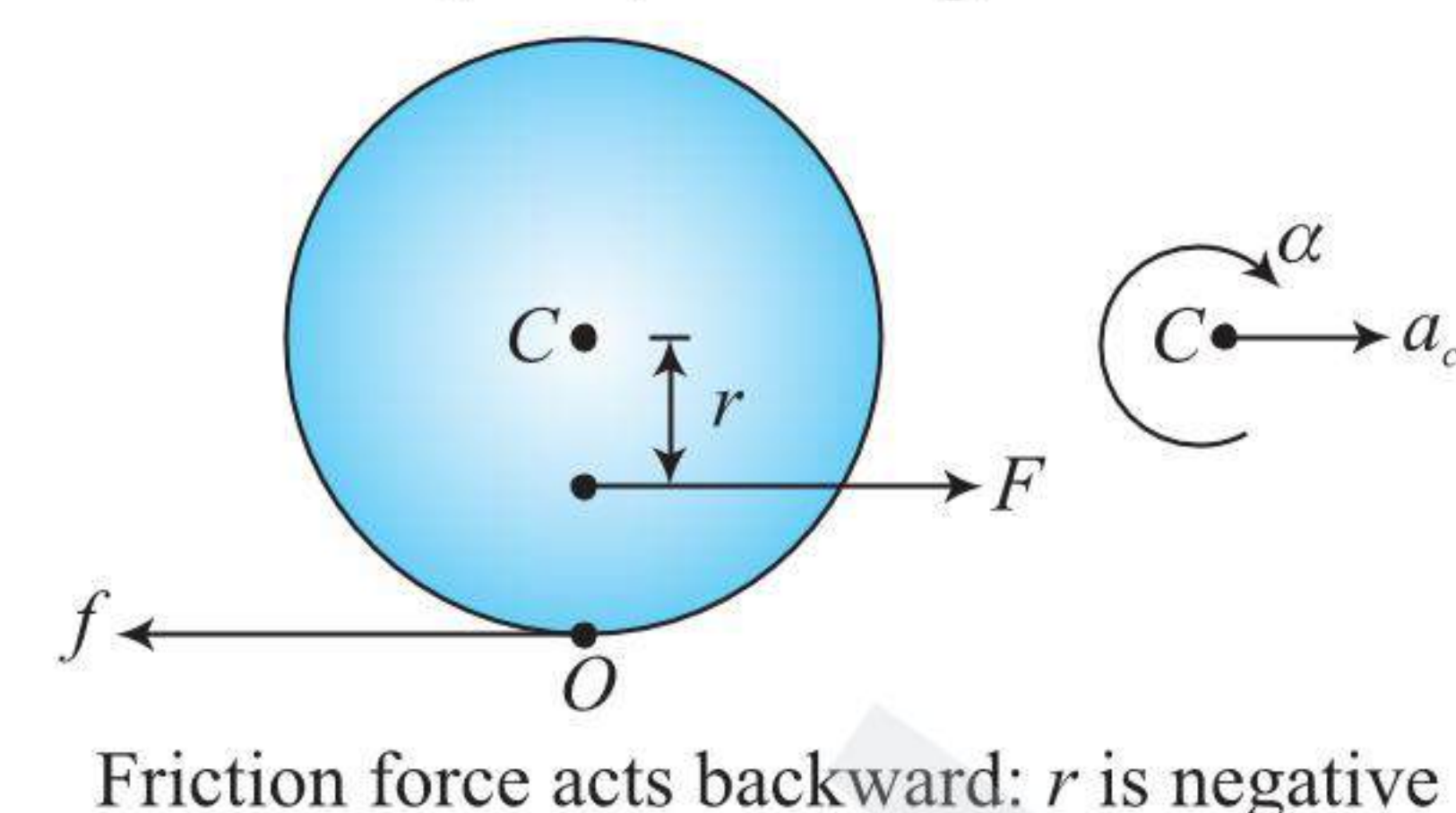
- the body moves forward
- rotation about the centre of mass is clockwise
- the friction force may act forward or backward

Backward, if $r < \frac{I_c}{MR}$

Forward, if $r > \frac{I_c}{MR}$

No friction force acts when $r = \frac{I_c}{MR}$

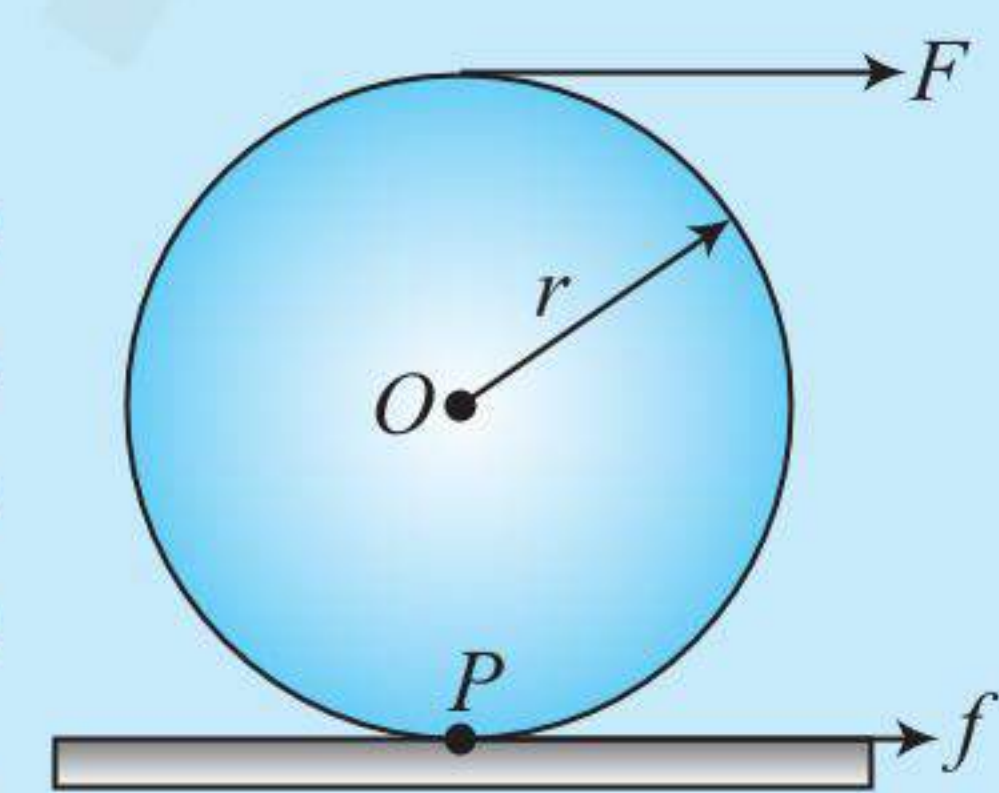
(c) Force acts below C , i.e., r is negative:



- The body moves forward.
- Rotation about the centre of mass is clockwise.
- Friction force acts backward ($f < F$).

ILLUSTRATION 4.30

A force F acts tangentially at the highest point of a sphere of mass m kept on a rough horizontal plane. If the sphere rolls without slipping, find the acceleration of the centre of the sphere.



Sol. The situation is shown in figure. As force F rotates the sphere, the point of contact has a tendency to slip towards left so that the static friction on the sphere will act towards right. Let r be the radius of the sphere and a be the linear acceleration of the centre of the sphere. The angular acceleration about the centre of the sphere is $\alpha = a/r$, as there is no slipping.

For the linear motion of the centre,

$$F + f = ma \quad \dots(i)$$

and for the rotational motion about the centre,

$$Fr - fr = I\alpha = \left(\frac{2}{5}mr^2\right)\frac{a}{r}$$

$$\text{or } F - f = \frac{2}{5}ma \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$2F = \frac{7}{5}ma \text{ or } a = \frac{10F}{7m}$$

Alternative method: As the sphere is rolling, we can apply torque equation about point of contact.

$$\text{i.e., } F(2r) = I_P \alpha = \left(\frac{2}{5}mr^2 + mr^2\right)\alpha$$

$$\Rightarrow F(2r) = \left(\frac{7}{5}mr^2\right)\alpha$$

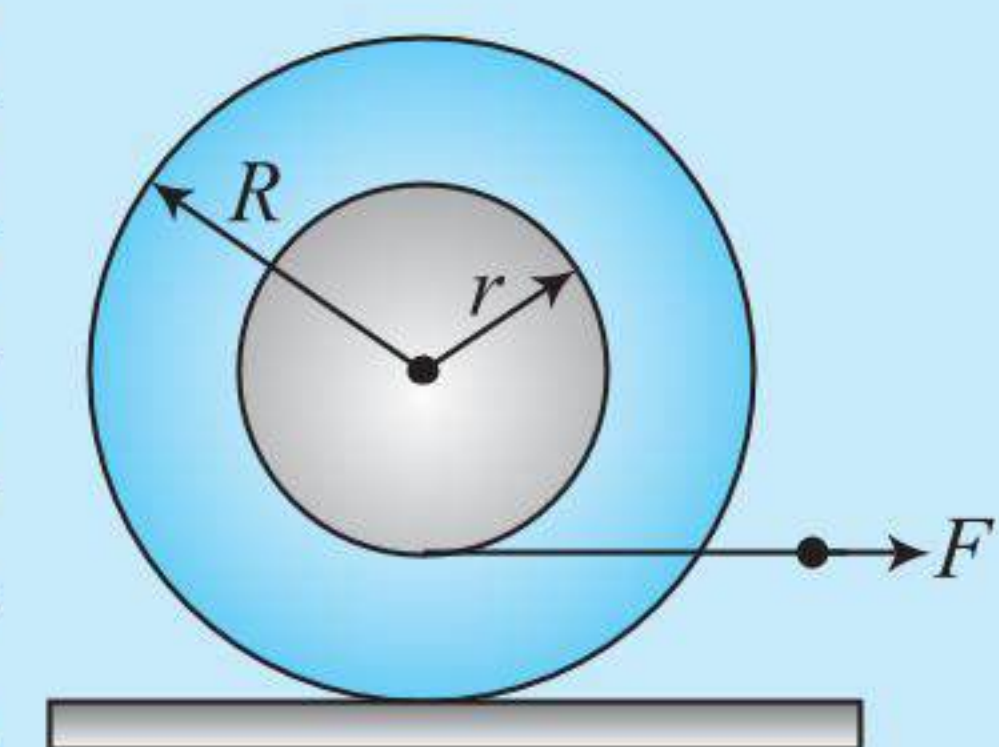
$$\text{Hence, angular acceleration } \alpha = \frac{10F}{7mr} \quad \dots(iii)$$

As sphere is rolling, acceleration of centre of mass is

$$a = \alpha r = \frac{10F}{7m}$$

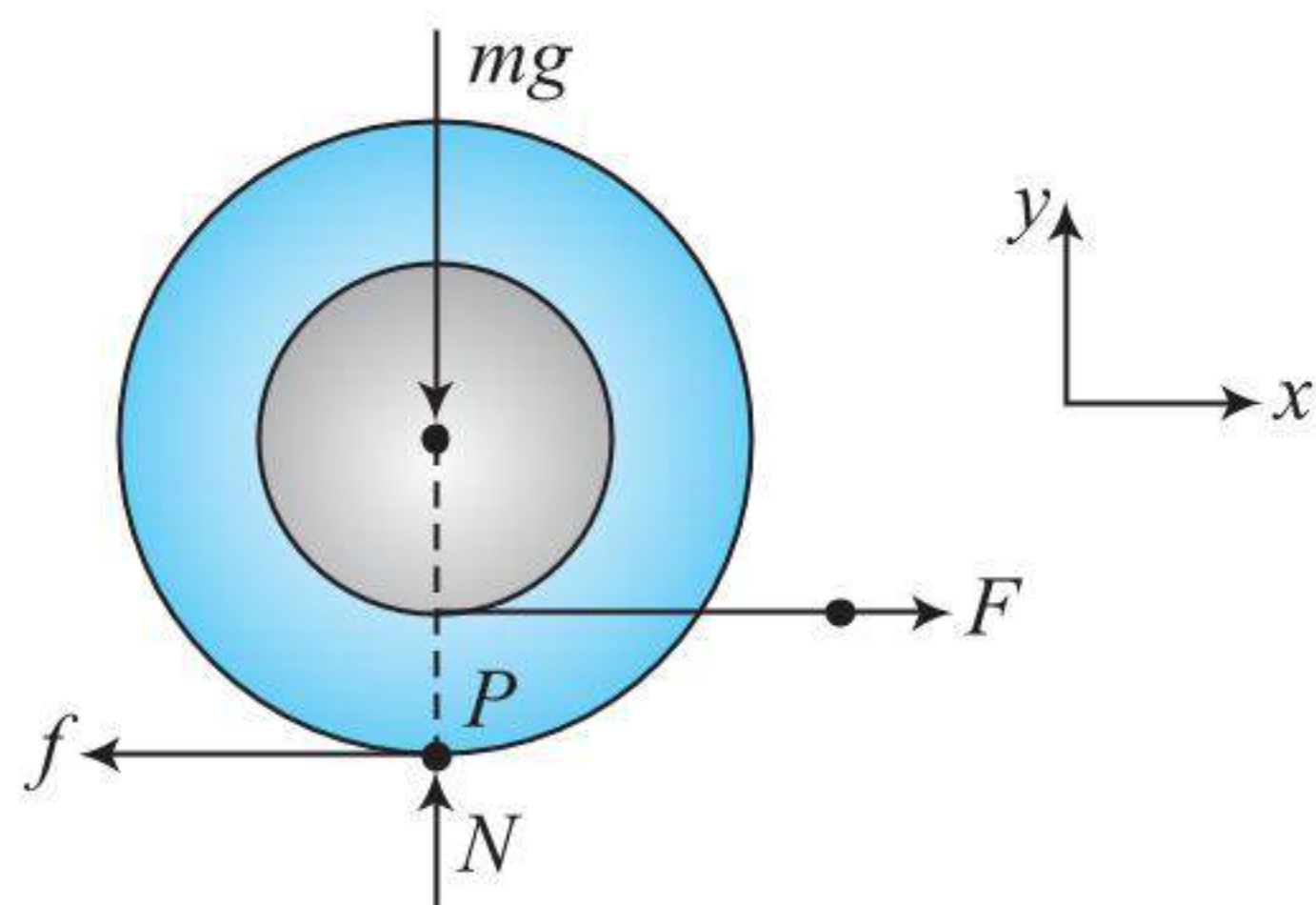
ILLUSTRATION 4.31

A cotton reel has inner radius r and outer radius R . Mass of the reel is M and moment of inertia about longitudinal rotational axis is I . A force F is applied at the free end of thread wrapped on the reel as shown in figure. If the reel rolls without sliding.



- (a) Determine the frictional force exerted by the table on the reel and the direction in which it acts.
 (b) In what direction does the reel begin to move?

Sol. Method 1: As the body rolls, point P will be instantaneous centre of rotation (ICR). There is clockwise torque about ICR and centre of mass will have acceleration in right direction. Hence the reel will rotate in clockwise direction and moves to the right. Force F exerts anticlockwise torque; therefore, friction force must exert a larger clockwise torque to produce clockwise rolling. The equations of the motion are:



$$\sum F_x = F - f = Ma \quad \dots(i)$$

$$\sum F_y = N - Mg = 0 \quad \dots(ii)$$

$$\sum \tau = f \times R - F \times r = I\alpha \quad \dots(iii)$$

As the reel rolls without slipping, $a = R\alpha$

From Eqs. (i) and (iii), we get

$$\frac{F - f}{M} = \frac{R(fR - Fr)}{I} \quad \text{or} \quad f = \frac{F(I + MR^2)}{I + MR^2}$$

Force of friction comes out to be positive, therefore our assumption about the direction of friction force was correct.

On substituting the expression for f in Eq. (i), we obtain

$$a = \frac{FR(R - r)}{I + MR^2}$$

As $R > r$, a is positive and towards right. Similarly, from Eq. (iii), we obtain

$$\alpha = \frac{F(R - r)}{I + MR^2}$$

which is positive, i.e., net torque on the reel is clockwise.

Method 2: We can apply torque equation about ICR

$$\Rightarrow F(R - r) = I_P \alpha$$

$$F(R - r) = [I + MR^2] \alpha$$

$$\Rightarrow \alpha = \frac{F(R - r)}{[I + MR^2]}$$

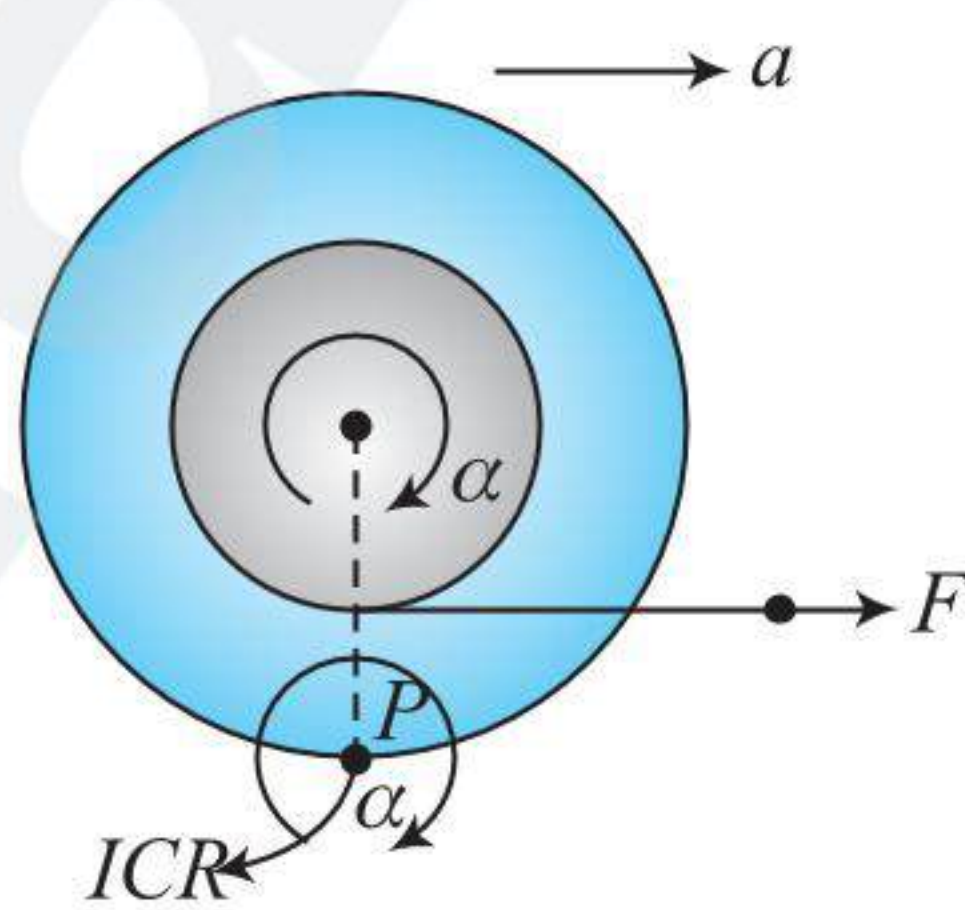
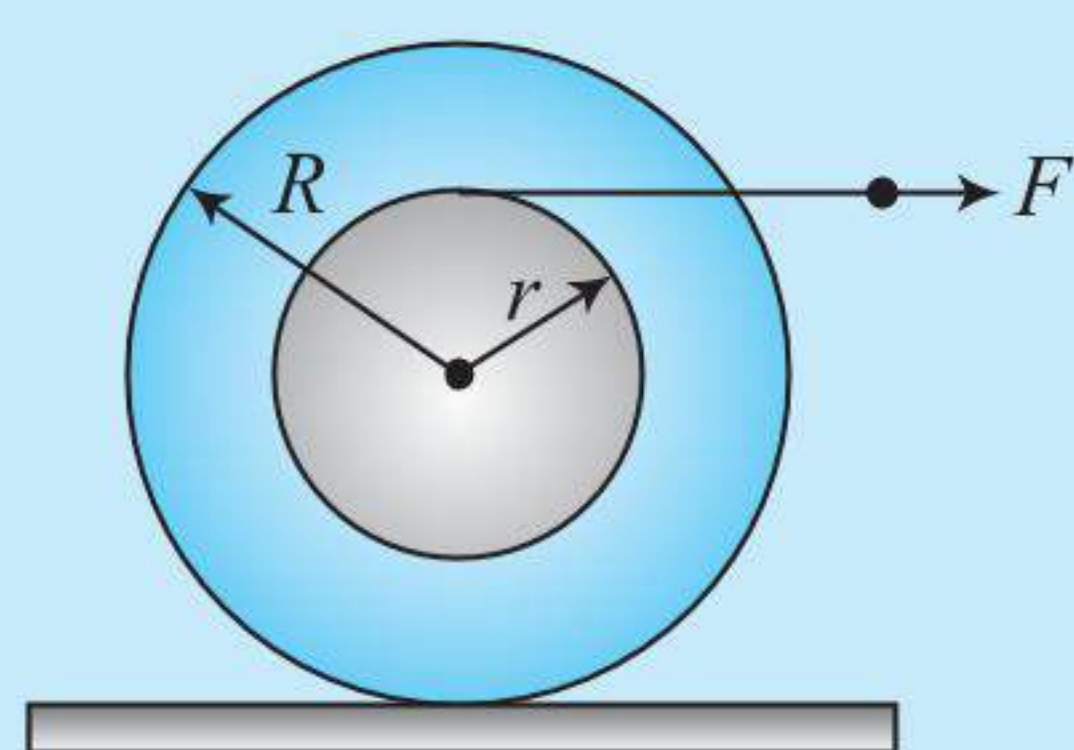


ILLUSTRATION 4.32

A wheel of radius R , mass m and moment of inertia I is pulled along a horizontal surface by application of force F to a rope unwinding from the axle of radius r as shown in figure. Friction is sufficient for pure rolling of the wheel.



- (a) What is the linear acceleration of the wheel?
 (b) Calculate the frictional force that acts on the wheel.

Sol. The equations of the motion of the wheel are:

$$F - f = ma \quad \dots(i)$$

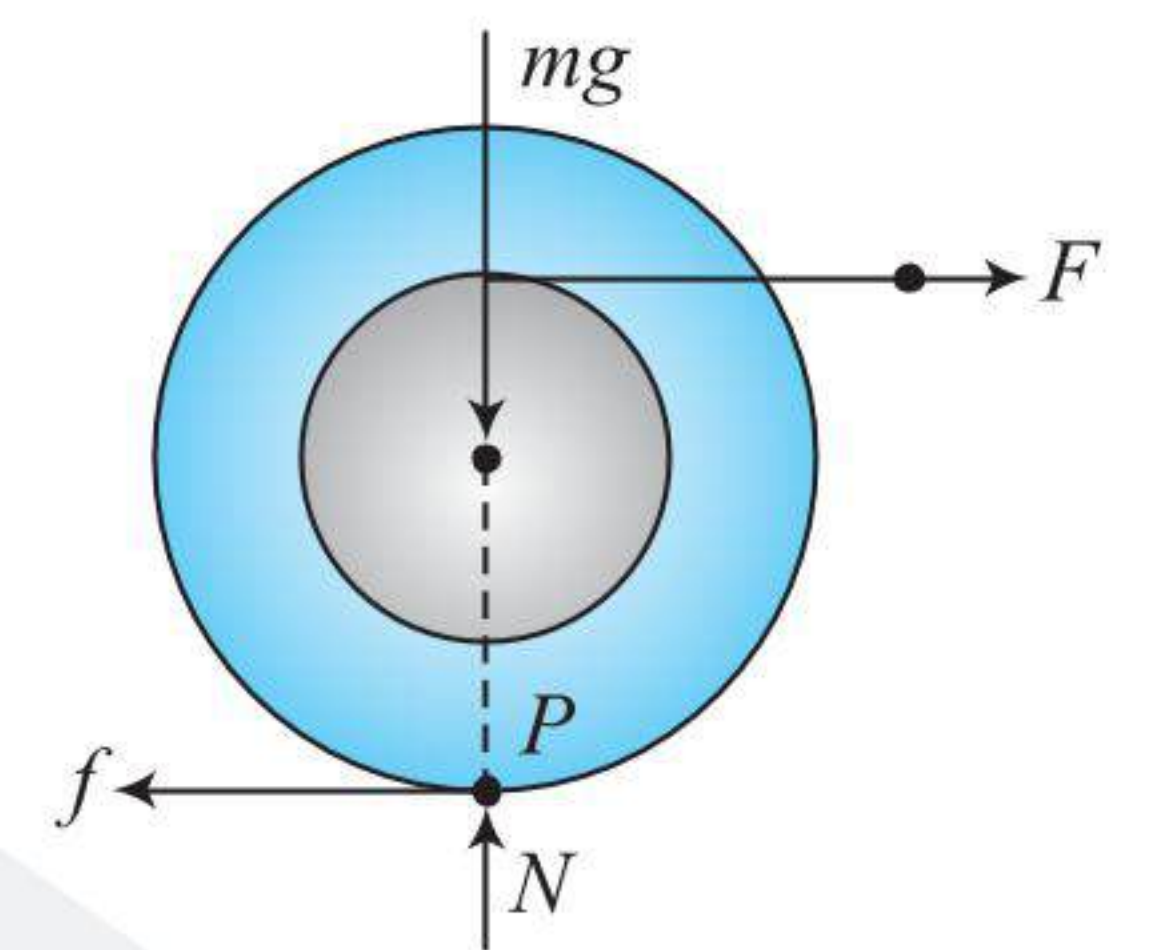
$$F \times r + f \times R = I\alpha \quad \dots(ii)$$

In case of pure rolling,

$$a = R\alpha \quad \dots(iii)$$

$$F \times r + (F - ma)R = \frac{I}{R} a$$

$$\text{or } a = \frac{F(r + R)R}{(I + mR^2)}$$



We can find acceleration of centre of wheel by another approach also. The point of contact will be instantaneous centre of rotation (ICR), and we apply torque equation about ICR.

$$F(R + r) = (I + mR^2)\alpha \Rightarrow \alpha = \frac{F(R + r)}{(I + mR^2)}$$

$$\text{Acceleration of centre of mass, } a = \alpha R = \frac{F(R + r)R}{(I + mR^2)}$$

and the frictional force is

$$f = F - ma = F \left[1 - \frac{(r + R)R}{\left(\frac{I}{m} + R^2\right)} \right] = \frac{F[(I/m) - Rr]}{[(I/m) + R^2]}$$

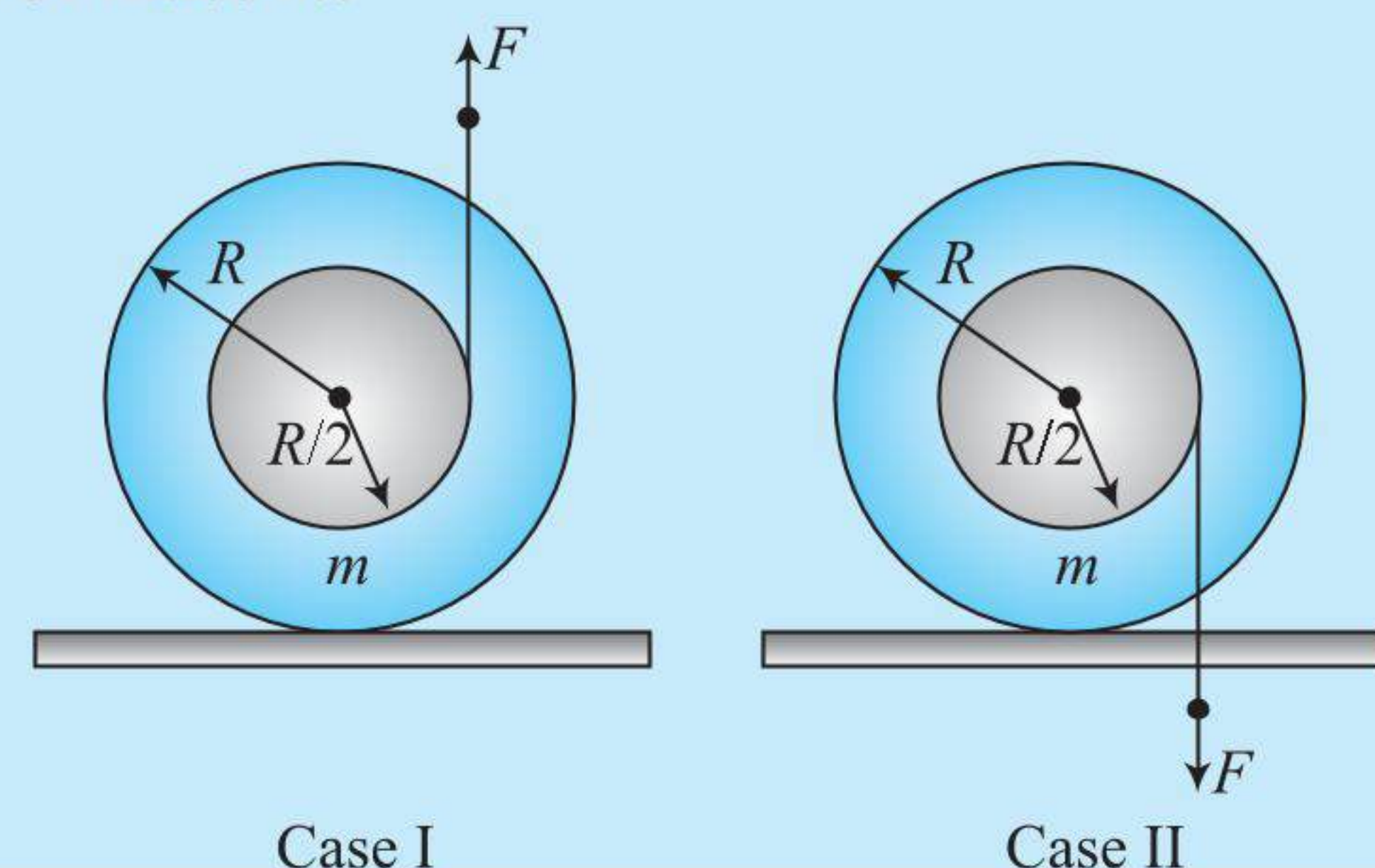
For $I/m = Rr$, frictional force is zero.

For $I/m > Rr$, frictional force is positive, i.e., it acts in backward direction.

For $I/m < Rr$, frictional force is negative, i.e., it acts in forward direction.

ILLUSTRATION 4.33

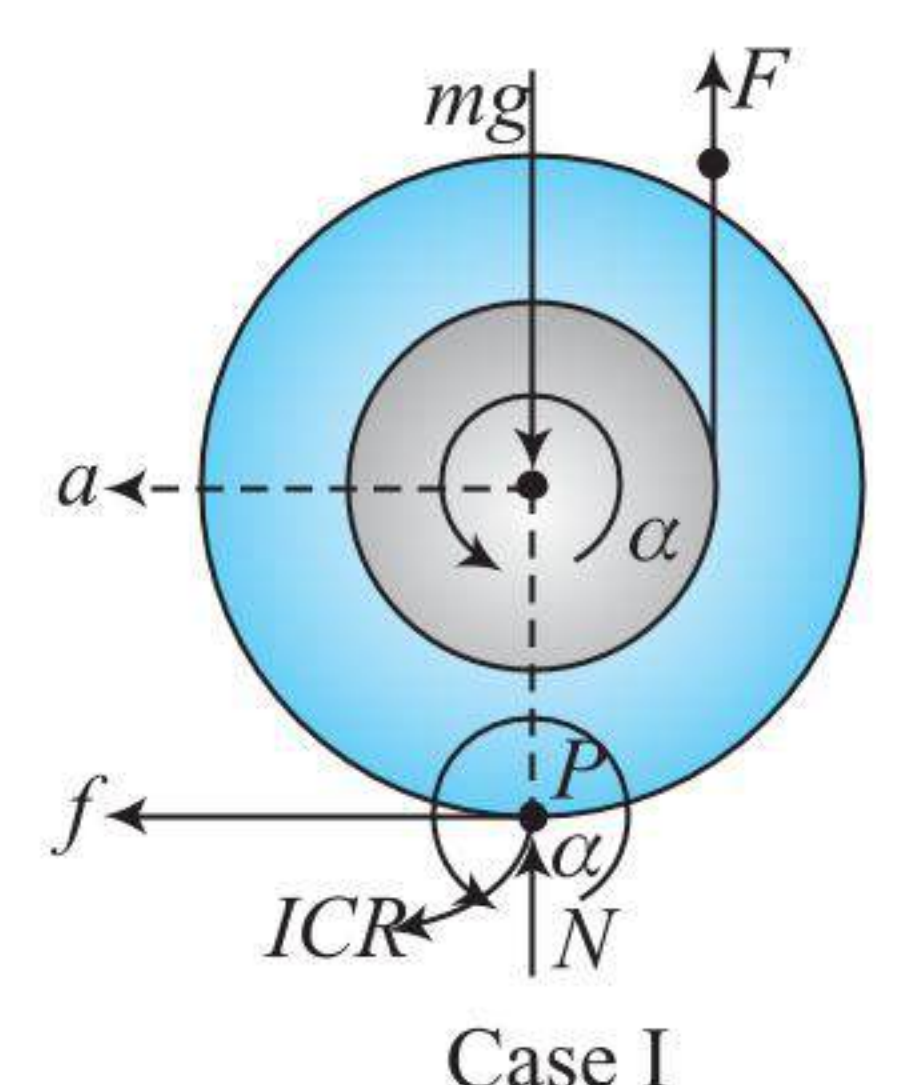
A force F is applied on a spool (mass m and radius R) in vertical direction as shown in the case (I) and case (II). The radius of gyration k of spool is $R/\sqrt{2}$. Assuming pure rolling, find the acceleration of the CM of the body, assuming it as a uniform cylinder of mass m .



Sol. As the cylinder is rolling. The point of contact will be instantaneous centre of rotation (ICR). The rolling body can be considered as pure rotation about the point of contact.

Case I: The force F provides anticlockwise torque about P . The cylinder will rotate in anticlockwise direction about P . The point P is instantaneous centre of rotation (ICR).

The cylinder will also rotate in anticlockwise direction about its centre of mass. The cylinder will accelerate in left direction. Hence friction will act in left direction.



Applying torque equation about P

$$F\left(\frac{R}{2}\right) = I_P \alpha = m(k^2 + R^2) \alpha$$

$$\text{or } F\left(\frac{R}{2}\right) = m\left(\frac{R^2}{2} + R^2\right) \alpha = \frac{3}{2} m R^2 \alpha$$

$$\text{which gives } R\alpha = \frac{F}{3m}$$

Hence acceleration of cylinder is $a = \frac{F}{3m}$ (towards left).

Case II: In this case the torque of F about instantaneous centre of rotation (ICR) is in clockwise direction.

The cylinder will also rotate in clockwise direction about its centre of mass. Hence, the acceleration of cylinder will be towards right and friction will act towards right.

Now the torque equation about P will be same as we got in previous case. We will get the same value of acceleration, but direction will be towards right.

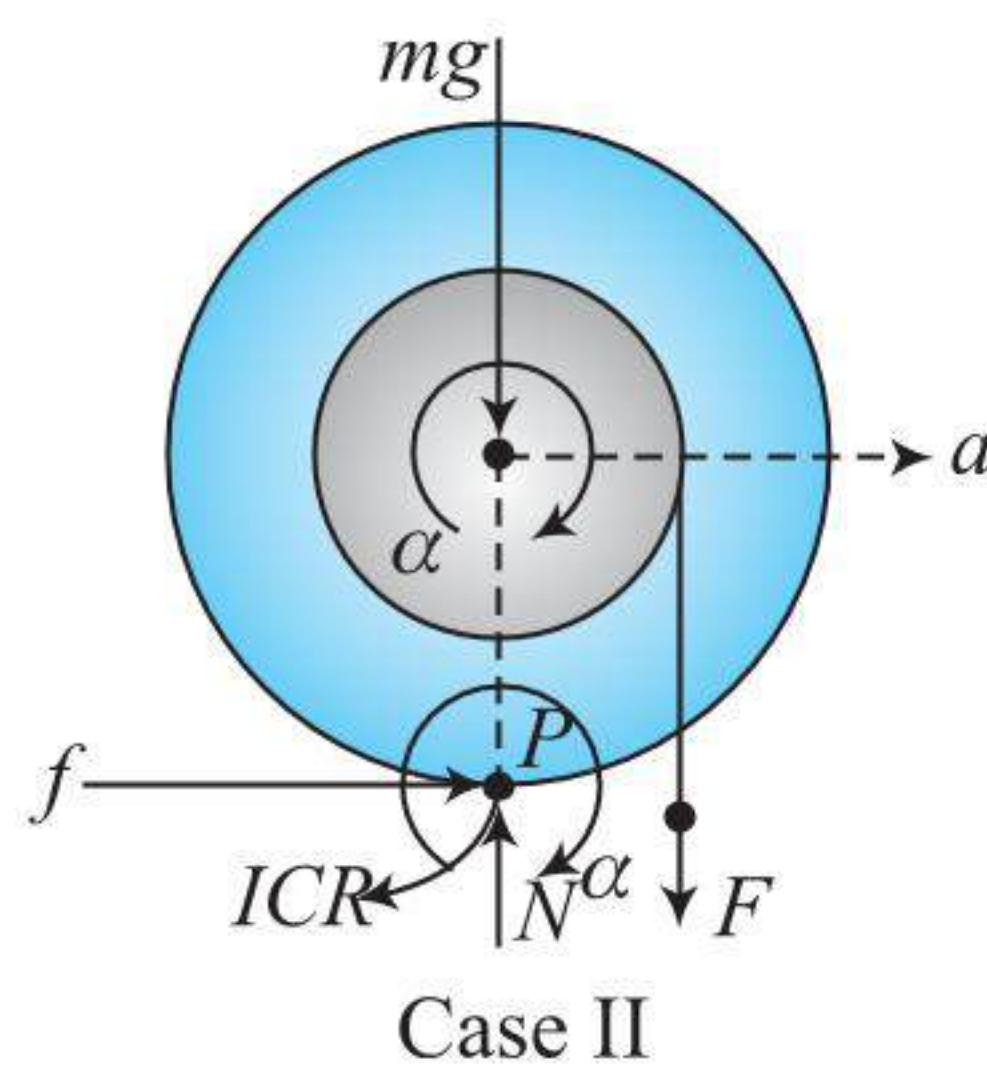
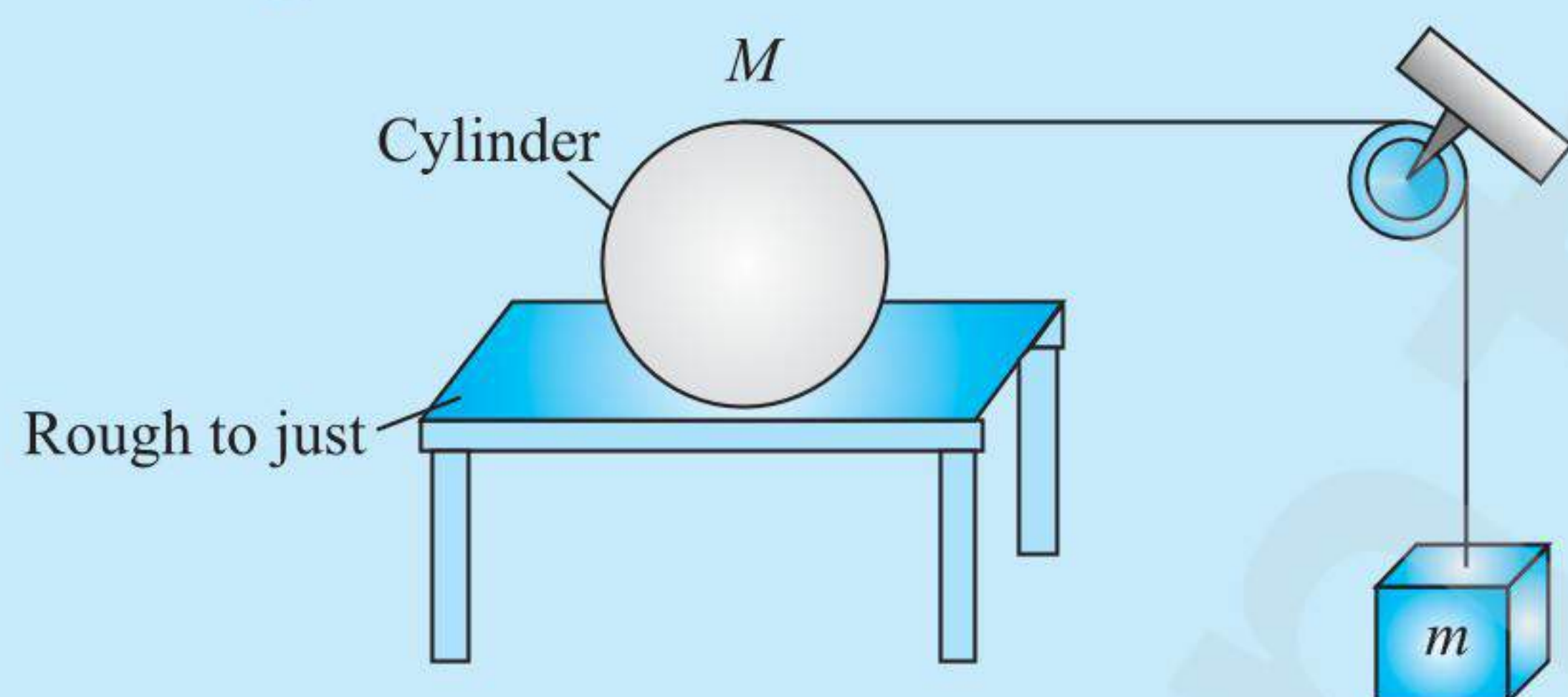


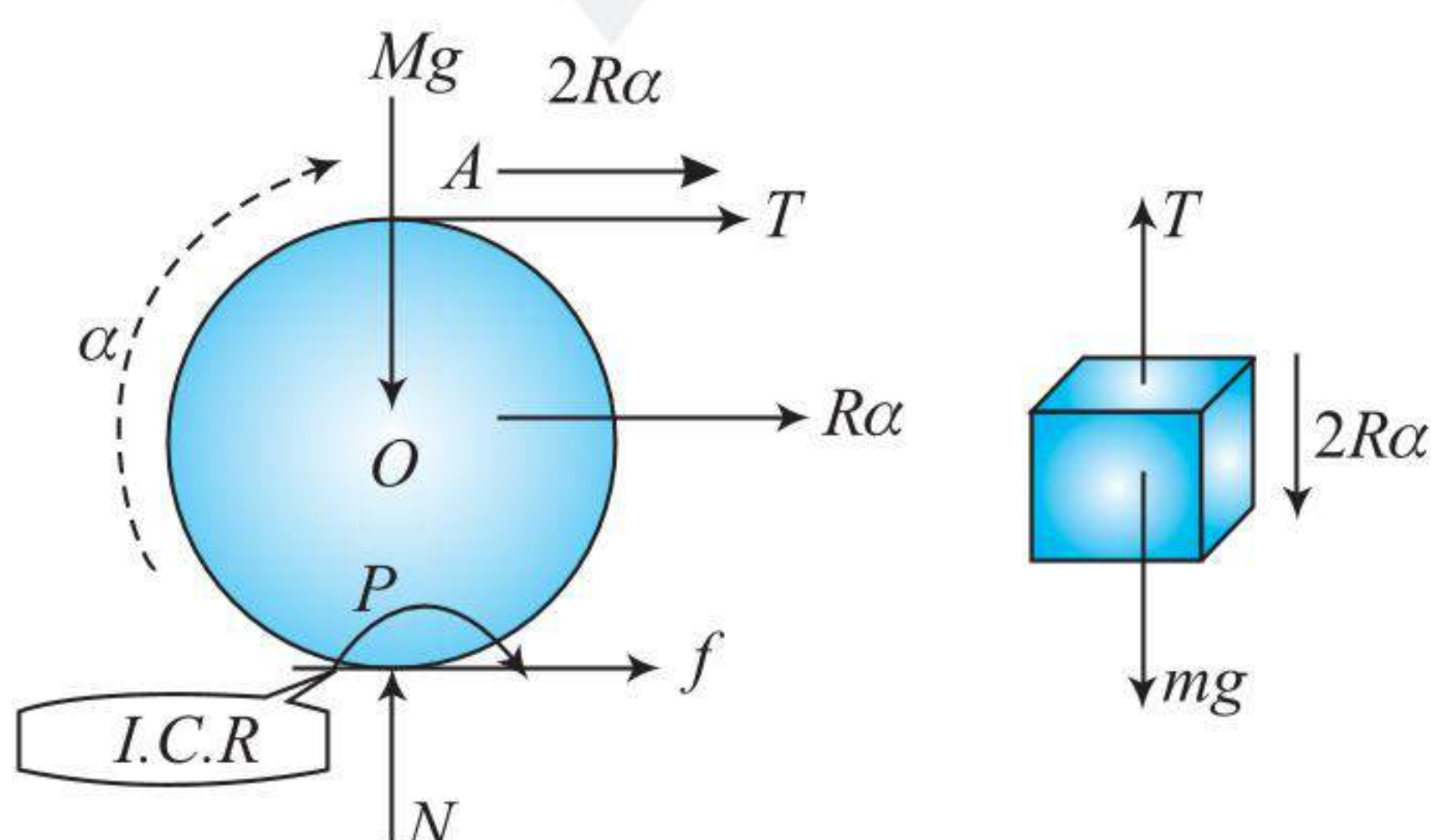
ILLUSTRATION 4.34

An inextensible thread is wound round a cylinder on mass M , the cylinder is placed over a rough horizontal surface and the thread is passed over massless and frictionless pulley such that the part of the thread between cylinder and pulley is horizontal as shown in figure.



When a block of mass m is attached with free end of the thread and system is released, it is observed that friction just prevents slipping of the cylinder over the horizontal surface. Find minimum coefficient of friction between horizontal surface and cylinder.

Sol. As the cylinder does not slip over the horizontal surface, the friction will be static in nature. Let us assume it is acting in forward direction. The instantaneous axis of rotation will pass through the lowest point P . Let angular acceleration of the cylinder is α , (clockwise) then acceleration of its centre will be equal to $R\alpha$ horizontally rightwards and that of block will be $2R\alpha$ vertically downward (refer figure).



We can apply torque equation about ICR. Writing torque equation about P ,

$$T \cdot 2R = I_P \cdot \alpha$$

where I_P is moment of inertia of the cylinder about P

$$T \cdot 2R = \left(\frac{3MR^2}{2}\right) \cdot \alpha \Rightarrow T = \frac{3}{4} MR\alpha \quad \dots(i)$$

Now considering FBD's

Force equation of the block,

$$mg - T = m(2R\alpha) \quad \dots(ii)$$

From Eqs. (i) and (ii)

$$R\alpha = \frac{mg}{\left(2m + \frac{3}{4}M\right)} \quad \dots(iii)$$

Force equation for cylinder,

$$T + f = M \cdot (R\alpha) \quad \dots(iv)$$

From (i), (iii) and (iv), we get

$$f = \frac{Mmg}{(8m + 3M)}$$

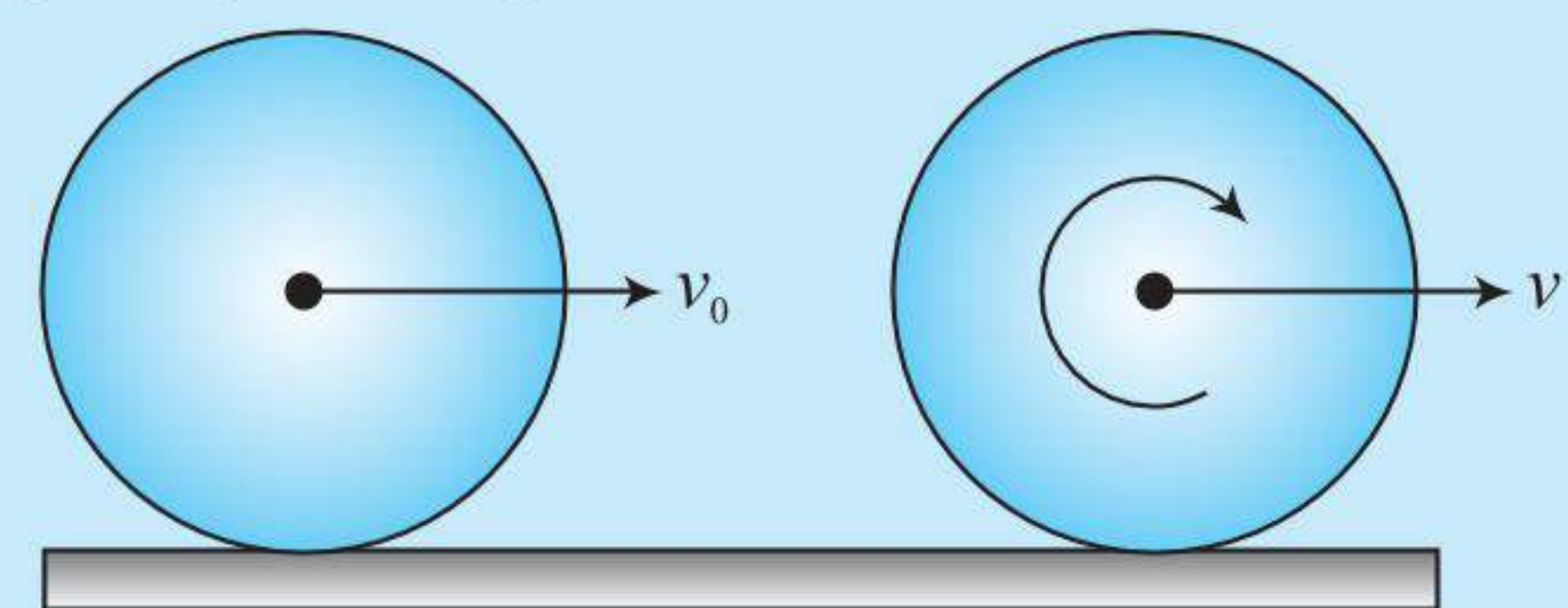
As friction is static nature,

$$f \leq \mu N \Rightarrow \frac{Mmg}{(8m + 3M)} \leq \mu Mg$$

$$\text{or } \mu \geq \frac{m}{(8m + 3M)}$$

ILLUSTRATION 4.35

A uniform disc of mass m and radius r is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at $t = 0$. At $t = t_0$ seconds it acquires a purely rolling motion.



(a) Calculate the velocity of the centre of mass of the disc at $t = t_0$.

(b) Assuming coefficient of friction to be μ , calculate t_0 .

Sol. When the disc is projected it starts sliding and hence there is a relative motion between the points of contact. Therefore, frictional force acts on the disc in the direction opposite to the motion.

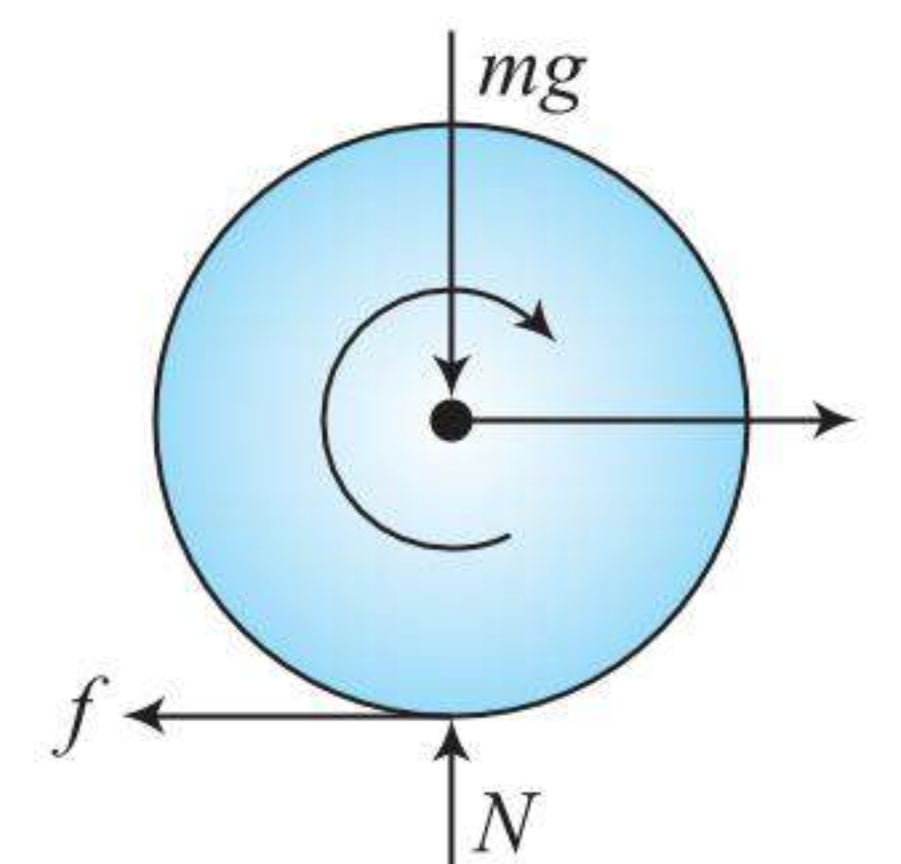
(a) Now for translational motion:

$$\vec{a}_{cm} = -\frac{\vec{f}}{m}$$

$$f = \mu N \text{ (as it slides)} \Rightarrow f = \mu mg$$

$$\Rightarrow a_{cm} = -\mu g, \text{ negative sign indicates that } a_{cm} \text{ is opposite to } v_{cm}$$

$$\Rightarrow v_{cm}(t) = v_0 - \mu g t$$



$$\text{At } t = t_0, t_0 = \frac{(v_0 - v)}{\mu g}, \text{ where } v_{\text{cm}}(t_0) = v \quad \dots (i)$$

For rotational motion about centre,

$$\tau_f = I_{\text{cm}} \alpha$$

$$\mu m g r = \frac{m r^2}{2} \alpha \Rightarrow \alpha = \frac{2 \mu g}{r} \quad \dots (ii)$$

$$\text{Therefore } \omega(t_0) = 0 + \frac{2 \mu g}{r} t_0,$$

$$\text{Using (i), } \omega = \frac{2(v_0 - v)}{r} \quad \dots (iii)$$

$$\text{For pure rolling } v_{\text{cm}} = \omega r \Rightarrow v = 2(v_0 - v) \text{ using (iii)}$$

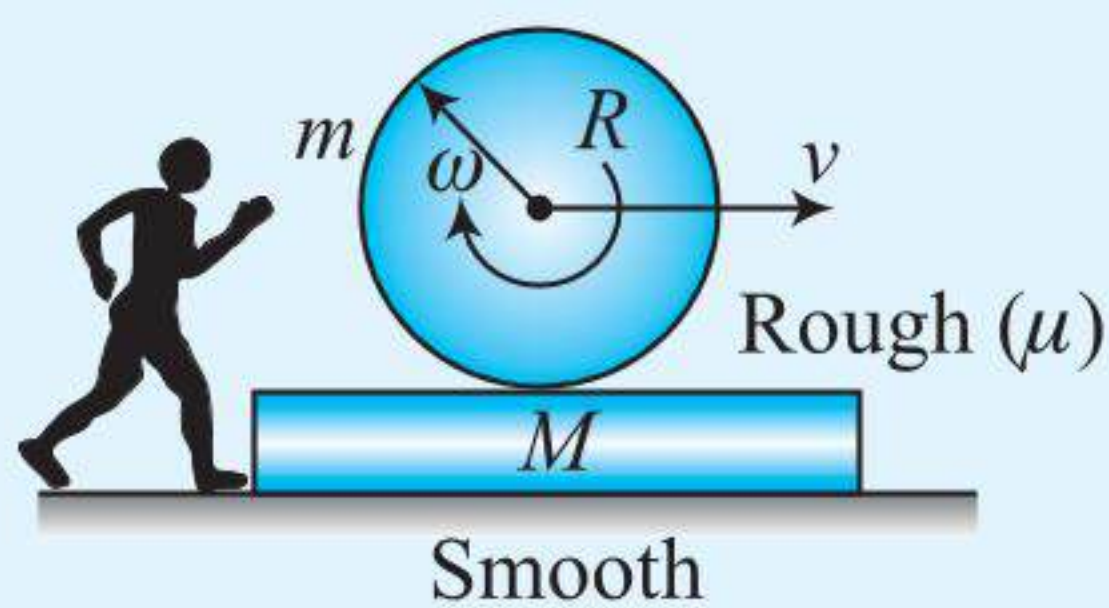
$$\Rightarrow v = \frac{2}{3} v_0$$

(b) Putting the value of v in Eq. (i), we get

$$t_0 = \frac{v_0}{3 \mu g}$$

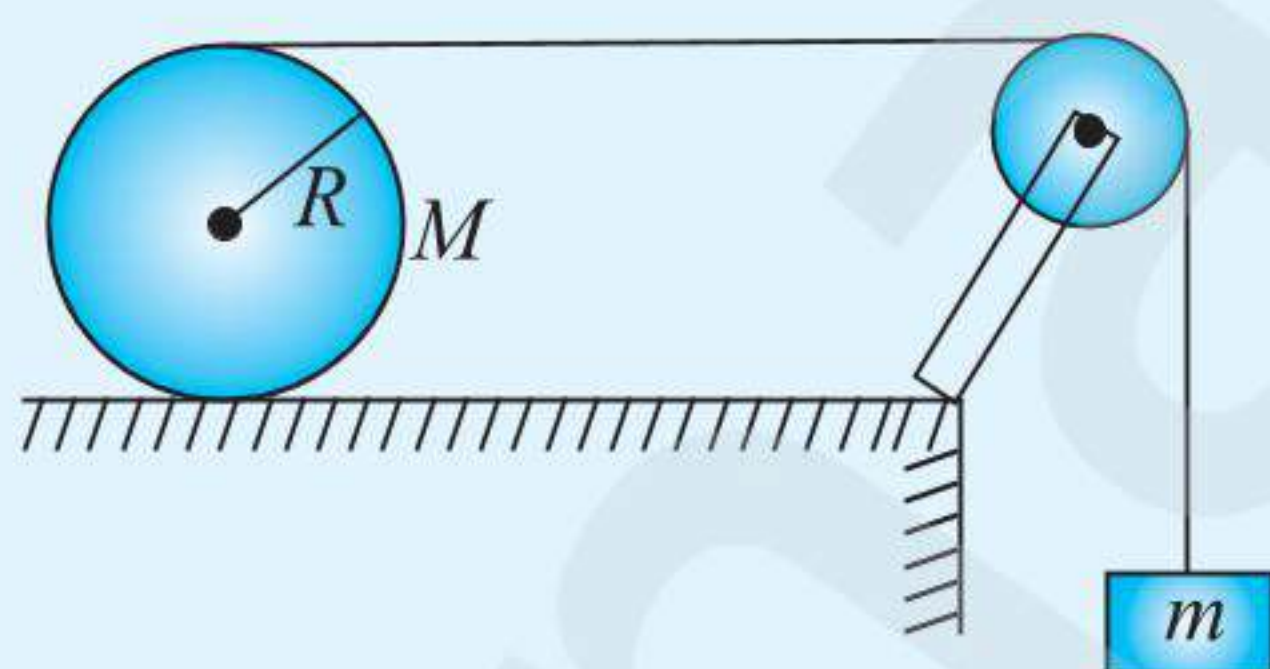
CONCEPT APPLICATION EXERCISE 4.4

1. A cylinder of mass m and radius R rolls on a stationary plank of mass M . The lower surface of the plank is smooth and the upper surface is sufficiently rough with a coefficient of friction μ . A man is to hold the plank stationary with respect to the ground, as shown figure.

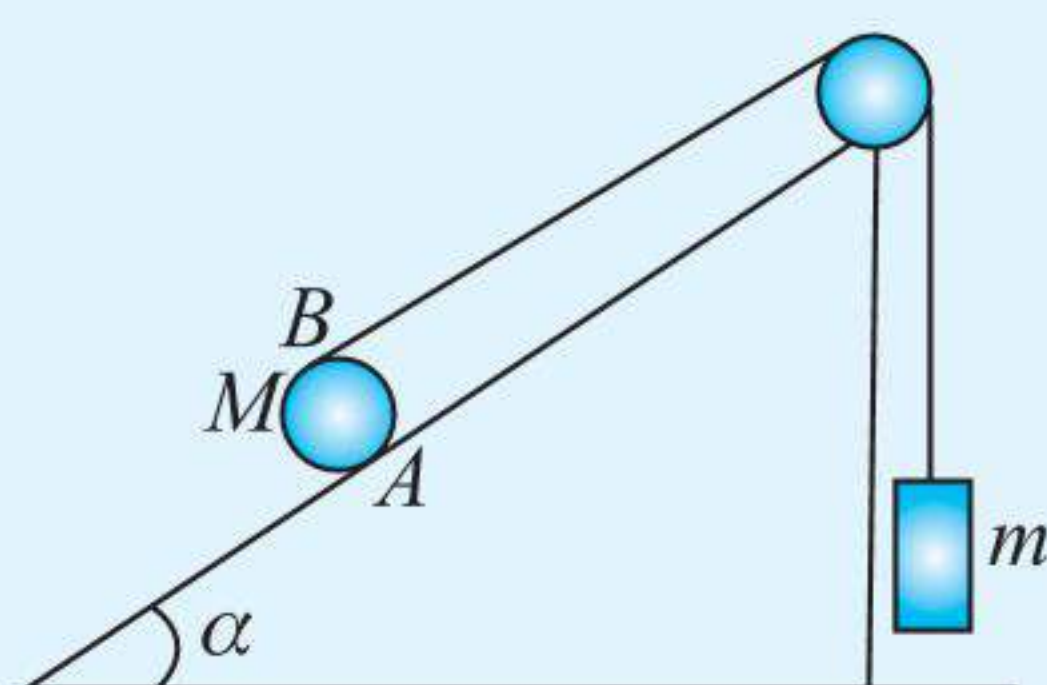


Find the force exerted by the man to keep the plank stationary.

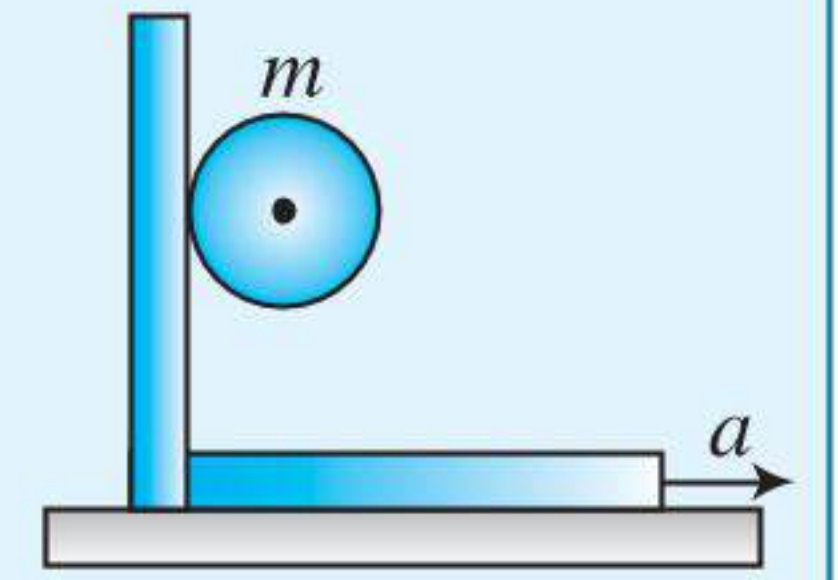
2. A load of mass m is attached to the end of a string wound on a cylinder of mass M and radius R . The string passes round a pulley.



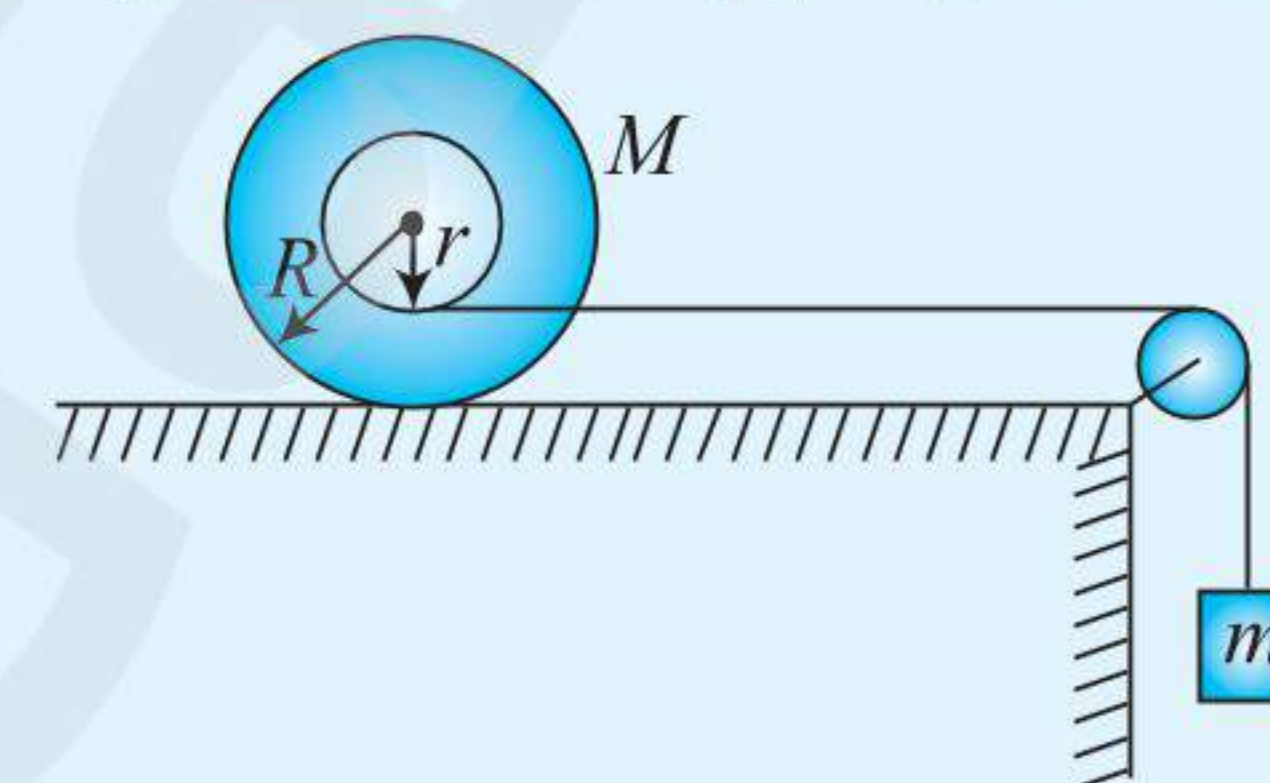
- (a) Find the acceleration of the cylinder M when it rolls.
 - (b) If the coefficient of friction is μ . Find the acceleration of the cylinder when it slips and rolls.
 - (c) Find the minimum value of coefficient of friction for which the cylinder rolls always.
3. Find the tension in the tape and the linear acceleration of the cylinder up the incline, assuming there is no slipping. (Take $M = 4m$ and $g = 10 \text{ m/s}^2$)



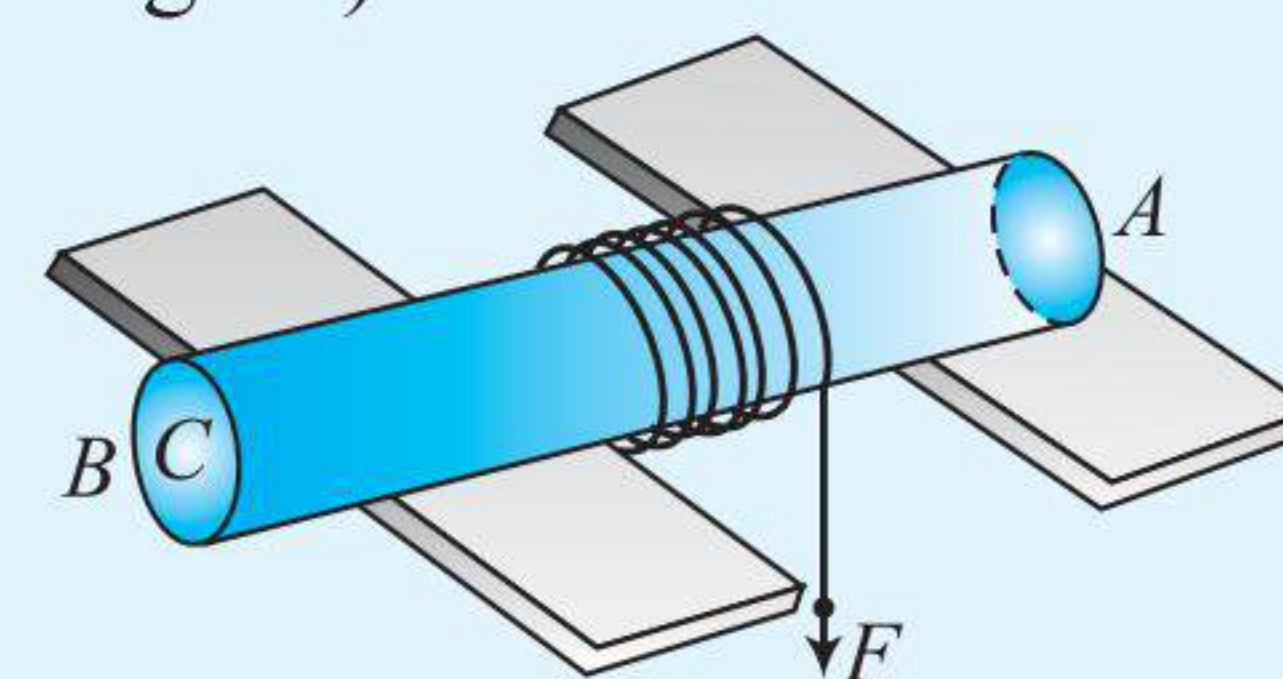
4. A uniform solid sphere rolls down a vertical surface without sliding. If the vertical surface moves with an acceleration $a = g/2$, find the minimum coefficient of friction between the sphere and vertical surfaces so as to prevent relative sliding.



5. A spool (consider it as a double disc system joined by a short tube at their centre) is placed on a horizontal surface as shown in figure. A light string wound several times over the short connecting tube leaves it tangentially and passes over a light pulley. A weight of mass m is attached to the end of the string. The radius of the connecting tube is r and mass of the spool is M and radius is R . Find the acceleration of the falling mass m . Neglect the mass of the connecting tube and slipping of the spool.

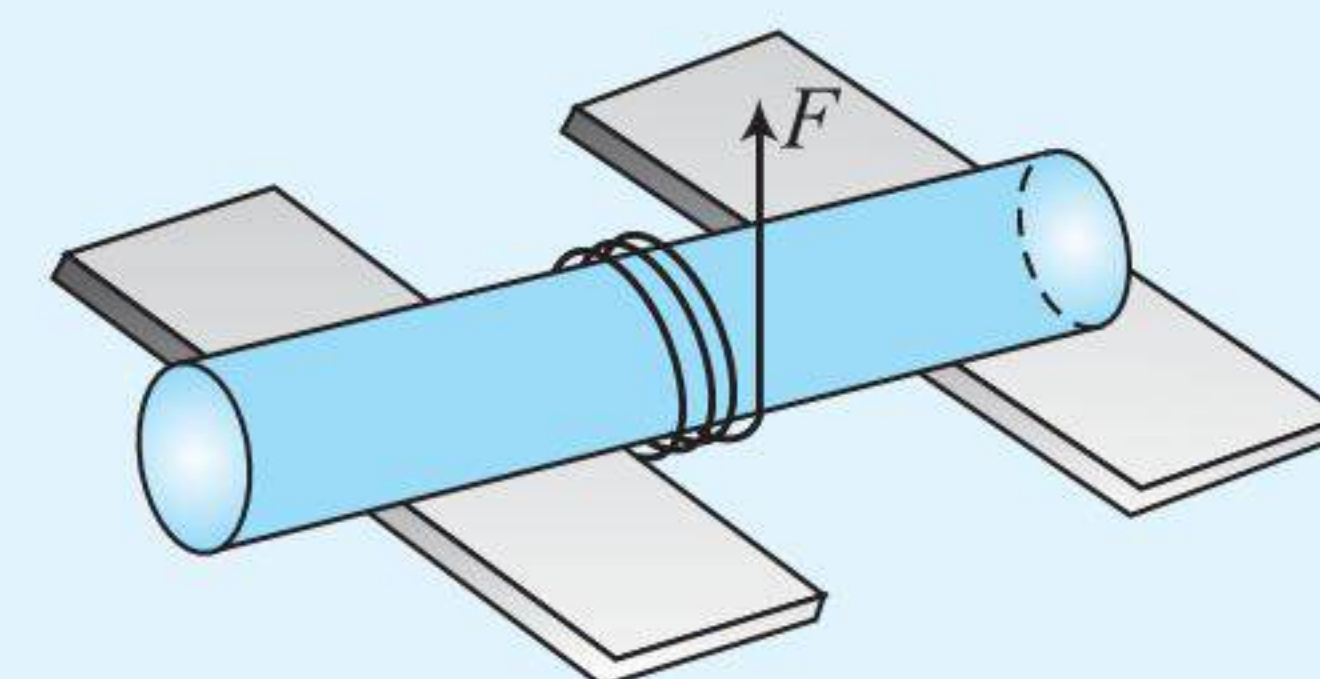


6. A solid cylinder wheel of mass M and radius R is pulled by a force F applied to the centre of the wheel at 37° to the horizontal. If the wheel is to roll without slipping, what is the maximum value of $|F|$? The coefficients of static and kinetic friction are $\mu_s = 0.40$ and $\mu_k = 0.30$. ($\sin 37^\circ = 3/5$).
7. A uniform solid cylinder of mass m rests on two horizontal planks. A thread is wound on the cylinder. The hanging end of the thread is pulled vertically down with a constant force F (see figure).

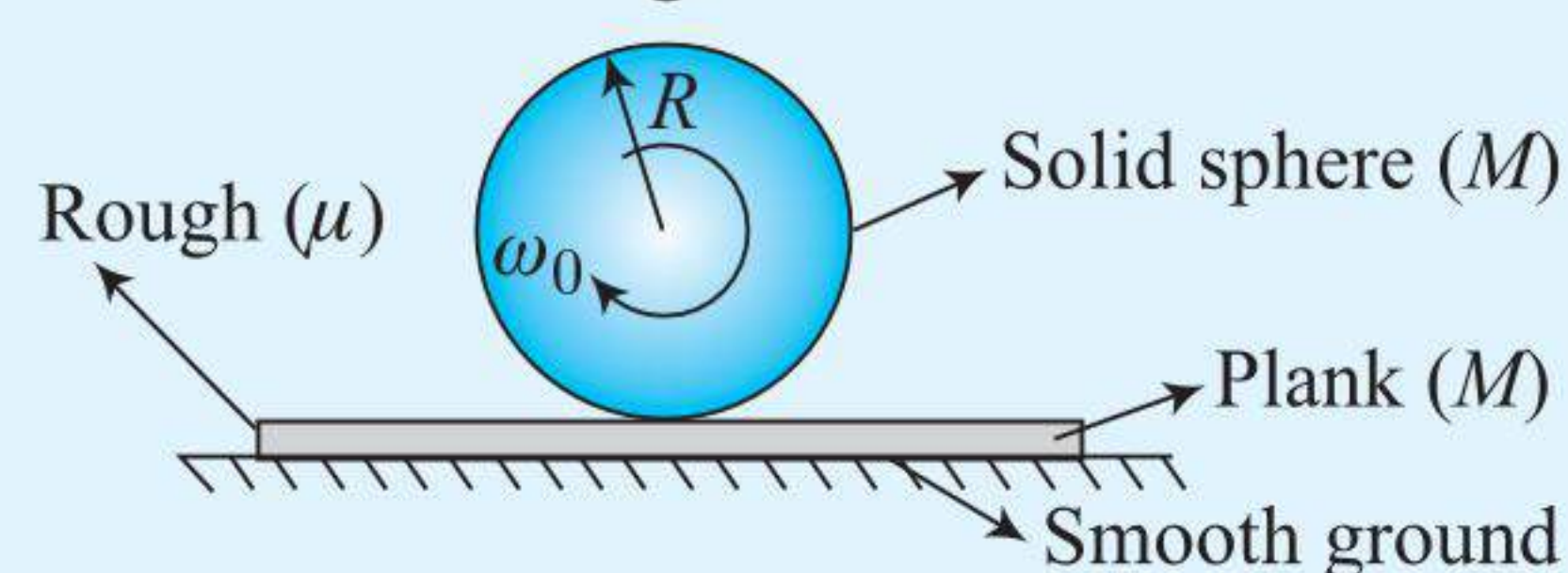


Find the maximum magnitude of the force F which may be applied without bringing about any sliding of the cylinder, if the coefficient of friction between the plank and the cylinder is equal to μ . What is the maximum acceleration of the centre of mass over the planks?

8. A uniform solid cylinder of mass m rests on two horizontal planks as shown in figure. A thread is wound on the cylinder. The hanging end of the thread is pulled vertically up with a force F . What is the maximum magnitude of the force F which still does not bring about any sliding of the cylinder, if the coefficient of friction between the cylinder and the planks is equal to μ . What is the maximum acceleration of the axis of the cylinder?



9. A solid sphere of mass M is placed on the top of a plank of the same mass, after giving an angular velocity ω_0 at $t = 0$. Find the velocity of the plank and the sphere when the sphere starts rolling.

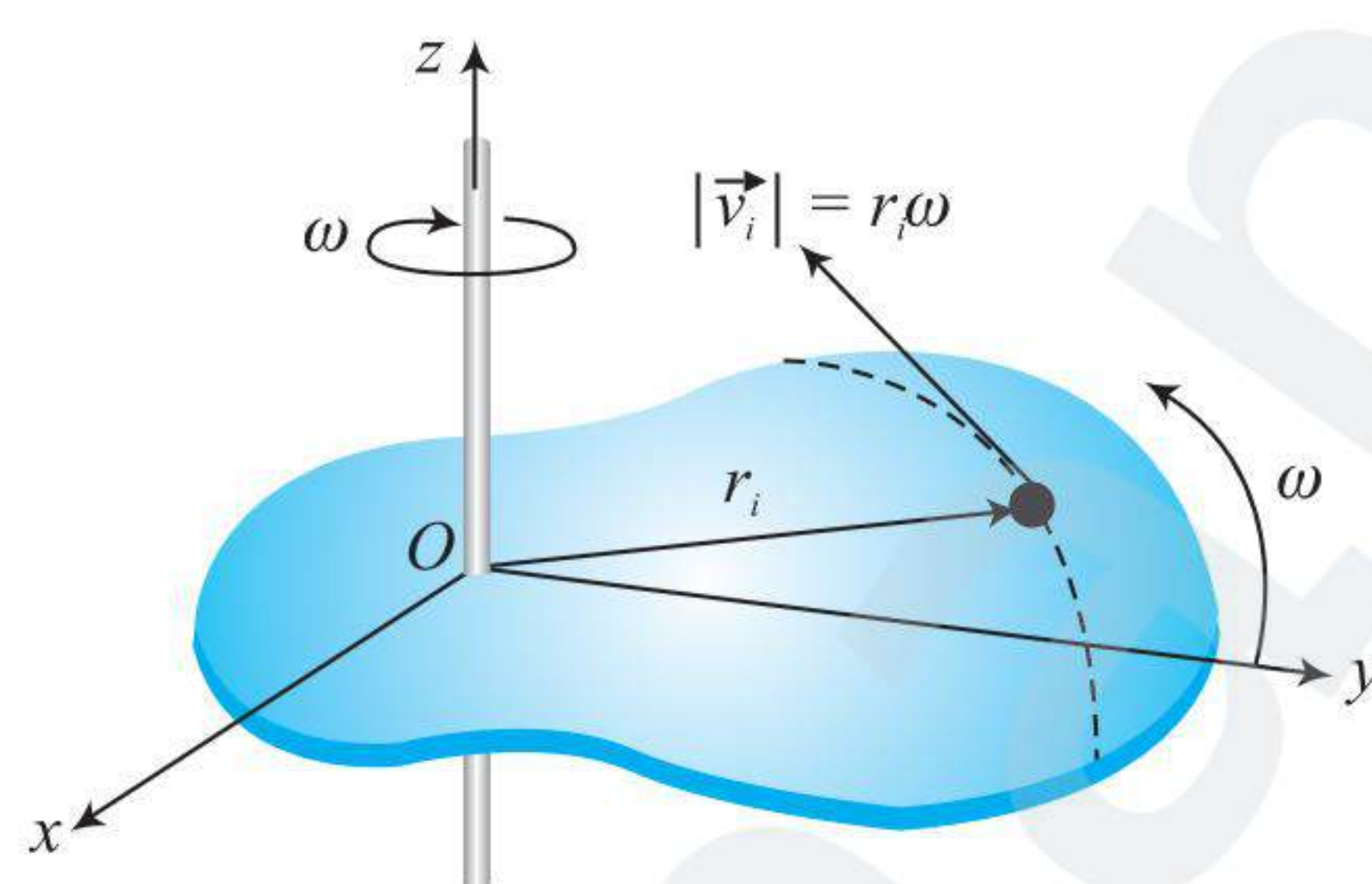


ANSWERS

1. zero 2. (a) $\frac{mg}{m + \frac{3}{8}M}$ (b) $\frac{3mg - \mu Mg}{M + 3m}$ (c) $\frac{M}{8m + 3M}$
3. 12 N, 4 m/s² 4. $\frac{4}{7}$
5. $\frac{2mg}{2m + 3M[R/(R-r)]^2}$ 6. $0.79Mg$ 7. $\frac{3\mu mg}{2-3\mu}, \frac{2\mu g}{2-3\mu}$
8. $\frac{3\mu mg}{2+3\mu}, \frac{2Mg}{2+3\mu}$ 9. $\frac{2}{9}\omega_0 R$

KINETIC ENERGY OF ROTATING BODY ABOUT A FIXED AXIS

We define the kinetic energy of an object as the energy associated with its motion through space. An object rotating about a fixed axis remains stationary in space, so there is no kinetic energy associated with translational motion. The individual particles making up the rotating object, however, are moving through space; they follow circular paths. Consequently, there is kinetic energy associated with rotational motion.



Let us consider an object as a system of particles and assume it rotates about a fixed z axis with an angular speed ω . Figure shows the rotating object and identifies one particle on the object located at a distance r_i from the rotation axis. If the mass of the i^{th} particle is m_i and its tangential speed is v_i , its kinetic energy is

$$K_i = \frac{1}{2} m_i v_i^2 \quad \dots(i)$$

To proceed further, recall that although every particle in the rigid object has the same angular speed ω , the individual tangential speeds depend on the distance r_i from the axis of rotation. The total kinetic energy of the rotating rigid object is the sum of the kinetic energies of the individual particles:

$$K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^2 = \frac{1}{2} \sum_i m_i r_i^2 \omega^2$$

We can write this expression in the form

$$K_R = \frac{1}{2} \left(\sum_i m_i r_i^2 \right) \omega^2 \quad \dots(ii)$$

where we have factored ω^2 from the sum because it is common to every particle. We recognize the quantity in parentheses as the moment of inertia of the object about the rotational axes.

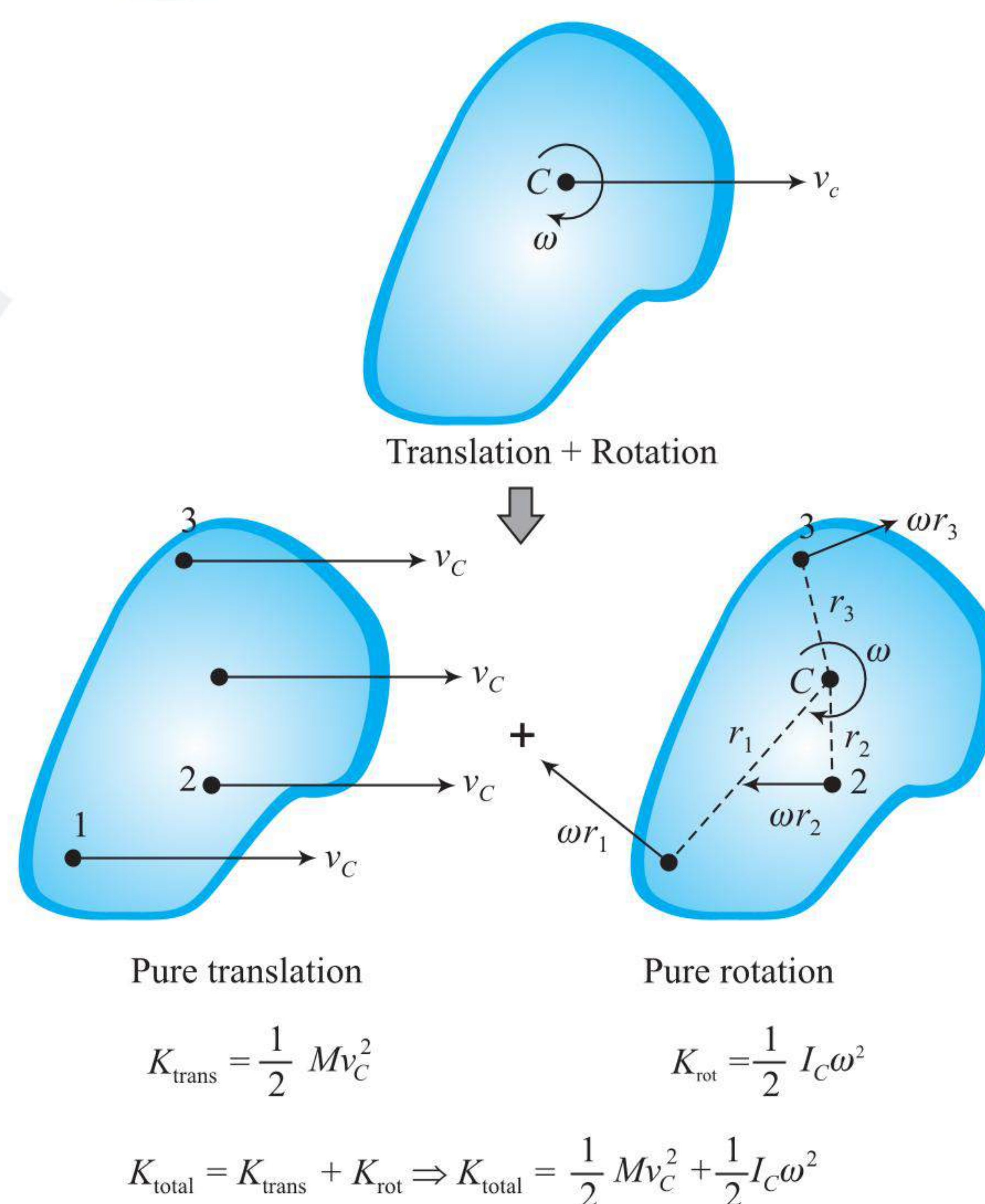
Therefore, Eq. (ii) can be written

$$K_R = \frac{1}{2} I \omega^2 \quad \dots(iii)$$

Although we commonly refer to the quantity $\frac{1}{2} I \omega^2$ as **rotational kinetic energy**, it is not a new form of energy. It is ordinary kinetic energy because it is derived from a sum over individual kinetic energies of the particles contained in the rigid object hence total kinetic energy of the body.

KINETIC ENERGY OF A BODY IN COMBINED ROTATION AND TRANSLATION

If during rotation, the center of mass of the rigid body undergoes translational motion, it will have both translational and rotational kinetic energy.



In pure translation, each and every particle of the body moves with same velocity as centre of mass, we can define kinetic energy of the object in case of pure translation.

$$K_{\text{trans}} = \frac{1}{2} M v_C^2 \quad \dots(i)$$

In case of pure rotation, the body rotate about its axes passing through its centre of mass, we define kinetic energy of the object as rotational kinetic energy

$$K_{\text{rot}} = \frac{1}{2} I_C \omega^2 \quad \dots(ii)$$

where I_C = moment of inertia of the object about an axis passing through its centre of mass.

Hence total kinetic energy of the object

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}}$$

$$\Rightarrow K_{\text{total}} = \frac{1}{2} M v_C^2 + \frac{1}{2} I_C \omega^2$$

The first term signifies the translational KE of the rigid body which can be understood as the KE of the centre of mass of the body. The second term gives the KE of rotational of the body about the centre of mass. Hence, we can call the term “ $\frac{1}{2} I_C \omega^2$ ” as rotational kinetic energy of the rigid body.

Important Points:

- **KE for pure translation:** $K_{\text{translational}} = \frac{1}{2} m v_C^2$

As a perfectly translating body has KE formula identical to that of a point mass, we can say that a translating body behaves as a point mass.

- **KE for fixed axis rotation:** $K = \frac{1}{2} I_P \omega^2$

I_P = moment of inertia of body about fixed axis

- **Kinetic energy for combined motion:**

$$K_{\text{total}} = \frac{1}{2} m v_C^2 + \frac{1}{2} I_C \omega^2$$

If a body is rotating about a fixed axis and spinning (or rotating) about its own axis we write the total kinetic energy of the body as

$$K_{\text{total}} = K_{\text{orbital}} + K_{\text{spin}}$$

Solving Eqs. (i) and (ii), we have

$$\omega = \frac{3v}{l} \quad \text{and} \quad v_C = \frac{v}{2}$$

- (b) Then substituting v_C and ω in the equation

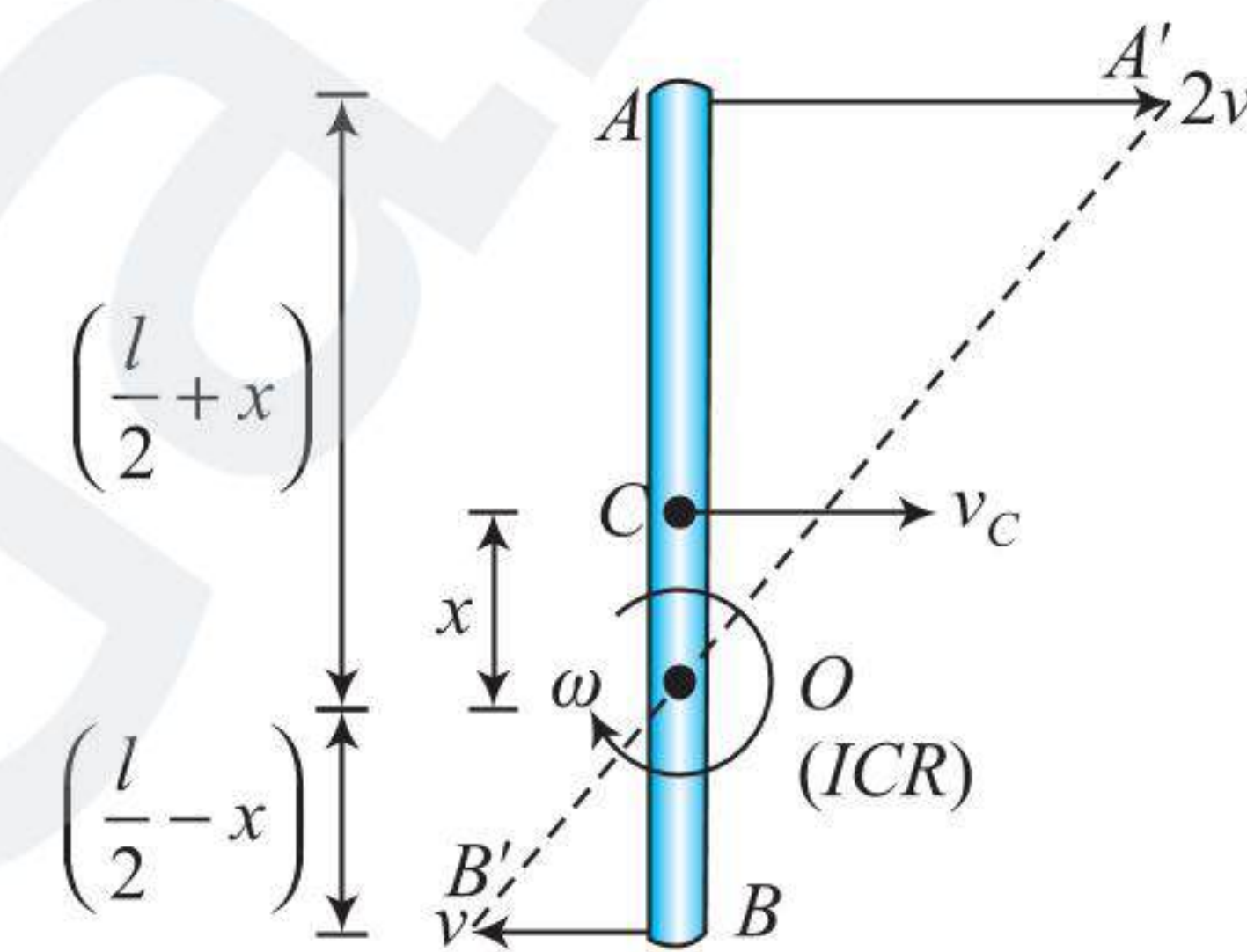
$$K = \frac{1}{2} m v_C^2 + \frac{1}{2} I_C \omega^2$$

$$\text{We have } K = \frac{1}{2} m \left(\frac{v}{2} \right)^2 + \frac{1}{2} \left(\frac{m l^2}{12} \right) \left(\frac{3v}{l} \right)^2$$

$$\text{This gives } K = \frac{1}{2} m v^2$$

Alternative: We can calculate angular velocity of the rod and velocity of centre of mass by ICR method.

We can find the ICR as shown in figure.



The triangles $AA'O$ and $BB'O$ are similar triangles

$$\frac{AA'}{AO} = \frac{BB'}{BO} \Rightarrow \frac{2v}{(l/2 + x)} = \frac{v}{(l/2 - x)} = \omega$$

$$\text{which gives } x = \frac{l}{6}$$

$$\text{Hence angular velocity } \omega = \frac{v}{\frac{l}{2} - x} = \frac{v}{\frac{l}{2} - \frac{l}{6}} \Rightarrow \omega = \frac{3v}{l}$$

$$\text{and velocity of centre of mass; } v_C = \omega x = \frac{3v}{l} \times \frac{l}{6} = \frac{v}{2}$$

Rotational kinetic energy of the rod

$$K_{\text{rot}} = \frac{1}{2} I_C \omega^2 = \frac{1}{2} \times \frac{m l^2}{12} \times \left(\frac{3v}{l} \right)^2 \Rightarrow K_{\text{rot}} = \frac{3}{8} m v^2$$

Translational kinetic energy of the rod

$$K_{\text{trans}} = \frac{1}{2} m v_C^2 = \frac{1}{2} m \left(\frac{v}{2} \right)^2 = \frac{1}{8} m v^2$$

Total kinetic energy $K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}}$

$$\Rightarrow K_{\text{total}} = \frac{1}{8} m v^2 + \frac{3}{8} m v^2 = \frac{1}{2} m v^2$$

We can calculate total kinetic energy of the rod by using the fact that the rod has pure rotation about ICR. It means total kinetic energy of the rod should be equal to rotational kinetic energy about ICR.

$$K_o = \frac{1}{2} I_o \omega^2 = \frac{1}{2} [I_C + m x^2] \omega^2$$

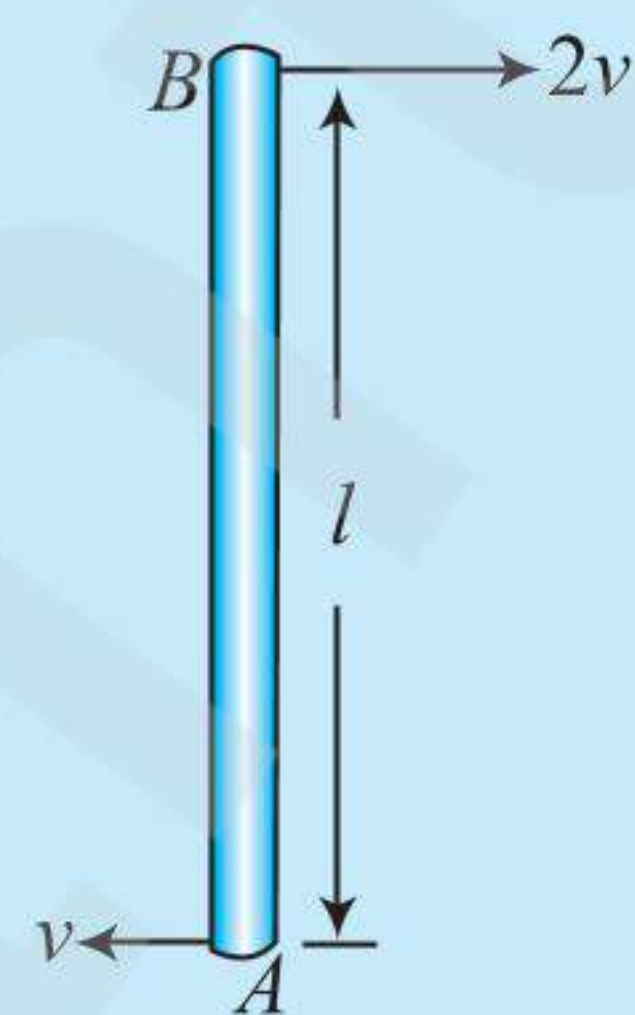
$$= \frac{1}{2} \left[\frac{m l^2}{12} + m \left(\frac{l}{6} \right)^2 \right] \left(\frac{3v}{l} \right)^2$$

$$\Rightarrow K_o = \frac{1}{2} m v^2$$

ILLUSTRATION 4.36

A uniform rod of mass $m = 2 \text{ kg}$ and length l is kept on a smooth horizontal plane. If the ends A and B of the rod move with speeds v and $2v$ respectively perpendicular to the rod, find the:

- (a) angular velocity of the rod and linear velocity of CM of the rod.
(b) kinetic energy of the rod



Sol.

- (a) As the velocities of the ends A and B of the rod are given as $2v$ and v respectively, we need to find velocity of centre of mass of rod and its angular velocity as follows.

$$\text{Since } \vec{v}_A = \vec{v}_{AC} + \vec{v}_C$$

$$\text{We know } \vec{v}_{AC} = -\frac{l}{2} \omega \hat{i}$$

$$\text{and } \vec{v}_C = v_C \hat{i}$$

But we have velocity of A ;

$$v_A (= -v) = v_C - \frac{l}{2} \omega \quad \dots(i)$$

Similarly, we have

$$v_B (= 2v) = v_C + \frac{l}{2} \omega \quad \dots(ii)$$

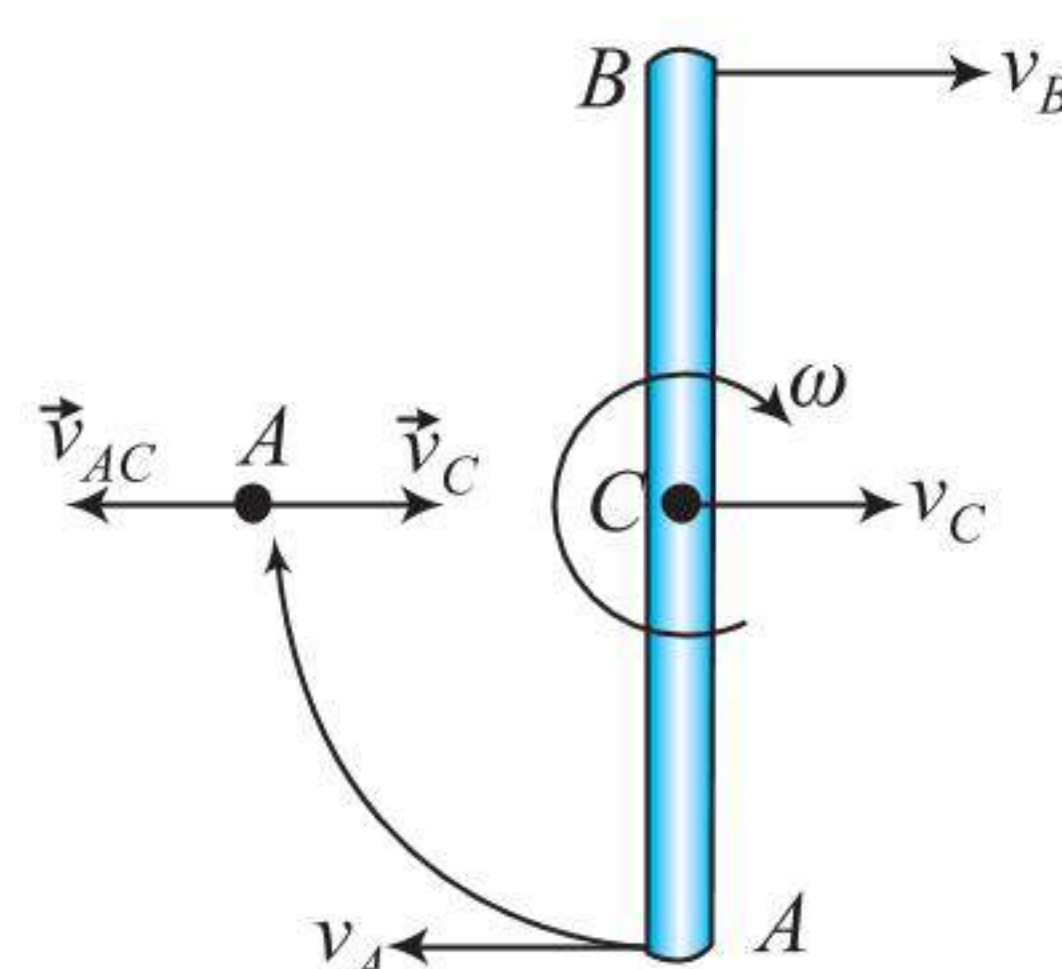
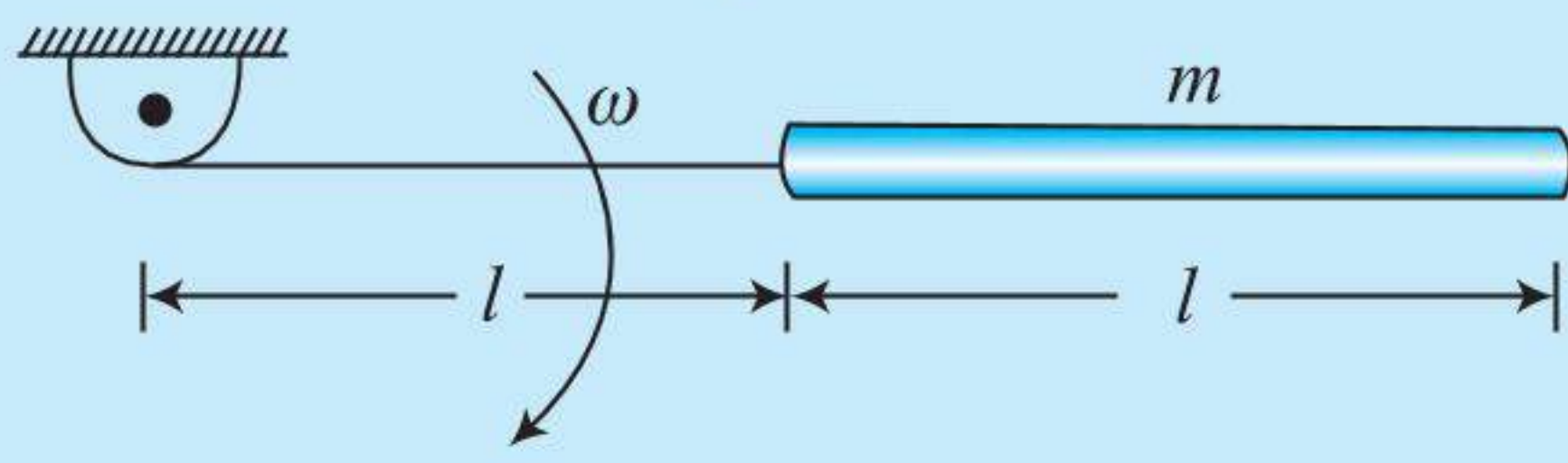


ILLUSTRATION 4.37

A rod of mass m and length l is connected with a light rod of length l . The composite rod is made to rotate with angular velocity ω as shown in the figure. Find the



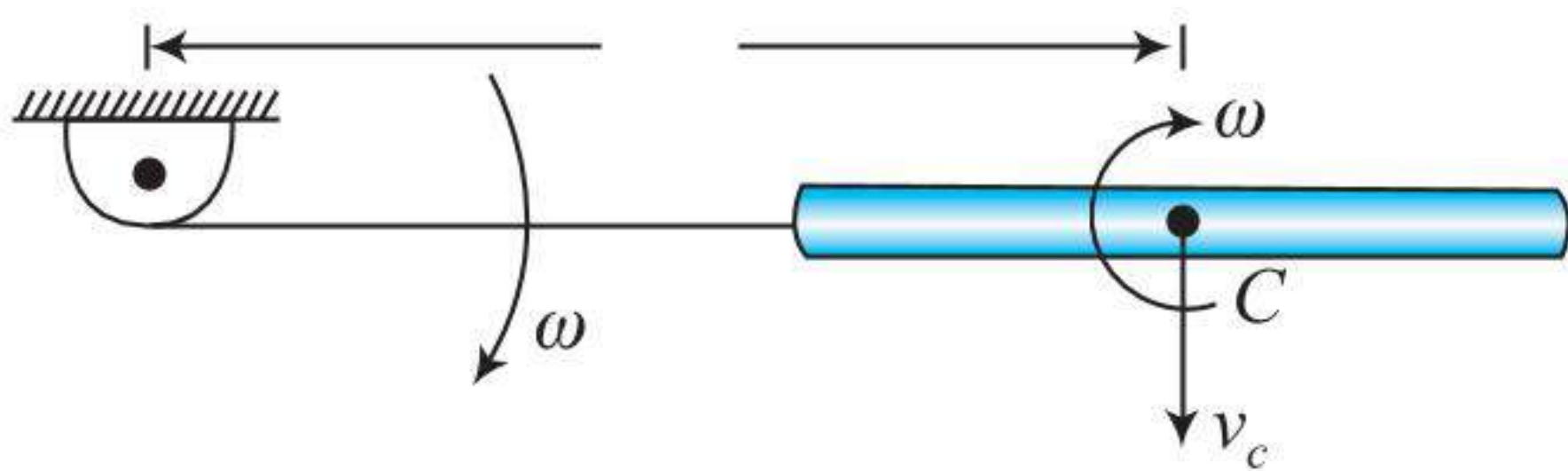
- translational kinetic energy.
- rotational kinetic energy.
- total kinetic energy of rod.

Sol.

(a) **Method 1:** Translational kinetic energy of rod,

$$K_{\text{translational}} = \frac{1}{2} m v_c^2$$

$$\text{Velocity of centre of mass } v_c = \omega \left(\frac{3}{2} l \right)$$



$$\text{Hence } K_{\text{translational}} = \frac{1}{2} m \left[\frac{3\omega l}{2} \right]^2 = \frac{9}{8} m \omega^2 l^2$$

(b) Rotational kinetic energy of rod,

$$K_{\text{rotational}} = \frac{1}{2} I_c \omega^2 = \frac{1}{2} \left[\frac{ml^2}{12} \right] \omega^2 = \frac{1}{24} m \omega^2 l^2$$

(c) Total kinetic energy,

$$\begin{aligned} K_{\text{total}} &= K_{\text{translational}} + K_{\text{rotational}} \\ &= \frac{9}{8} m \omega^2 l^2 + \frac{1}{24} m \omega^2 l^2 = \frac{7}{6} m \omega^2 l^2 \end{aligned}$$

Method 2: Total energy of a body rotating about fixed axis = rotational kinetic energy about the fixed axis.

$$K_{\text{total}} = \frac{1}{2} I_P \omega^2$$

Now using parallel axis theorem,

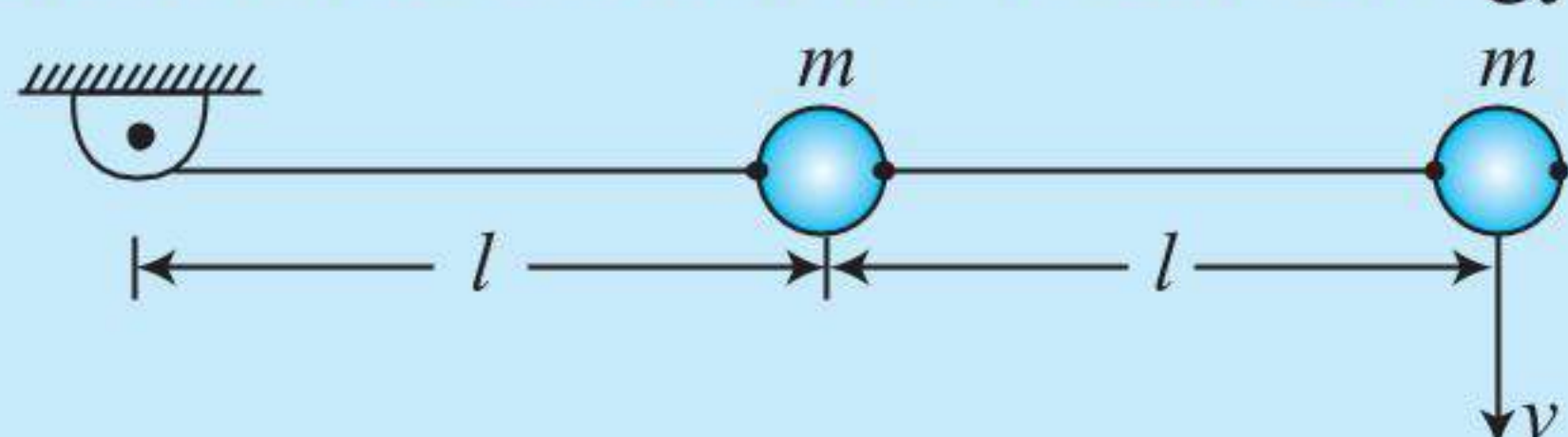
$$\begin{aligned} I_P &= I_c + m \left(\frac{3}{2} l \right)^2 \\ &= \frac{ml^2}{12} + \frac{9}{4} ml^2 = \frac{7}{3} ml^2 \end{aligned}$$

Hence total kinetic energy

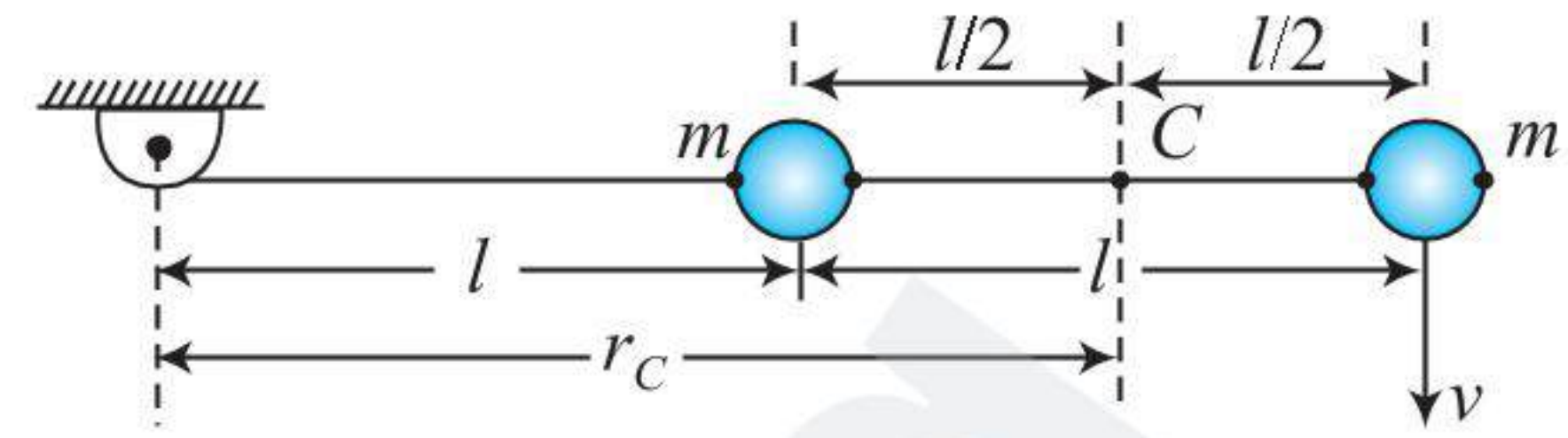
$$K_{\text{total}} = \frac{1}{2} \left(\frac{7}{3} ml^2 \right) \omega^2 = \frac{7}{6} m \omega^2 l^2$$

ILLUSTRATION 4.38

Two beads each of mass m are welded at the ends of two light rigid rods each of length l . If the pivots are smooth, find the ratio of translational and rotational kinetic energy of system.



Sol. We know translational kinetic energy $K_t = \frac{1}{2} m v_c^2$ and rotational kinetic energy $K_r = \frac{1}{2} I_c \omega^2$,



Distance of centre of mass from pivot end

$$r_c = \frac{ml + m(2l)}{m + m} = \frac{3}{2} l$$

Moment of inertia of system about centre of mass

$$\text{and } I_c = m \left(\frac{l}{2} \right)^2 + m \left(\frac{l}{2} \right)^2 = \frac{ml^2}{2}$$

$$\text{Angular velocity of rod } \omega = \frac{v}{2l}$$

Velocity of centre of mass

$$v_c = \omega \left(\frac{3}{2} l \right) = \left(\frac{v}{2l} \right) \left(\frac{3}{2} l \right) = \frac{3}{4} v$$

Translational kinetic energy of system

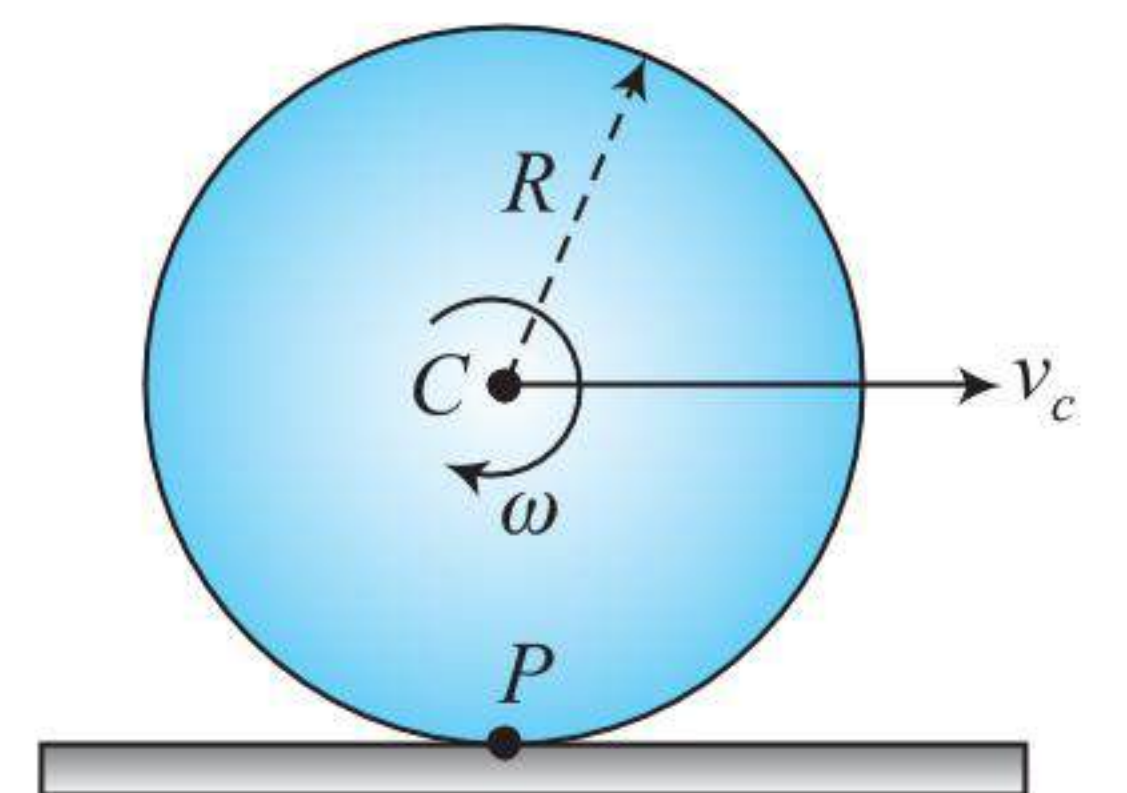
$$K_t = \frac{1}{2} M_{\text{total}} \cdot v_c^2 = \frac{1}{2} (2m) \left(\frac{3}{4} v \right)^2 = \frac{9}{16} m v^2$$

$$K_r = \frac{1}{2} I_c \omega^2 = \frac{1}{2} \left(\frac{ml^2}{2} \right) \left(\frac{v}{2l} \right)^2 = \frac{1}{16} m v^2$$

$$\text{This gives } \frac{K_t}{K_r} = 9$$

KINETIC ENERGY OF A BODY ROLLING ON A FIXED SURFACE

Let a body of mass m rolling on a fixed horizontal surface. The rolling motion is the combined rotational and translational motion. Here we can define kinetic energy of the rolling body



$$K_{\text{total}} = \frac{1}{2} m v_c^2 + \frac{1}{2} I_c \omega^2 \quad \dots(i)$$

Here m is the mass of the body and I_c is the moment of inertia of the body about an axis passing through centre of mass.

$$I = m k^2, \quad k = \text{radius of gyration}$$

$$\text{As the body is rolling, } v_c = \omega R \text{ or } \omega = \frac{v_c}{R} \quad \dots(ii)$$

From (i) and (ii)

$$K_{\text{total}} = \frac{1}{2} m v_c^2 + \frac{1}{2} (m k^2) \left(\frac{v_c}{R} \right)^2$$

$$\Rightarrow K_{\text{total}} = \left(1 + \frac{k^2}{R^2} \right) \cdot \frac{1}{2} m v_c^2 = (1 + c) \cdot \frac{1}{2} m v_c^2 \quad \dots(iii)$$

Here c is a constant depends upon the shape of the rolling body. From (i) and (ii) we can write

$$K_{\text{total}} = \frac{1}{2} m (\omega R)^2 + \frac{1}{2} I_c \omega^2 = \frac{1}{2} (m R^2 + I_c) \omega^2$$

Using parallel axis theorem, we can write, $mR^2 + I_c = I_P$
 I_P is the moment of inertia the rolling body about point of contact with the ground, which is also instantaneous centre of rotation (ICR).

$$\text{Hence } K_{\text{total}} = \frac{1}{2} I_P \omega^2 \quad \dots(\text{iv})$$

Therefore, we conclude that the combined effects of translation of centre of mass of a body and its rotation about an axis passing through the CM are equivalent to its pure rotation about an axis passing through the point of contact P , of the rolling body.

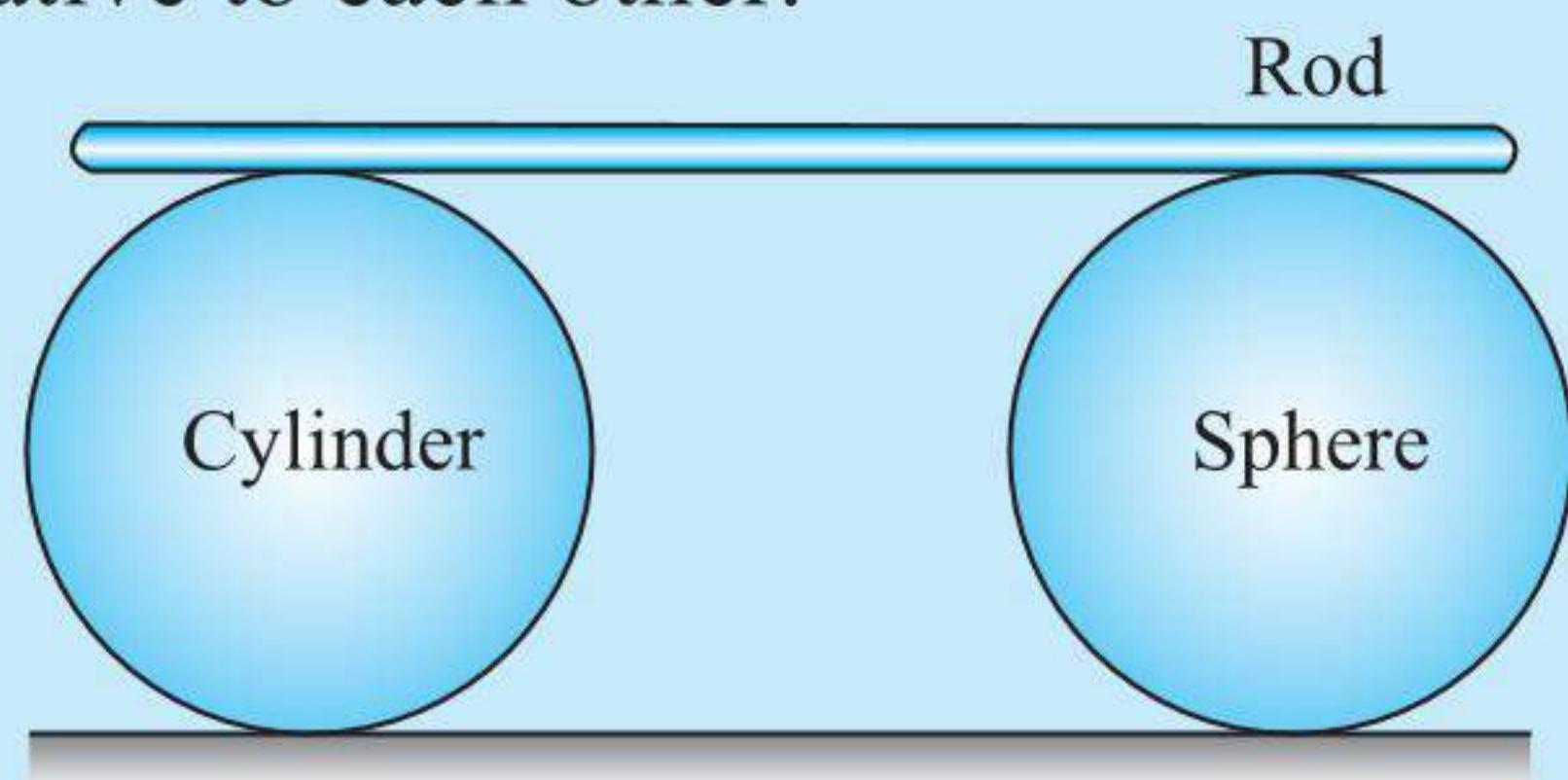
$$\% \text{ of energy of translation} = \frac{KE_t}{KE} \times 100\%$$

$$= \frac{\frac{1}{2} m v_c^2}{\frac{1}{2} m v_c^2 (1+c)} \times 100\% = \frac{1}{1+c} \times 100\%$$

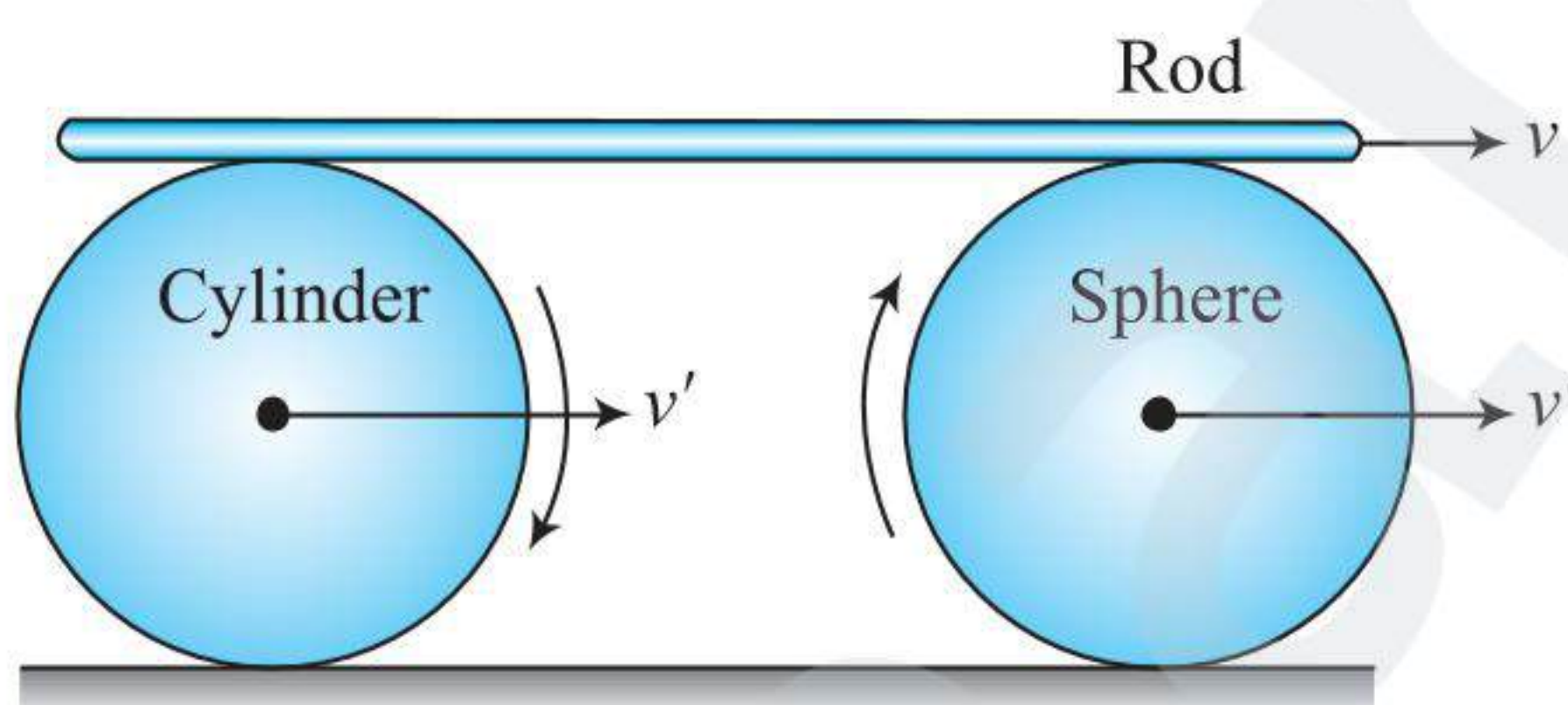
$$\text{Similarly, \% of energy of rotation} = \frac{c}{1+c} \times 100\%$$

ILLUSTRATION 4.39

A rod of mass m is kept on a cylinder and sphere each of radius R . The masses of the sphere and cylinder are m_1 and m_2 respectively. If the speed of the rod is v , find the KE of the system (rod + cylinder + sphere). Assume that the surfaces do not slide relative to each other.



Sol. Kinetic energy in case of rolling $K = (1+c) \frac{mv^2}{2}$



where, $c = \frac{k^2}{R^2}$; k = radius of gyration and R = radius

$$\text{For cylinder, } c = \frac{\frac{1}{2} R^2}{R^2} = \frac{1}{2}$$

$$\text{For sphere, } c = \frac{\frac{2}{5} R^2}{R^2} = \frac{2}{5}$$

In case of rolling on ground or on a fixed surface, the velocity of the top point of the rolling body is double the velocity of its centre, hence $v' = \frac{v}{2}$, here v' is the velocity of the centre of mass of the cylinder and sphere.

Kinetic energy of the cylinder,

$$K_{\text{cylinder}} = \left(1 + \frac{1}{2}\right) \cdot \frac{1}{2} m_1 v'^2$$

$$\Rightarrow K_{\text{cylinder}} = \frac{3}{4} m_1 \left(\frac{v}{2}\right)^2 = \frac{3}{16} m_1 v^2$$

Kinetic energy of the sphere,

$$K_{\text{sphere}} = \left(1 + \frac{2}{5}\right) \cdot \frac{1}{2} m_2 v'^2$$

$$\Rightarrow K_{\text{sphere}} = \frac{7}{10} m_2 \left(\frac{v}{2}\right)^2 = \frac{7}{40} m_2 v^2$$

The motion of the rod is pure translational, hence kinetic energy of rod is,

$$K_{\text{rod}} = \frac{1}{2} m v^2$$

Then total kinetic energy of the system

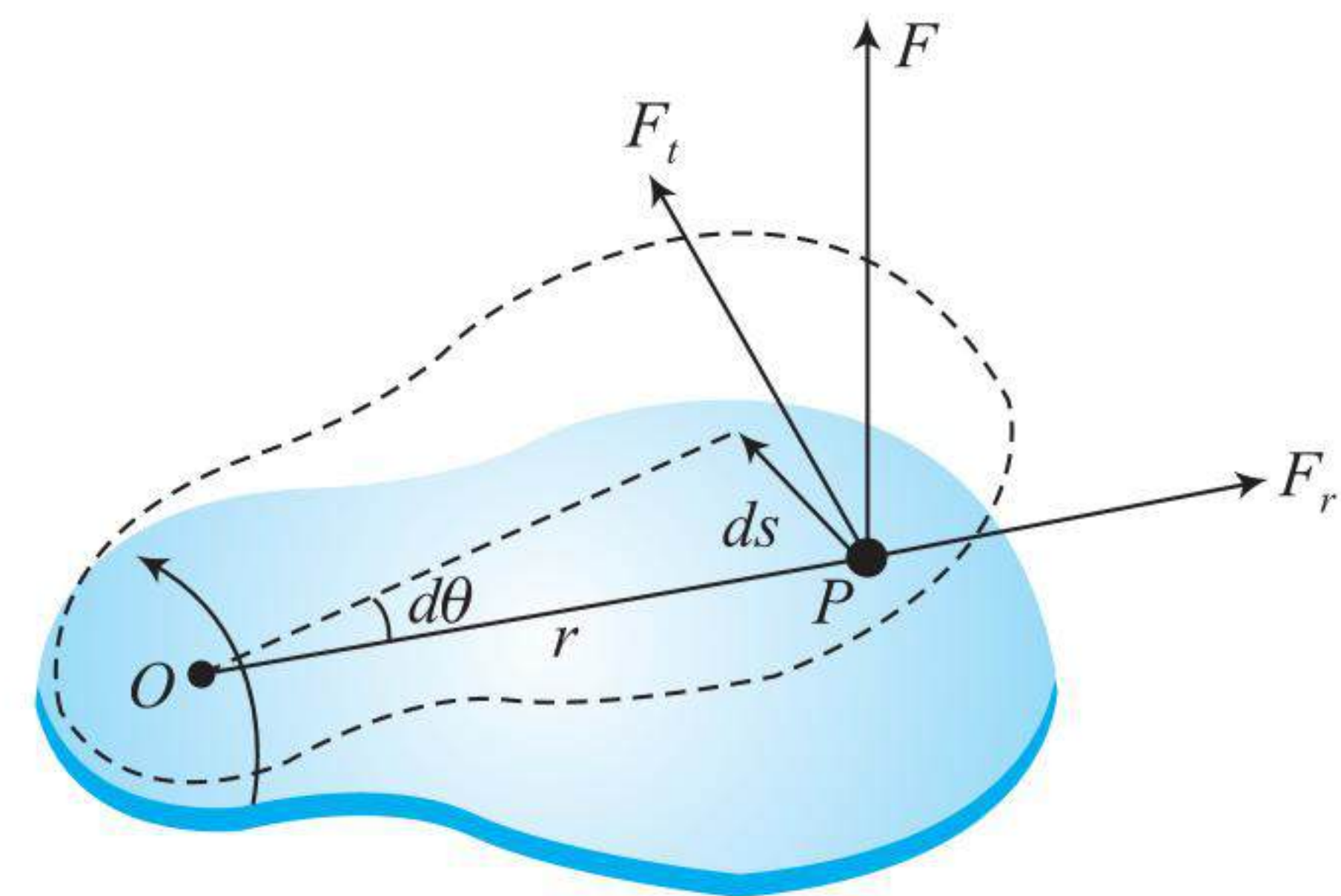
$$K_{\text{total}} = K_{\text{rod}} + K_{\text{cylinder}} + K_{\text{sphere}} = \frac{1}{2} m v^2 + \frac{3}{16} m_1 v^2 + \frac{7}{40} m_2 v^2$$

ROTATIONAL WORK AND POWER

Consider the rigid object pivoted at O in figure. It is acted on by an external force F that has a radial component F_r and a tangential component F_t . In an infinitesimal time interval dt , the point of application of force moves in a circular arc of length $r d\theta$. Therefore, the work done is

$$dW_{\text{rot}} = F_t (r d\theta) = (F_t r) d\theta = \tau d\theta$$

Notice that the radial component vector of $\vec{F}$ does no work on the object because it is perpendicular to the displacement of the point of application of $\vec{F}$.



For a finite angular displacement,

$$W_{\text{rot}} = \int_{\theta_i}^{\theta_f} \tau d\theta \quad \dots(\text{i})$$

The rate at which work is being done by $\vec{F}$ as the object rotates about the fixed axis through the angle $d\theta$ in a time interval dt is

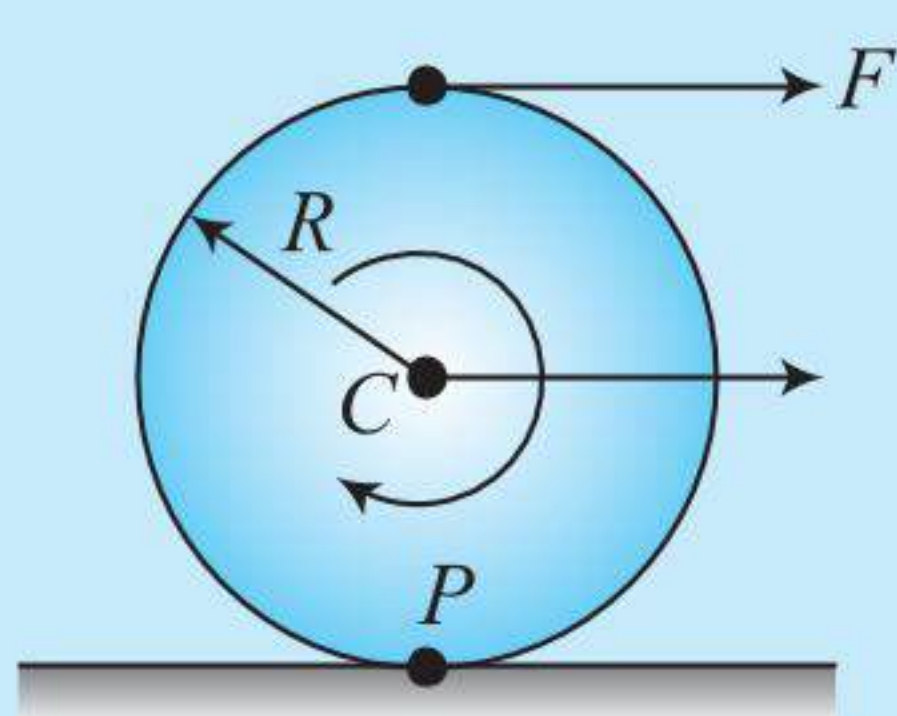
$$\frac{dW}{dt} = \tau \frac{d\theta}{dt}$$

Because dW/dt is the instantaneous power P delivered by the force and $d\theta/dt = \omega$, this expression reduces to

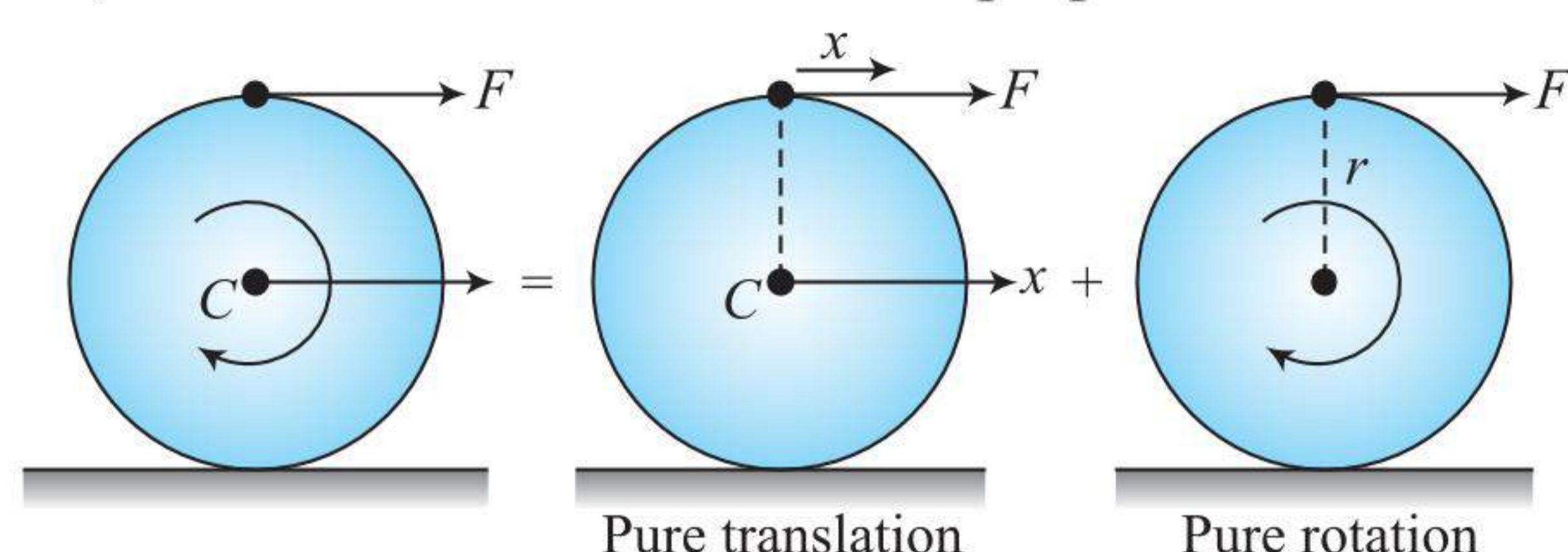
$$P = \frac{dW}{dt} = \tau \omega$$

ILLUSTRATION 4.40

A disc of mass m and radius R is placed on a smooth horizontal surface. A horizontal force F is acting at the top most position of the disc. Find the work done by the force F in rotating and translating the disc, if its centre of mass moves through distance x and the body rotates through an angle θ .



Sol. The motion of the disc is combined translation and rotation, here we can use method of superposition.



The work done by the force F in translating the disc

$$W_{\text{translation}} = F \cdot x$$

Work done by the force F in rotating the disc

$$W_{\text{rotation}} = \tau \cdot \theta = (F \cdot R)\theta = FR\theta$$

Total work done by F in combined rotation and translation

$$W_{\text{total}} = Fx + FR\theta$$

ILLUSTRATION 4.41

Calculate the torque developed by an airplane engine whose output is 2000 HP at an angular velocity of 2400 rev/min.

Sol. Given angular velocity, $\omega = 2\pi(2400/60) = 80\pi$ rad/s
 Power output of airplane engine = 2000 HP = 2000×746 W
 Work done by torque = (torque) $\times$ (angular displacement)

$$\text{Power} = \text{work done per second} = \tau \frac{\Delta\theta}{\Delta t} = \tau\omega$$

$$\text{As } P = \tau\omega \Rightarrow \tau = \frac{2000 \times 746}{80\pi} = 5937 \text{ Nm}$$

WORK-KINETIC ENERGY THEOREM FOR ROTATIONAL MOTION

Like the work-kinetic energy theorem in translational motion, this theorem states that the net work done on the system of the rigid object, which represents a transfer of energy across the boundary of the system that appears as an increase in the object's rotational kinetic energy.

To prove that fact, let us begin with the rigid object under a net torque model, whose mathematical representation is given by,

$$\tau_{\text{ext}} = I\alpha = I \frac{d\omega}{dt} \quad \dots(i)$$

Using the chain rule from calculus, we can express the net torque as

$$\tau_{\text{ext}} = I \frac{d\omega}{d\theta} \frac{d\theta}{dt} = I \frac{d\omega}{d\theta} \omega \quad \dots(ii)$$

Rearranging this expression and noting that $\tau_{\text{ext}} d\theta = dW$ gives

$$\tau_{\text{ext}} d\theta = I\omega d\omega$$

Integrating this expression, we obtain for the work W done by the net external force acting on a rotating system

$$W_{\text{rotation}} = \int_{\omega_i}^{\omega_f} I\omega d\omega = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2 \quad \dots(iii)$$

Important Points:

- The work done by a force acting on a rigid body may be classified into two parts: translational work and rotational work.
- A force can do translational work if the centre of mass of the body is free to move. Mathematically, it is defined as

$$W_{\text{trans}} = \int F ds_c$$

where ds_c is the infinitesimal displacement of the centre of mass.

- A force can do rotational work if the body is free to rotate. Mathematically, it is defined as

$$W_{\text{rot}} = \int \tau_c d\theta$$

where $d\theta$ is the infinitesimal angular displacement about the centre of mass and τ_c is the torque produced by the force about the centre of mass of the body.

- In an inertial frame, the work-energy theorem may be stated as

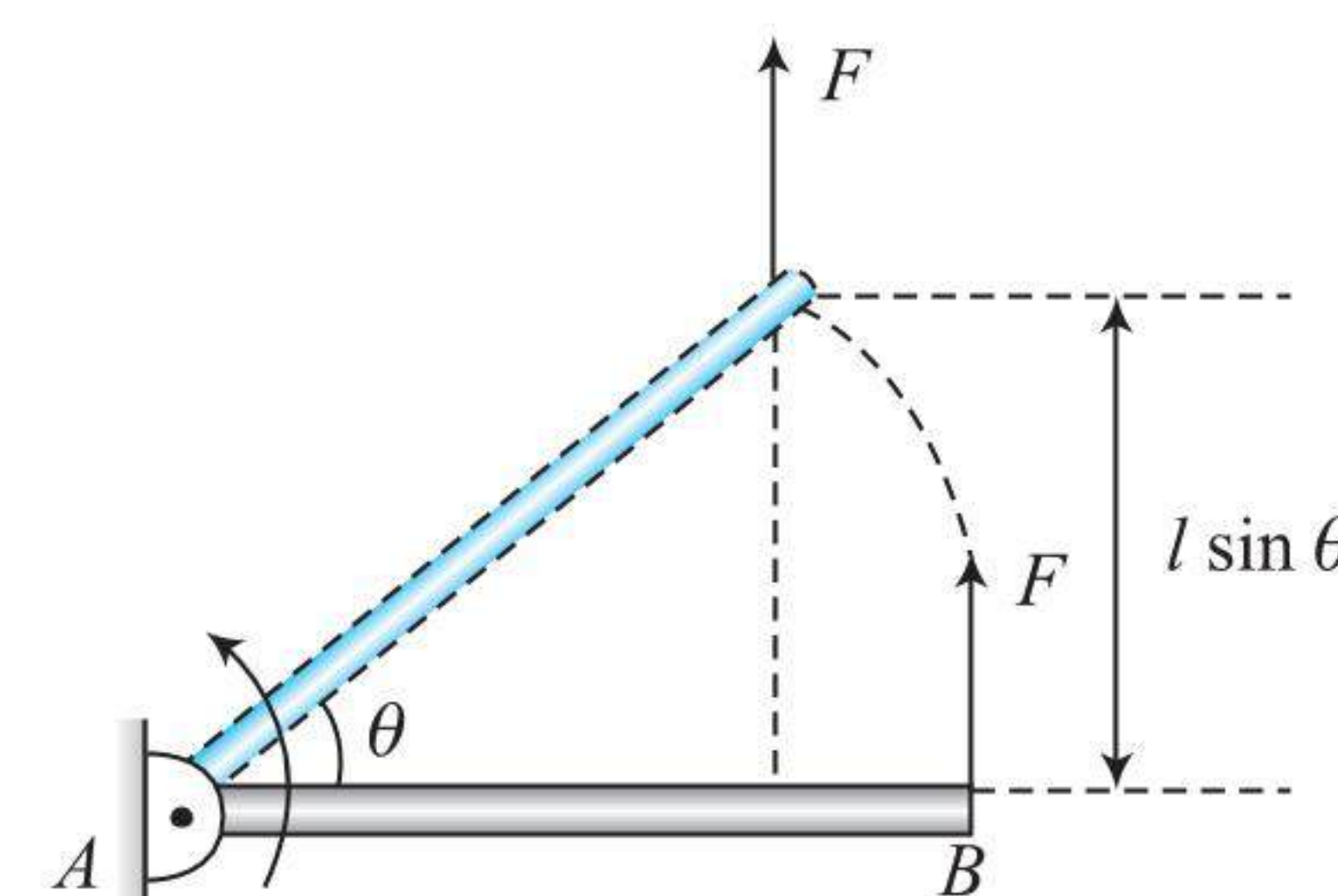
$$W_C + W_{NC} + W_{\text{oth}} = \Delta K$$

where W_C is the work done by conservative forces, W_{NC} is the work done by non-conservative forces, W_{oth} is the work done by all other forces which are not included in the above two categories. Each term of work includes both the translational and rotational work. In the right-hand side term of the equation the kinetic energy also includes both the translational and rotational kinetic energies.

ILLUSTRATION 4.42

A thin horizontal uniform rod AB of mass m and length l can rotate freely about a vertical axis passing through its end A . At a certain moment, the end B starts experiencing a constant force F which is always perpendicular to the original position of the stationary rod and directed in a horizontal plane. Find the angular velocity of the rod as a function of its rotation angle θ measured relative to the initial position.

Sol. Work done by the torque,



$$W_{\text{rotation}} = \int \tau d\theta = \int_0^\theta Fl \cos \theta d\theta$$

$$\Rightarrow W_{\text{rotation}} = Fl \sin \theta$$

or we can calculate the work done by force as

$$W_{\text{rotation}} = F \times \text{displacement in the direction of force} = Fl \sin \theta$$

Now using work energy theorem, $W_{\text{rotation}} = \Delta K_{\text{rotational}}$

$$\therefore Fl \sin \theta = \left[\frac{1}{2} \left(\frac{ml^2}{3} \right) \omega^2 - 0 \right],$$

$$\text{which gives, } \omega = \sqrt{\frac{6F \sin \theta}{ml}}$$

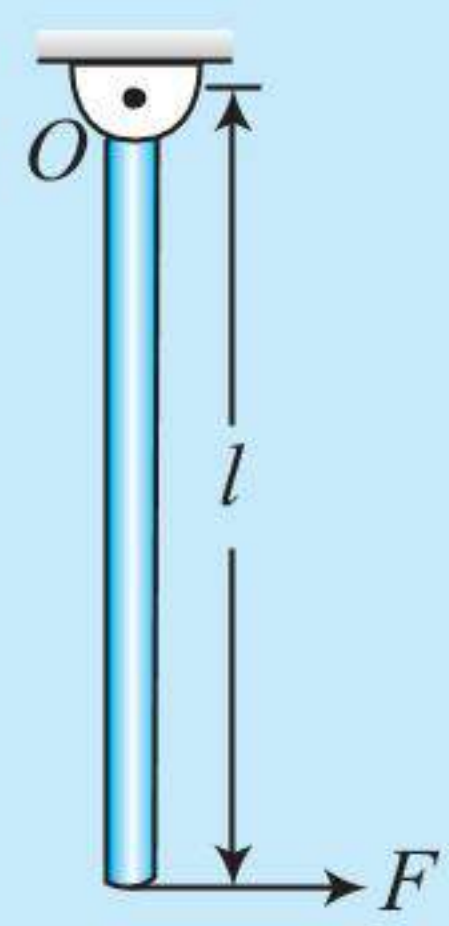
Important Points:

- The rotational kinetic energy about a fixed axis is the total kinetic energy. It includes both translational as well as rotational kinetic energy about centroidal axes.
- Care should be taken in calculating the work done. Either calculate work done by rotational method (torque method) or by translational method (force method).

ILLUSTRATION 4.43

A uniform rod of mass m and length l is pivoted smoothly at O . A horizontal force acts at the bottom of the rod.

- Find the angular velocity of the rod as the function of angle of rotation θ .
- What is the maximum angular displacement of the rod?



Sol. Here we can calculate work done in rotating the rod either by torque method or by force method, but in this case, we can easily use force method for calculating the work by force and gravity.

(a) Using work energy theorem

$$W_F + W_{gr} = \Delta K$$

$$\text{Hence, } W_F + W_{gr} = \left(\frac{1}{2} I_P \omega^2 - 0 \right)$$

$$\frac{1}{2} I_P \omega^2 = \text{Rotational kinetic energy}$$

about the rotational axes passing through hinge, it is also total kinetic energy of the rod

$$\text{Work done by 'F': } W_F = F \times l \sin \theta = Fl \sin \theta$$

$$\text{Work done by gravity: } W_{gr} = -mg \times \frac{l}{2} (1 - \cos \theta)$$

$$\Rightarrow Fl \sin \theta - mg \frac{l}{2} (1 - \cos \theta) = \frac{1}{2} \left(\frac{ml^2}{3} \right) \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{6F}{ml} \sin \theta - \frac{3g}{l} (1 - \cos \theta)} \quad \dots(i)$$

(b) At maximum angular displacement put $\omega = 0$ in Eq. (i)

$$0 = \sqrt{\frac{6F}{ml} \sin \theta - \frac{3g}{l} (1 - \cos \theta)}$$

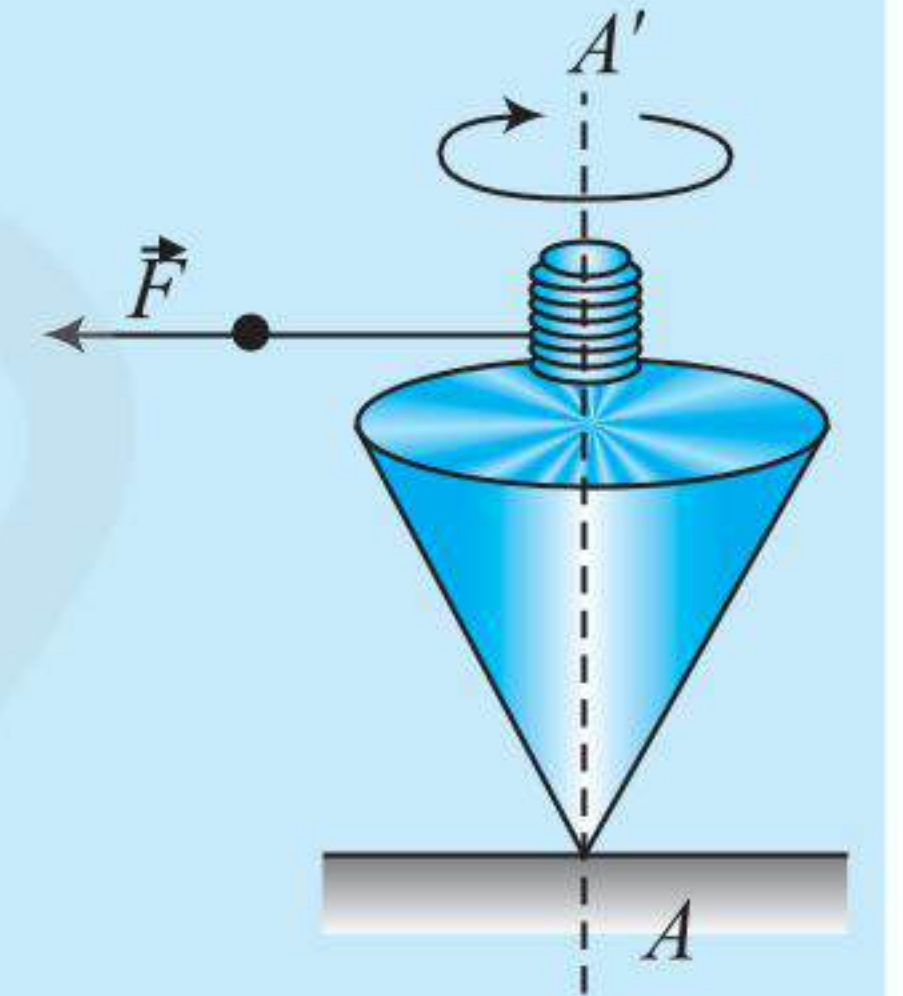
$$\Rightarrow \frac{6F}{ml} \sin \theta = \frac{3g}{l} (1 - \cos \theta)$$

$$\Rightarrow \frac{2F}{m} \left(2 \sin \left(\frac{\theta}{2} \right) \cdot \cos \left(\frac{\theta}{2} \right) \right) = g \left(2 \sin^2 \left(\frac{\theta}{2} \right) \right)$$

$$\Rightarrow \tan \frac{\theta}{2} = \frac{2F}{mg} \Rightarrow \theta = 2 \tan^{-1} \left(\frac{2F}{mg} \right)$$

ILLUSTRATION 4.44

The top in the figure has a moment of inertia equal to $4.0 \times 10^{-4} \text{ kg m}^2$ and is initially at rest. It is free to rotate about the stationary axis AA' . A string wrapped around a peg along the axis of the top is pulled in such a manner as to maintain a constant tension of 10.0 N. If the string does not slip while it is unwound from the peg, what is the angular speed of the top after 80.0 cm of string has been pulled off the peg?



$$\text{Sol. Work done} = F \Delta r = (10.0 \text{ N})(0.80 \text{ m}) = 8.0 \text{ J}$$

$$\text{and work} = \Delta K = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$$

$$\text{Thus, } 8.0 = \frac{1}{2} (4.0 \times 10^{-4} \text{ kg m}^2) \omega_f^2 - 0$$

(The last term is zero because the top starts from rest.)
and from this $\omega_f = 200 \text{ rad/s}$.

CONSERVATION OF MECHANICAL ENERGY

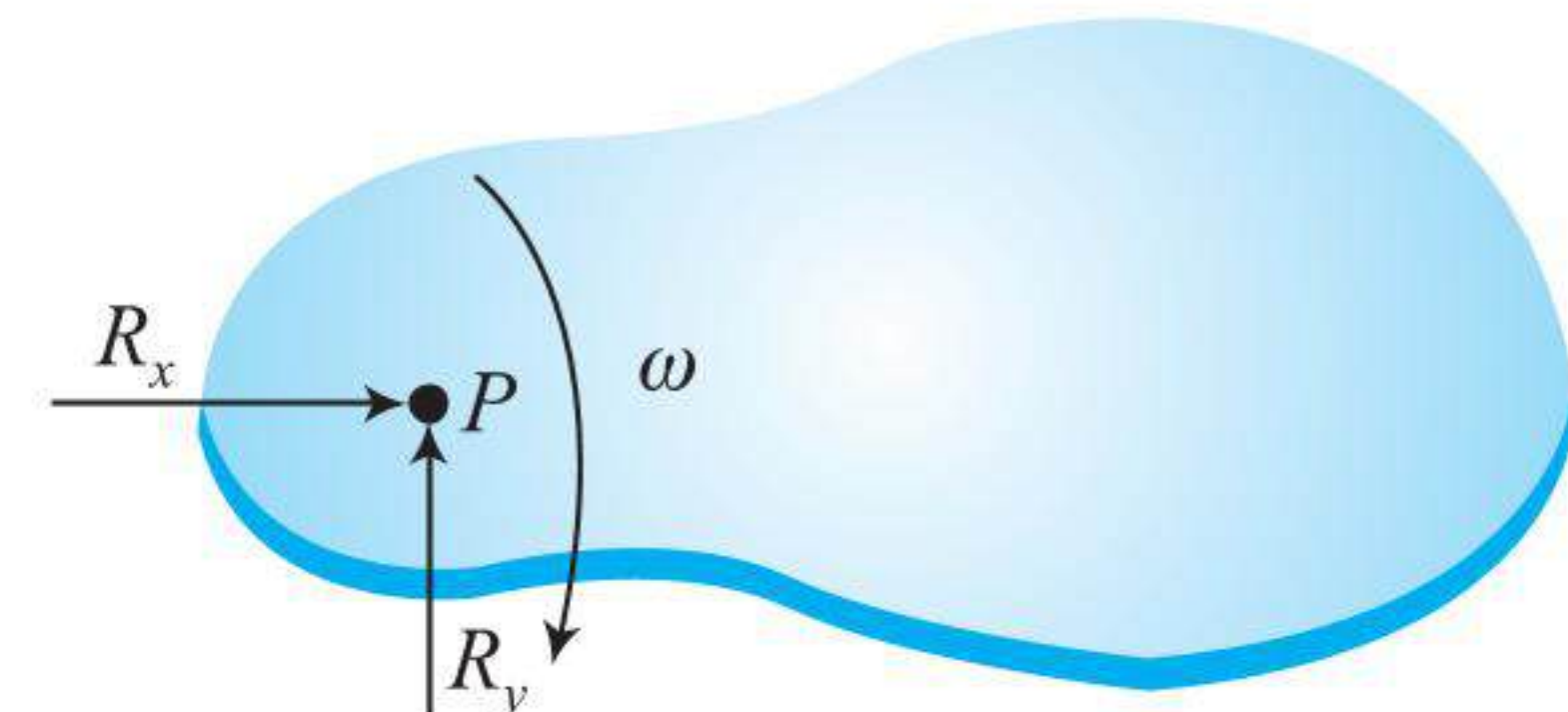
In an inertial frame, the work–energy theorem may be stated as

$$W_C + W_{NC} = \Delta K$$

If a rigid body does not experience any non-conservative force or even though a non-conservative force like force of static friction acts, if it does not perform any work (as described in pure rolling etc.) i.e., $W_{NC} = 0$ the work–energy equation assumes the form $W_C = \Delta K$.

Since conservative forces perform work at the expense of their potential energy, we can write $W_C = -\Delta U$.

Then, we have $-\Delta U = \Delta K$. This gives $\Delta K + \Delta U = 0$.

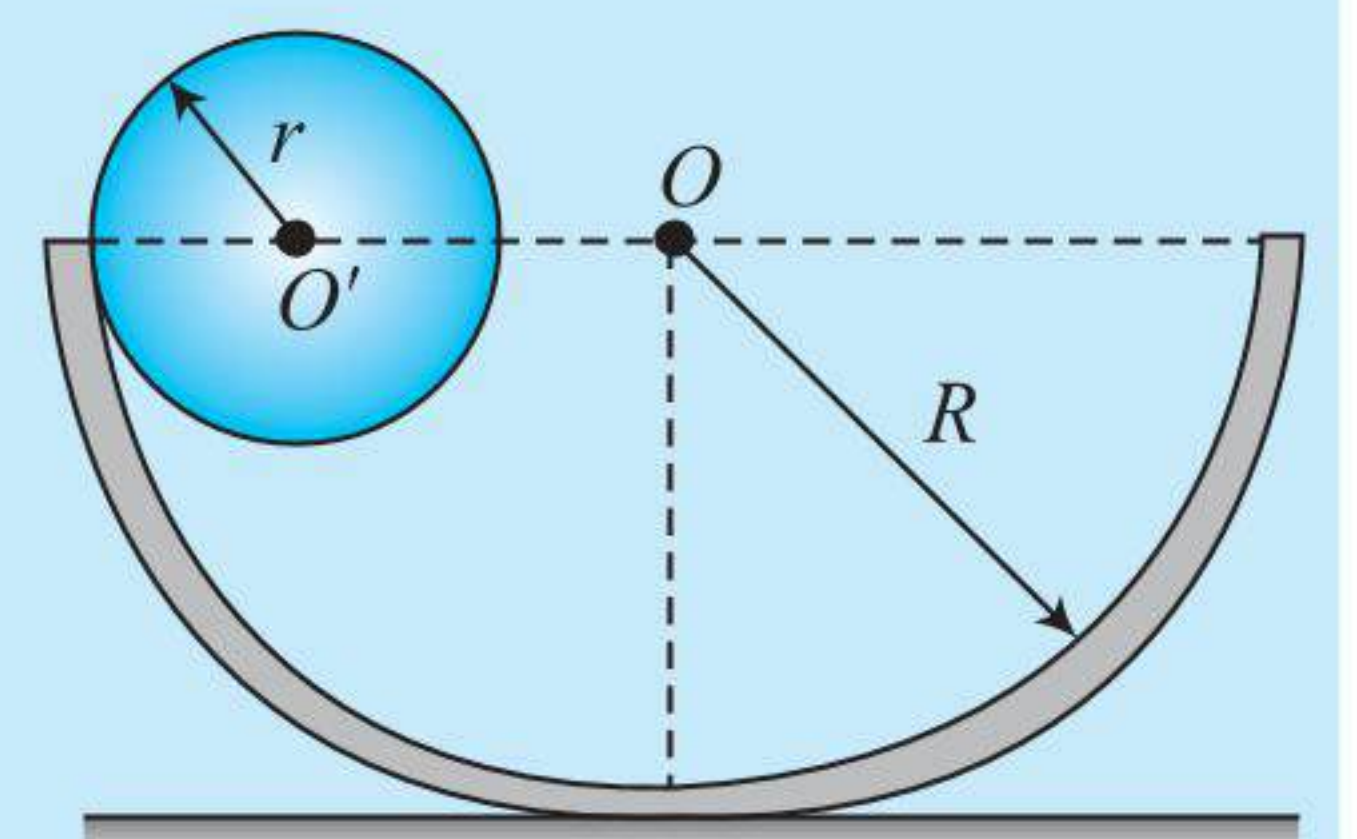


When a rigid body rotates about a fixed axis, the constraint forces are reactions at the pivot. The normal reaction sometimes does not perform work if the point of application of the forces does not move. For instance, when a rigid body rotates about a fixed axis, the reaction forces R_x and R_y do not perform work.

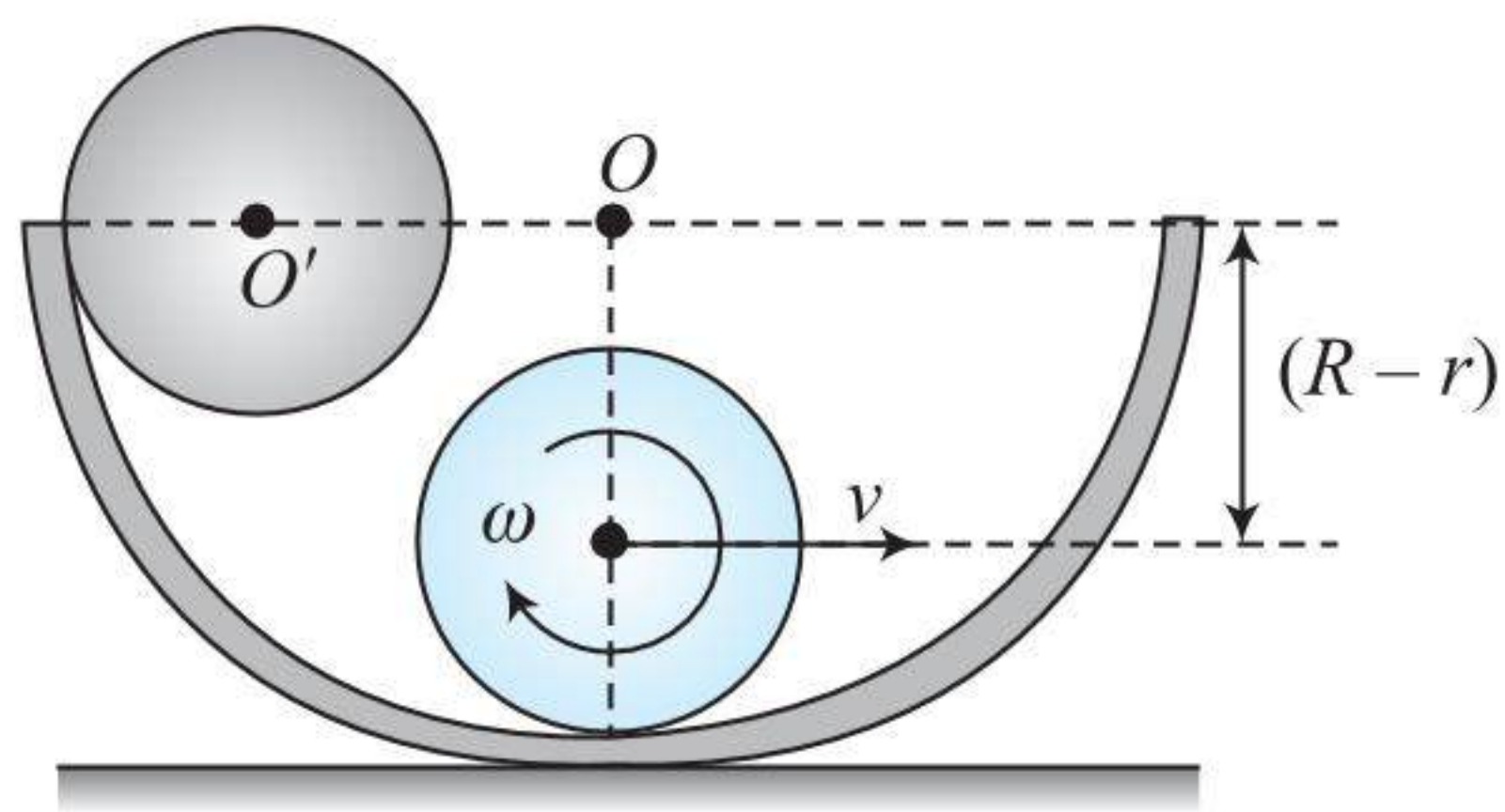
In such situations also, we can apply conservation of mechanical energy principle.

ILLUSTRATION 4.45

A solid sphere of mass m and radius r is released from rest from the given position. If it rolls without sliding on the circular track of radius R , find its speed when it reaches its lowest position.



Sol. As the sphere is rolling, the static friction does not perform any work. Hence mechanical energy is conserved



According to conservation of energy, $\Delta K + \Delta U = 0$.

$$\text{or } (K_f - K_i) + \Delta U = 0 \quad \dots(i)$$

When the sphere reaches its lowest position, it loses gravitational potential energy by ΔU and gains a kinetic energy by ΔK , where

$$\Delta U = -mg(R-r)$$

The sphere is released from rest, its initial kinetic energy $K_i = 0$.

And kinetic energy when the sphere reaches at its lowest position

$$K_f = (1+c) \frac{1}{2} mv^2$$

$$\text{For sphere, } c = \frac{k^2}{R^2} = \frac{2/5 R^2}{R^2} = \frac{2}{5}$$

$$K_f = \frac{1}{2} mv^2 \left(1 + \frac{2}{5}\right) = \frac{7}{10} mv^2$$

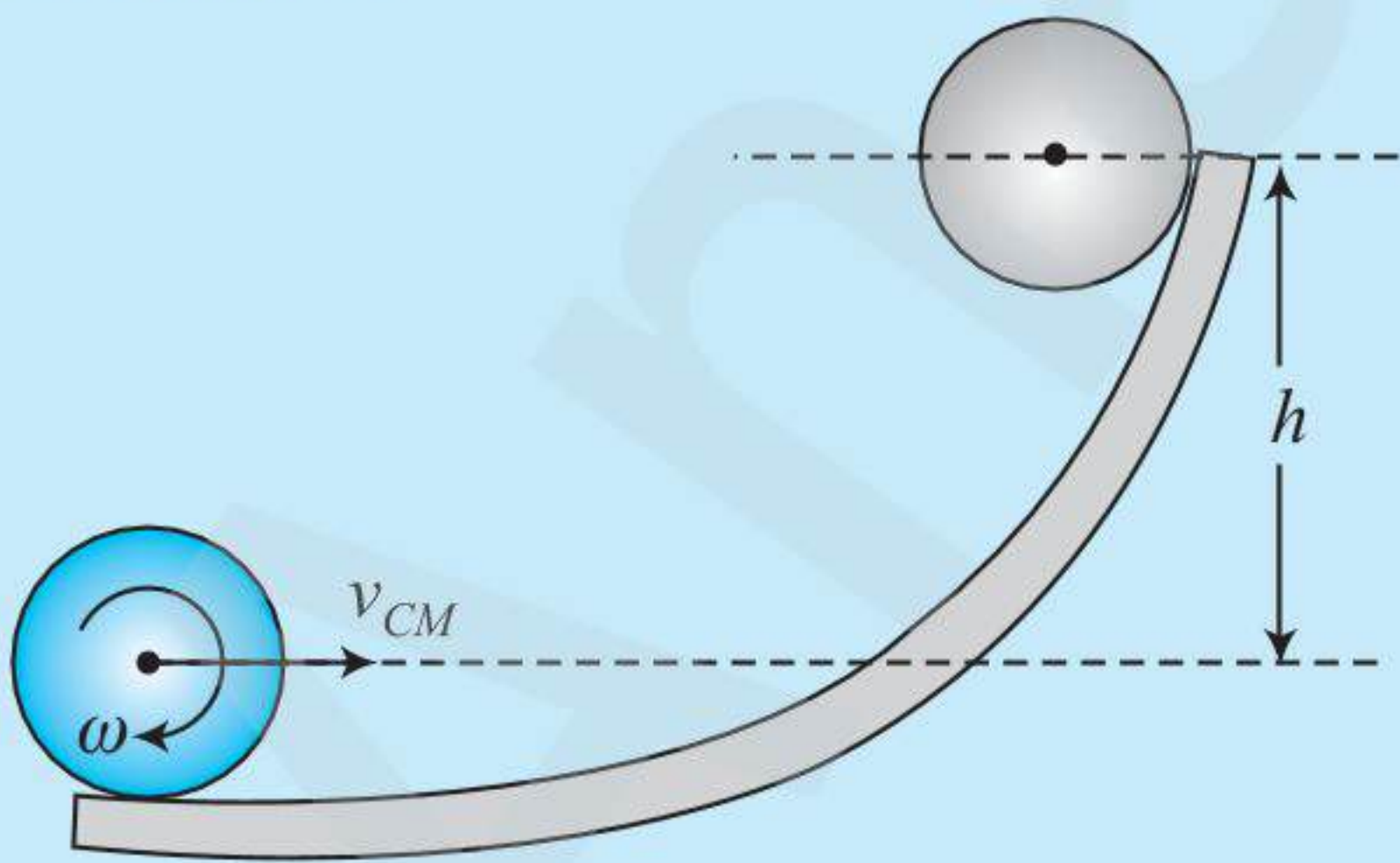
Hence from (i),

$$\left(\frac{7}{10} mv^2 - 0\right) + (-mg(R-r)) = 0$$

$$\text{We get, } v = \sqrt{\frac{10}{7} g(R-r)}$$

ILLUSTRATION 4.46

A ball of radius R and mass m is rolling without slipping on a horizontal surface with velocity of its centre of mass v_0 . It then rolls without slipping up a hill to a height h before momentarily coming to rest. Find h .



Sol. Mechanical energy is conserved because static friction does not perform any work. As the ball is rolling, then initial kinetic energy of the ball,

$$K_{\text{initial}} = (1+c) \frac{1}{2} mv^2 = \left(1 + \frac{2}{5}\right) \frac{1}{2} mv^2 = \frac{7}{10} mv_0^2$$

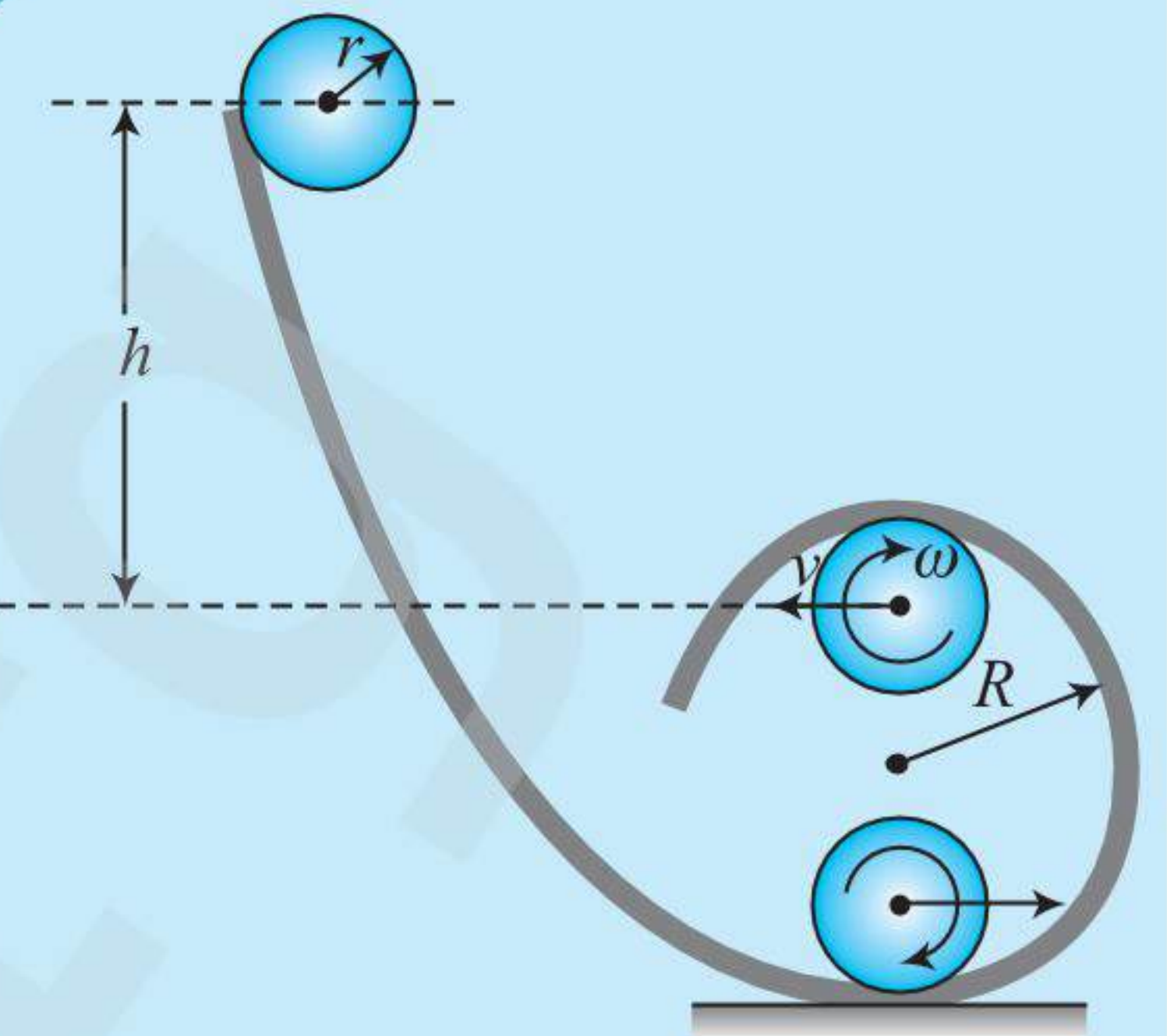
As mechanical energy is conserved, the initial kinetic energy will be converted to potential energy

$$\Delta K + \Delta U = 0$$

$$\Rightarrow \left(0 - \frac{7}{10} mv_0^2\right) + (mgh) = 0 \text{ or } h = \frac{7v_0^2}{10g}$$

ILLUSTRATION 4.47

A spherical ball of mass m and radius r is released on the path as shown in figure. If friction is sufficient to prevent the sliding of the ball on every point on the track, find the minimum height h (above the top position in the loop) that will permit the ball to maintain constant contact with the surface of the loop.



Sol. According to conservation of mechanical energy, the kinetic energy of the sphere at the top position in the loop is equal to the decrease in potential energy (mgh) as it falls from the initial position to this position.

As the ball is rolling, the kinetic energy of the ball,

$$K = (1+c) \frac{1}{2} mv^2 = \left(1 + \frac{2}{5}\right) \frac{1}{2} mv^2 = \frac{7}{10} mv^2 \quad \dots(i)$$

In the critical case when the ball just negotiates the loop, the force exerted by the loop on the ball is zero when the latter reaches the top of the loop. The centripetal force needed for the circular motion of the ball is supplied entirely by gravity:

$$\frac{mv^2}{(R-r)} = mg$$

$$\Rightarrow v^2 = (R-r)g \quad \dots(ii)$$

From (i) and (ii), we get

$$K = \frac{7}{10} m(R-r)g \quad \dots(iii)$$

As mechanical energy is conserved, then $\Delta K + \Delta U = 0$

$$\left(\frac{7}{10} m(R-r)g - 0\right) + (-mgh) = 0$$

$$\Rightarrow h = \frac{7(R-r)}{10}$$

Hence $h = 7R/10$ is the minimum initial height required.

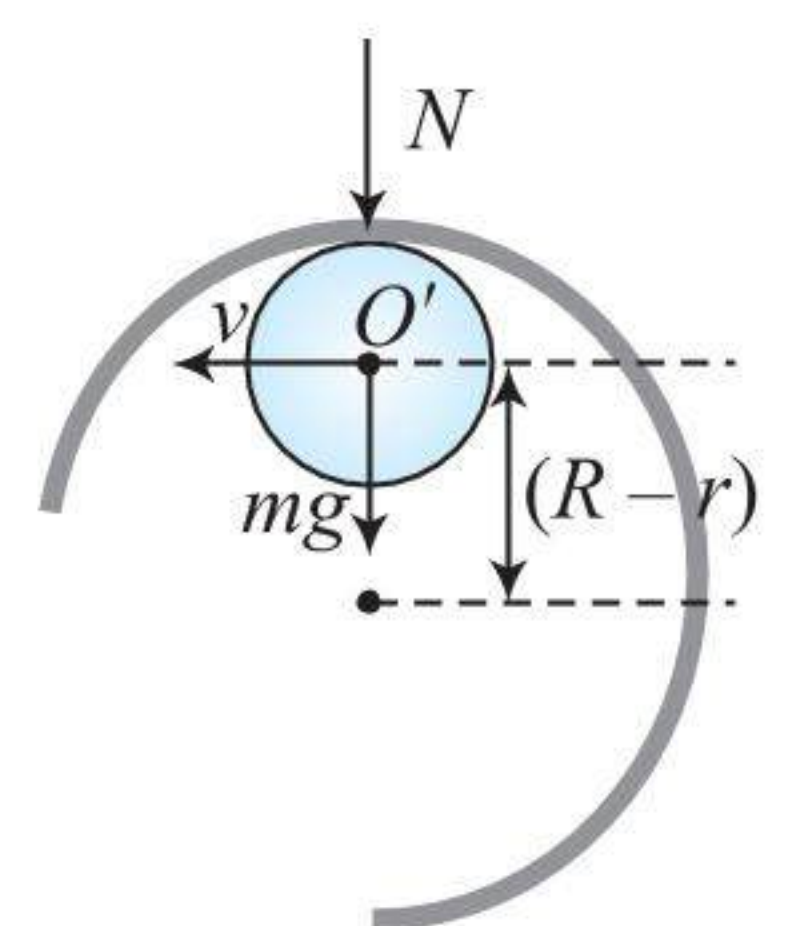
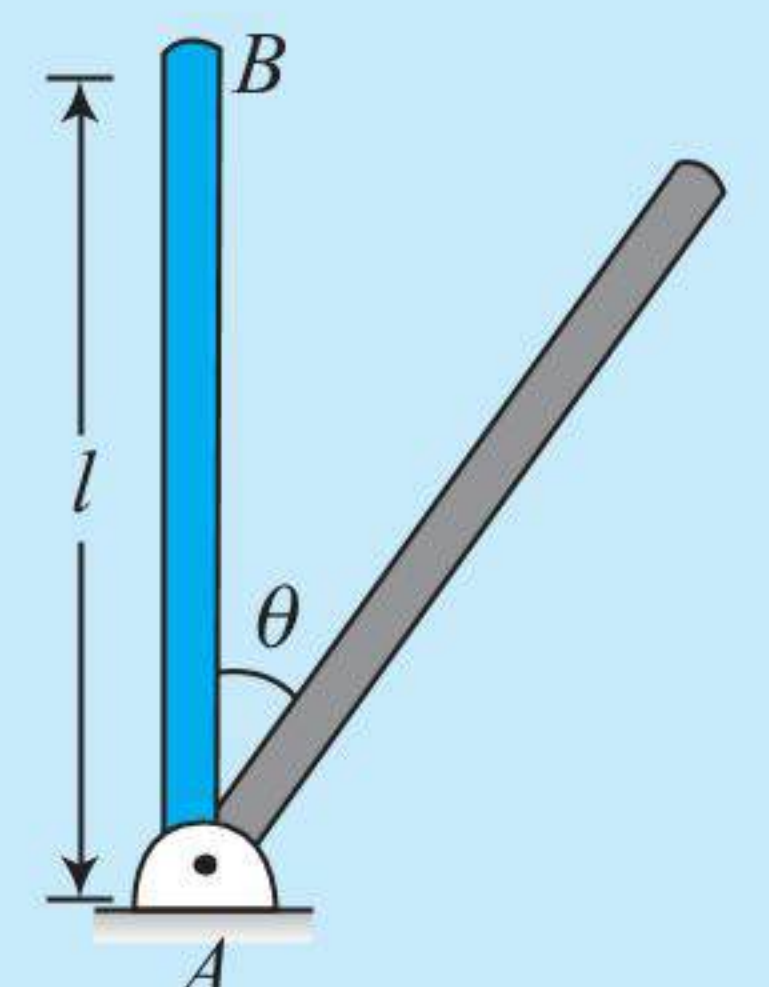
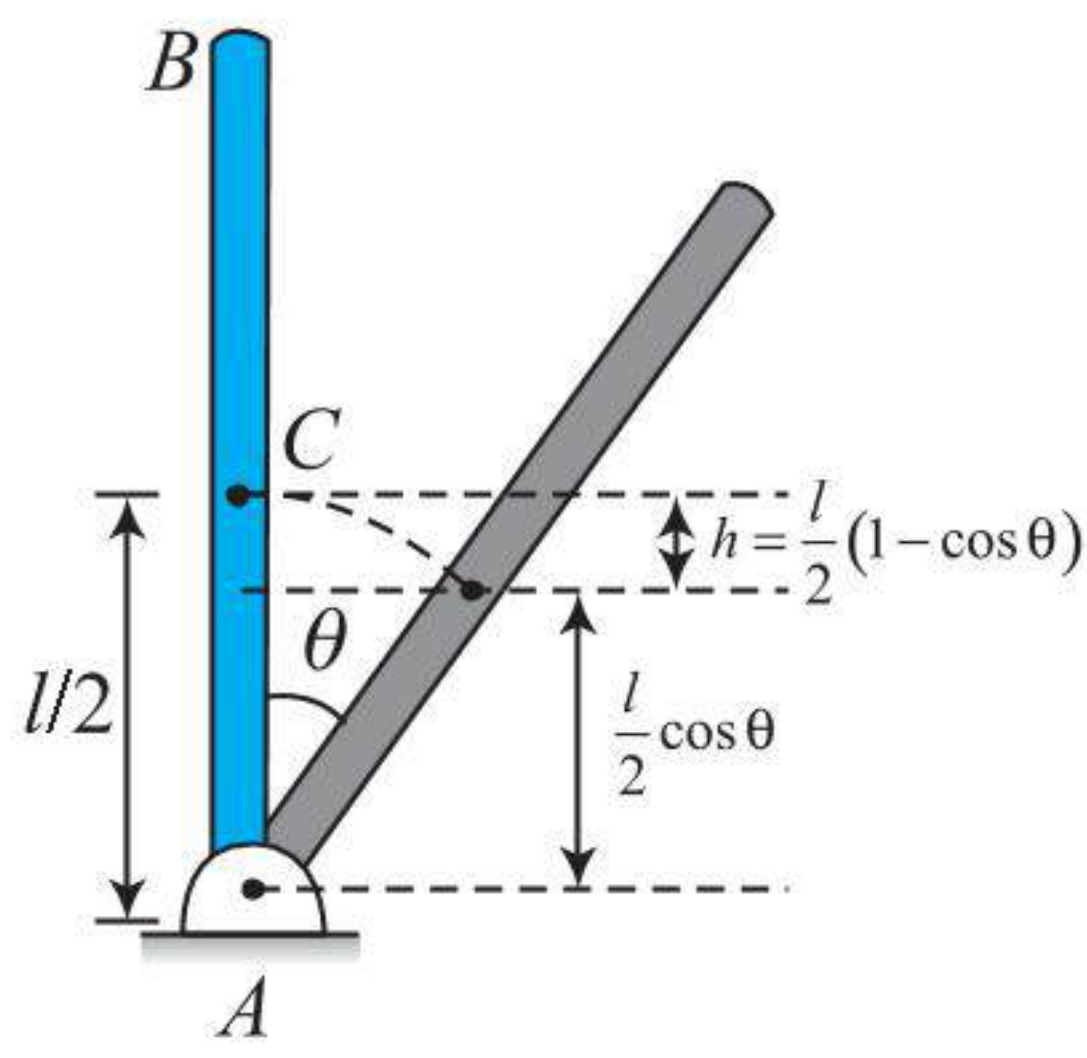


ILLUSTRATION 4.48

A rod of length l is pivoted about a horizontal, frictionless hinge through one end. The rod is released from rest in a vertical position. Find the velocity of the CM of the rod when the rod is inclined at an angle θ from the vertical.



Sol. The rod is rotating about frictionless hinge, its mechanical energy should be conserved.



According to conservation of energy,

$$\Delta K + \Delta U = 0$$

$$\Rightarrow (K_f - K_i) + \Delta U = 0 \quad \dots(i)$$

Initially the rod is released from rest $K_i = 0$.

Kinetic energy of the rod when it makes an angle θ with vertical $K_f = \frac{1}{2} I_A \omega^2$ = Rotational kinetic energy about the rotational axes passing through hinge, it is also total kinetic energy of the rod.

The fall in position of the CM of the rod, $h = \frac{l}{2}(1 - \cos \theta)$.

Hence change in potential energy of the rod,

$$\Delta U = -mgh = -mg \frac{l}{2}(1 - \cos \theta)$$

$$\text{From (i), } \left(\frac{1}{2} \left(\frac{ml^2}{3} \right) \omega^2 - 0 \right) + \left(-mg \frac{l}{2}(1 - \cos \theta) \right) = 0$$

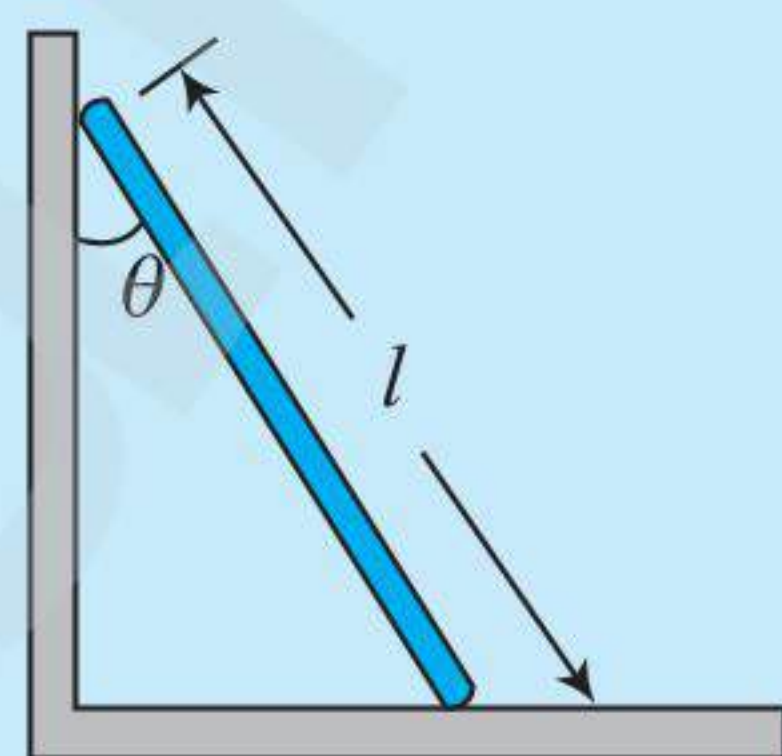
$$\therefore \omega = \sqrt{\frac{3g}{l}(1 - \cos \theta)}$$

The velocity of the CM of the rod, $v_{cm} = \omega R$ is

$$v_{cm} = \sqrt{\frac{3g}{l}(1 - \cos \theta)} \times \frac{l}{2} = \sqrt{\frac{3gl(1 - \cos \theta)}{4}}$$

ILLUSTRATION 4.49

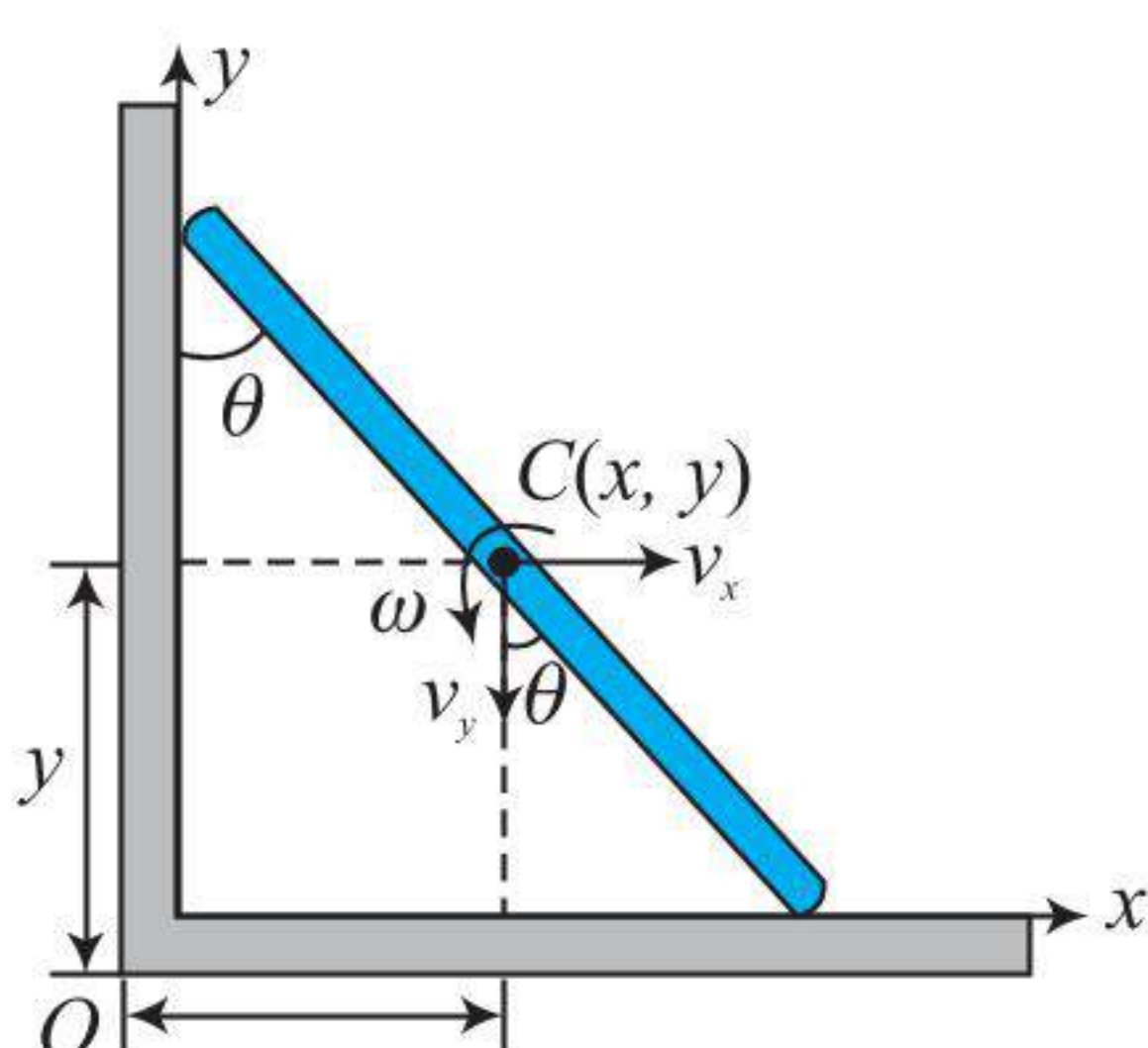
A uniform rod of length l is released from rest. In consequence, it rotates in vertical plane and its centre of mass moves down under the action of gravity and the reaction forces offered by horizontal and vertical smooth surfaces. Find the angular speed of the rod as the function of angle θ made by it with the vertical.



Sol. Method 1: The rod is sliding without friction, its mechanical energy should be conserved. According to conservation of energy,

$$\Delta K + \Delta U = 0$$

$$\Rightarrow (K_f - K_i) + \Delta U = 0 \quad \dots(i)$$



As the rod rotates through an angle θ its centre of mass falls through a vertical distance

$$h = \frac{l}{2}(1 - \cos \theta).$$

$$\text{Hence } \Delta U = -mgh = -mg \frac{l}{2}(1 - \cos \theta)$$

Initially the rod is released from rest, $K_i = 0$.

If the rod spins with an angular velocity ω and horizontal and vertical velocities of centre of mass of the rod be v_x and v_y respectively, the kinetic energy of the rod is given as

$$K_f = K_{\text{translational}} + K_{\text{rotational}} = \frac{1}{2} m v_c^2 + \frac{1}{2} I_c \omega^2$$

$$K_f = \frac{1}{2} m (v_x^2 + v_y^2) + \frac{1}{2} \left(\frac{ml^2}{12} \right) \omega^2 \quad \dots(ii)$$

Since $x = \frac{l}{2} \sin \theta$, we have

$$v_x = \frac{l}{2} \cos \theta \frac{d\theta}{dt} = \frac{l\omega}{2} \cos \theta$$

Similarly, as $y = \frac{l}{2} \cos \theta$, we have

$$v_y = \frac{l}{2} \left(-\sin \theta \frac{d\theta}{dt} \right) = -\frac{l\omega}{2} \sin \theta$$

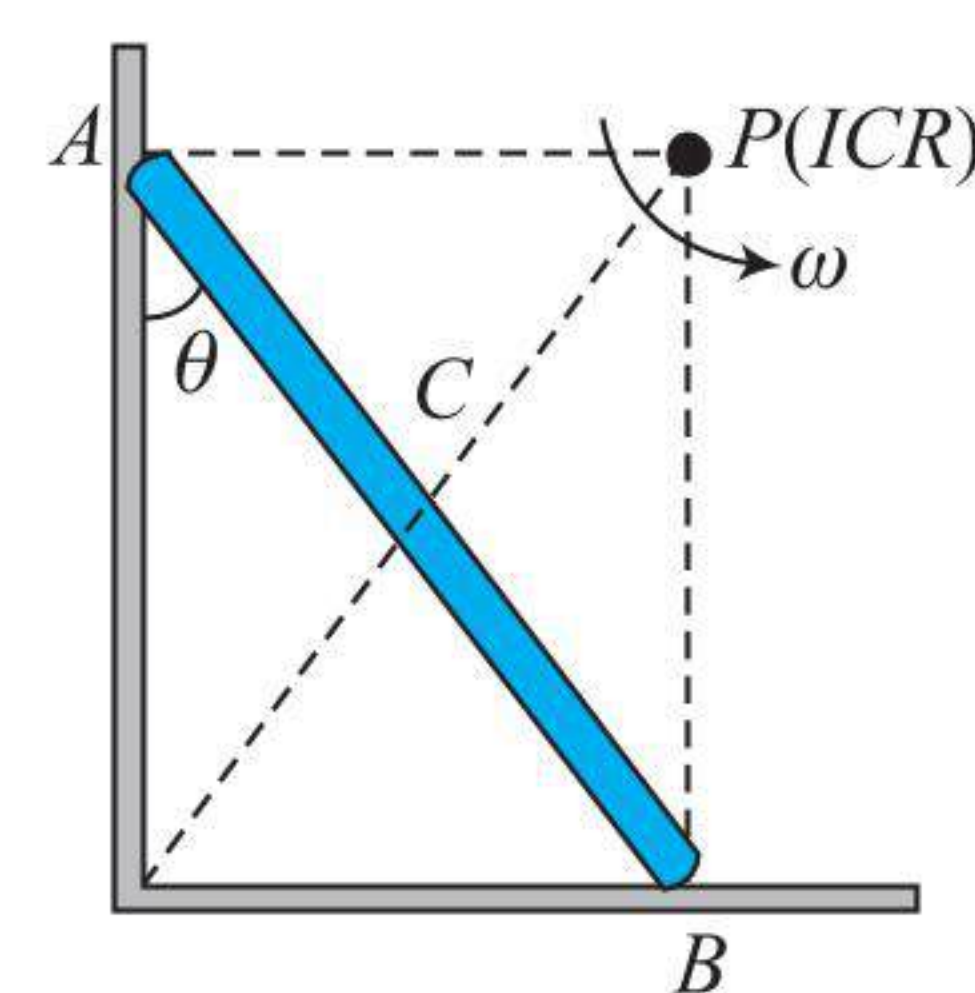
Substituting v_x and v_y in Eq. (ii), we have

$$K_f = \frac{ml^2 \omega^2}{6}$$

Finally substituting ΔU and ΔK in the Eq. (i), we have

$$\omega = \sqrt{\frac{3g}{l}(1 - \cos \theta)}$$

Method 2: Instantaneous axis of rotation (IAR) passes through P . There will be pure rotation of the rod about P .



From conservation of mechanical energy, $\Delta K + \Delta U = 0$.

$$\Rightarrow (K_f - K_i) + \Delta U = 0 \quad \dots(i)$$

Change in gravitational potential energy

$$\Delta U = -mgh = -mg \frac{l}{2}(1 - \cos \theta)$$

Initially the rod is released from rest, $K_i = 0$.

When the rod makes an angle θ with vertical let it spins with an angular velocity ω about IAR. It's kinetic energy at this position

$$K_f = \frac{1}{2} I_P \omega^2$$

$$\text{Here, } I_P = I_C + m(PC)^2 = \frac{ml^2}{12} + m \left(\frac{l}{2} \right)^2 = \frac{ml^2}{3}$$

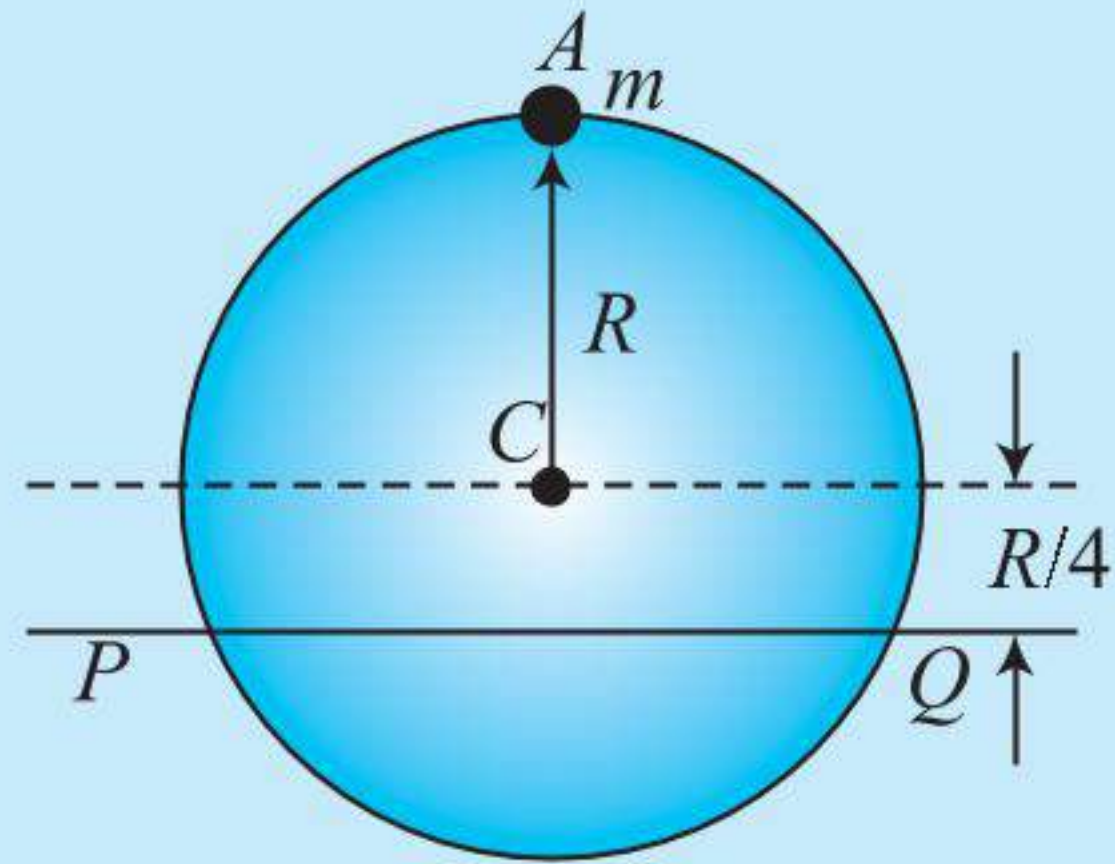
$$\text{Hence, } K_f = \frac{1}{2} \left(\frac{ml^2}{3} \right) \omega^2$$

Finally substituting ΔU and ΔK in the Eq. (i), we have

$$\omega = \sqrt{\frac{3g}{l}} (1 - \cos \theta)$$

ILLUSTRATION 4.50

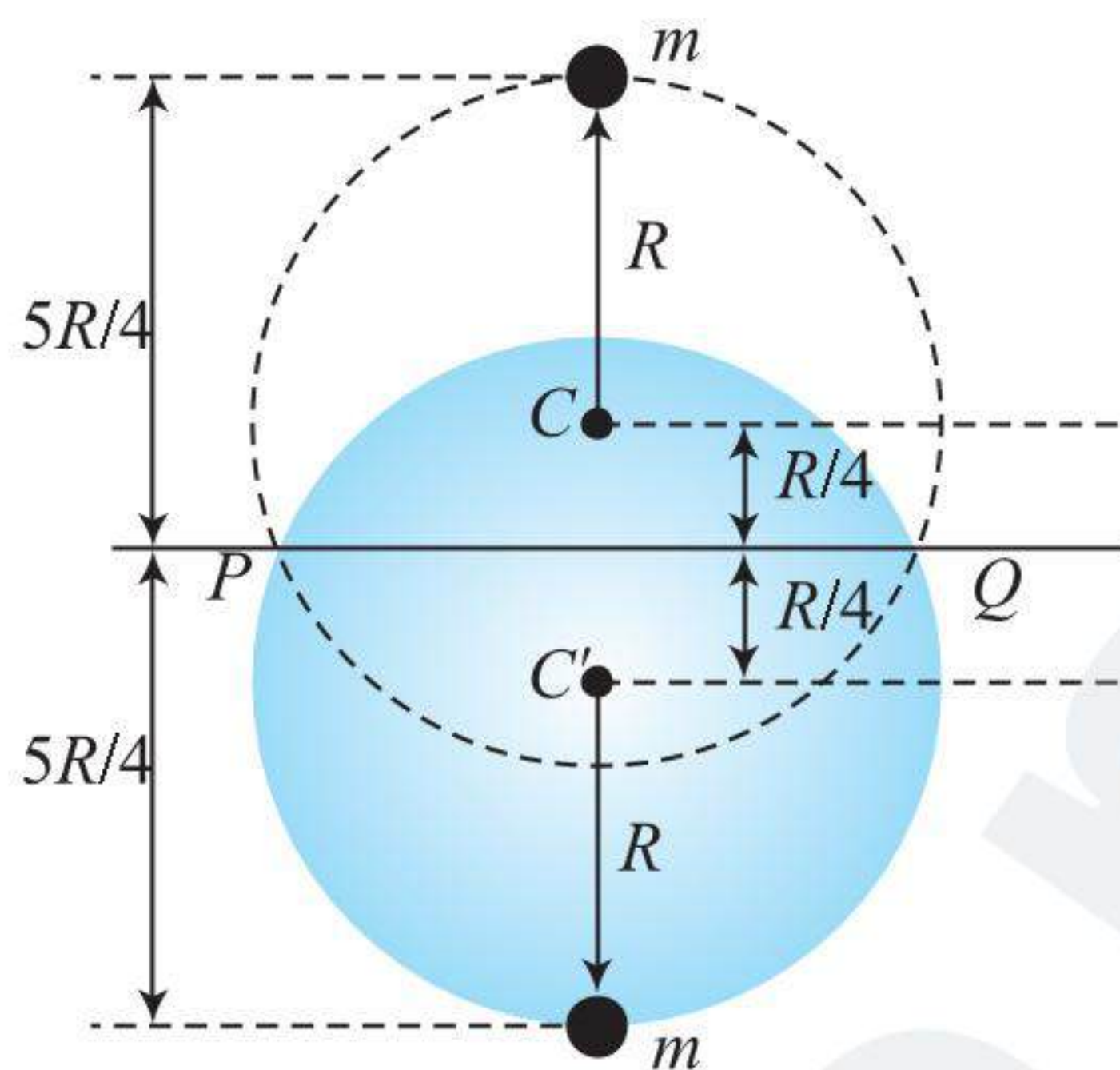
A uniform circular disc has radius R and mass m . A particle, also of mass m , is fixed at point A on the edge of the disc as shown in figure. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance $R/4$ from the centre C of the disc. The line AC is perpendicular to PQ . Initially, the disc is held vertical with point A at its highest position. It is then allowed to fall so that it starts rotating about PQ . Find the linear speed of the particle as it reaches its lowest position.



Sol. The system (disc + particle) is rotating about fixed axis without friction, its mechanical energy should be conserved. According to conservation of energy, $\Delta K + \Delta U = 0$

$$\Rightarrow (K_f - K_i) + \Delta U = 0 \quad \dots(i)$$

During the fall, the disc-particle system gains rotational kinetic energy. This is at the expense of potential energy.



Change in gravitational potential energy,

$$\Delta U = - \left(mg2 \left(R + \frac{R}{4} \right) + mg2 \left(\frac{R}{4} \right) \right) = -3mgR$$

$$\text{Change in kinetic energy, } \Delta K = \left(\frac{1}{2} I_{PQ} \omega^2 - 0 \right)$$

where I_{PQ} = MI of the disc + MI of particle about PQ

$$I_{PQ} = (I_{\text{disc}})_{PQ} + (I_{\text{par}})_{PQ} = \left[\frac{mR^2}{4} + m \left(\frac{R}{4} \right)^2 \right] + m \left(\frac{5R}{4} \right)^2$$

$$= \frac{15mR^2}{8}$$

$$\text{Therefore, } \Delta K = \frac{1}{2} \left(\frac{15mR^2}{8} \right) \omega^2$$

Finally substituting ΔU and ΔK in the Eq. (i), we have

$$\frac{1}{2} \left(\frac{15mR^2}{8} \right) \omega^2 + (-3mgR) = 0$$

$$\Rightarrow \omega = \sqrt{\frac{16g}{5R}}$$

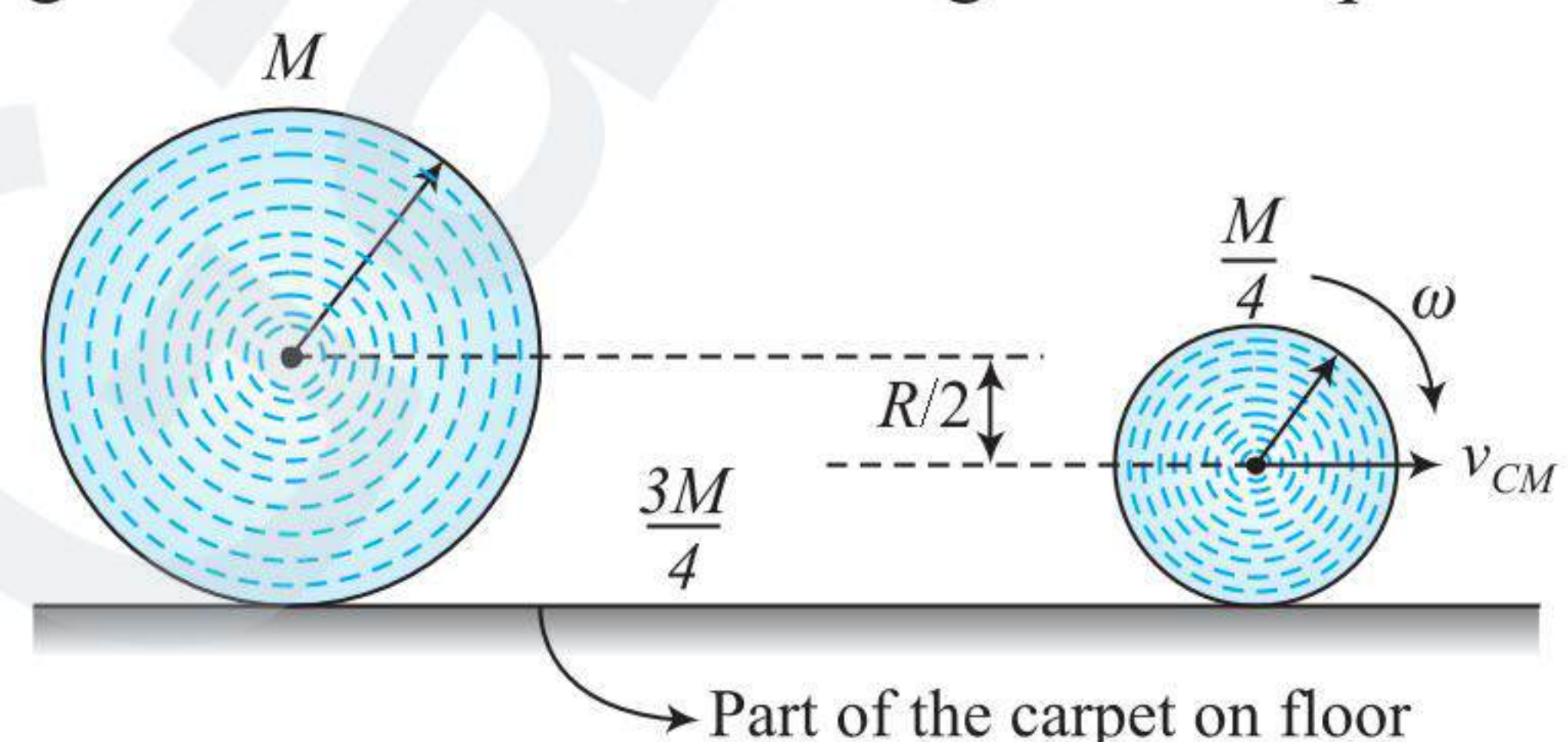
Let v be the velocity of mass m at the lowest point of rotation

$$v = \omega \left(R + \frac{R}{4} \right) = \sqrt{\frac{16g}{5R}} \times \frac{5R}{4} = \sqrt{5gR}$$

ILLUSTRATION 4.51

A carpet of mass M made of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to $R/2$.

Sol. Figure shows the forces acting on the carpet in two cases:



(i) when the radius of the roll is R

(ii) when the radius is $R/2$

Mass of the roll of radius $R/2 = M/4$

(Mass is proportional to the area of cross section)

The mass of the part of the carpet spread on the floor = $(3/4)M$.

The mechanical energy of the system will remain constant. The part of the carpet spread on the ground will have no kinetic energy.

According to conservation of energy, $\Delta K + \Delta U = 0$.

$$\Rightarrow (K_f - K_i) + \Delta U = 0. \quad \dots(i)$$

As the roll of the carpet is not sliding, we can consider that the roll of carpet as a rolling cylinder. We can calculate the kinetic energy of the moving cylindrical roll by the relation

$$K = (1 + c) \frac{mv^2}{2} = \left(1 + \frac{1}{2} \right) \frac{mv^2}{2} = \frac{3mv^2}{4}$$

M = mass of the roll at any time

Hence the kinetic energy of the roll when its radius is $R/2$

$$K_f = \frac{3(M/4)v^2}{4} = \frac{3Mv^2}{16}$$

Change in kinetic energy,

$$\Delta K = K_f - K_i = \frac{3Mv^2}{16}$$

Change in gravitational potential energy.

$$\Delta U = -MgR + \frac{1}{4} Mg \frac{R}{2} = -\frac{7}{8} MgR$$

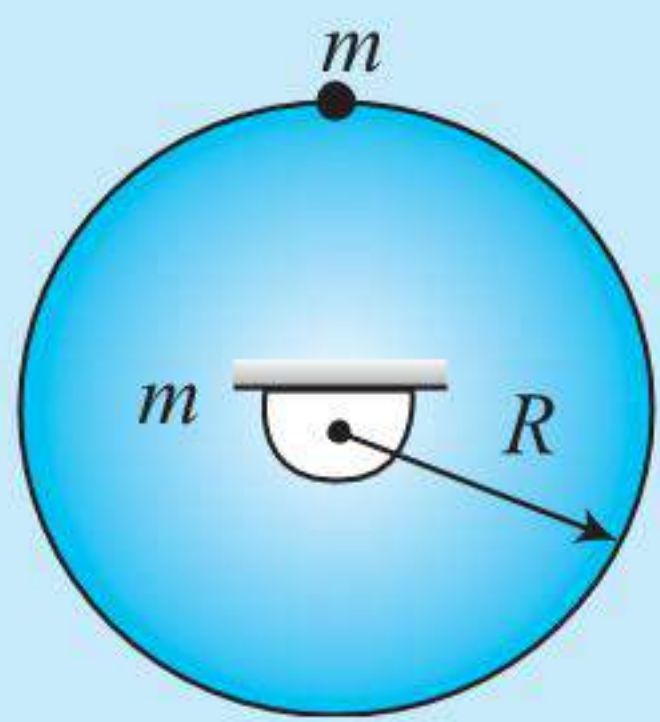
Finally substituting ΔU and ΔK in the Eq. (i), we have

$$\Rightarrow \frac{3Mv^2}{16} + \left(-\frac{7}{8} MgR \right) = 0$$

$$\Rightarrow v = \sqrt{\frac{14}{3} gR}$$

ILLUSTRATION 4.52

A bead of mass m is welded at the periphery of the smoothly pivoted disc mass m and radius R . Find the speed of the bead at its lowest position.



Sol. When the bead comes to its lowest position

$$\Delta U = -mgh = -mg(2R) = -2mgR \quad \dots(i)$$

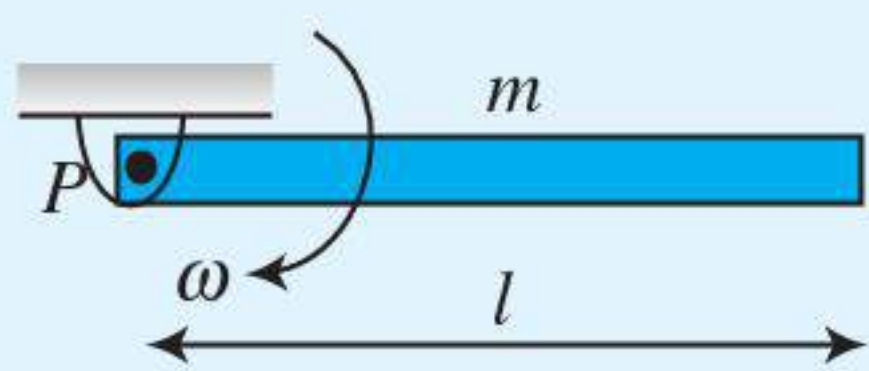
$$\begin{aligned} \Delta K' &= \Delta K'_{\text{bead}} + \Delta K_{\text{disc}} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\omega^2 \\ &= \frac{mv^2}{2} + \frac{mv^2}{4} = \frac{3}{4}mv^2 \quad \left(\text{because } \omega = \frac{v}{R}\right) \quad \dots(ii) \end{aligned}$$

Substituting ΔU from Eq. (i), ΔK from Eq. (ii) in the equation, $\Delta U + \Delta K = 0$, we have

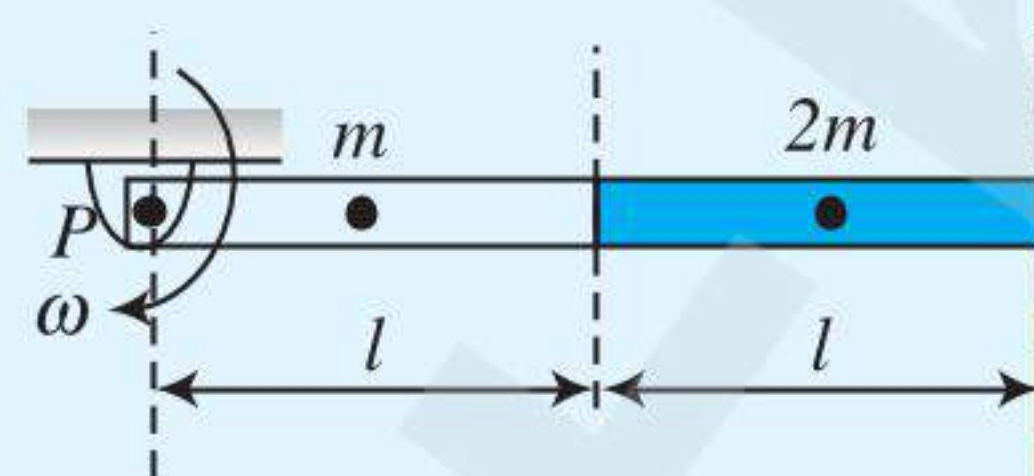
$$-2mgR + \frac{3}{4}mv^2 = 0 \quad \text{or} \quad v = \sqrt{\frac{8}{3}gR}$$

CONCEPT APPLICATION EXERCISE 4.5

1. A rod of mass m spins with an angular speed $\omega = \sqrt{g/l}$. Find its
(a) kinetic energy of rotation.
(b) kinetic energy of translation
(c) total kinetic energy.



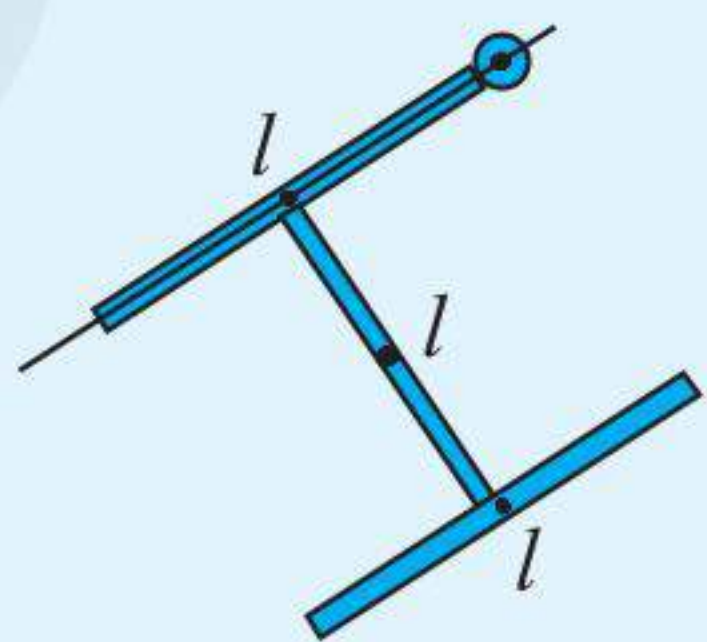
2. Find KE of the rotating composite rod comprising two rods of mass m and $2m$ and each of length $l = 1$ m. Assume $\omega = 10 \text{ rad s}^{-1}$ and $m = 1$ kg.



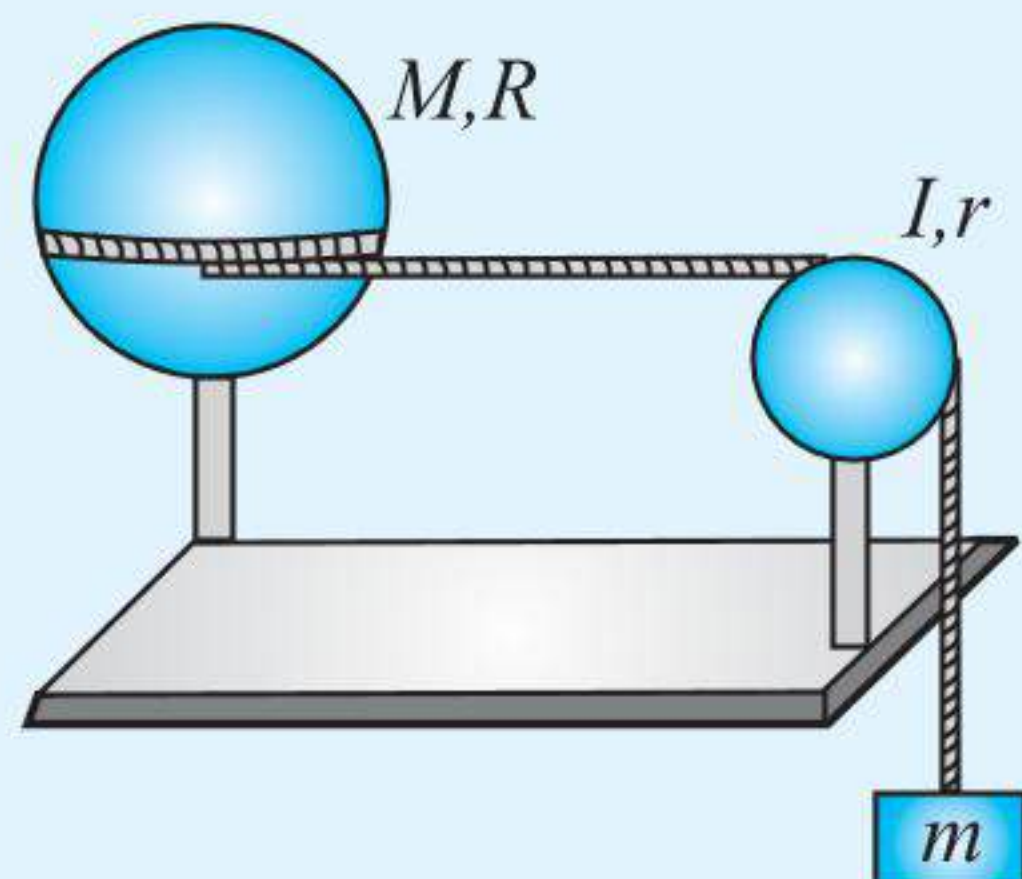
3. A uniform rod smoothly pivoted at one of its ends is released from rest. If it swings in vertical plane, find the maximum speed of the end P of the rod.



4. A rigid body is made of three identical thin rods, each with length l , fastened together in the form of letter H . The body is free to rotate about a horizontal axis that runs along the length of one of the arms of H . The body is allowed to fall from rest from a position in which the plane of the H is horizontal. What is the angular speed of the body when the plane of H is vertical?



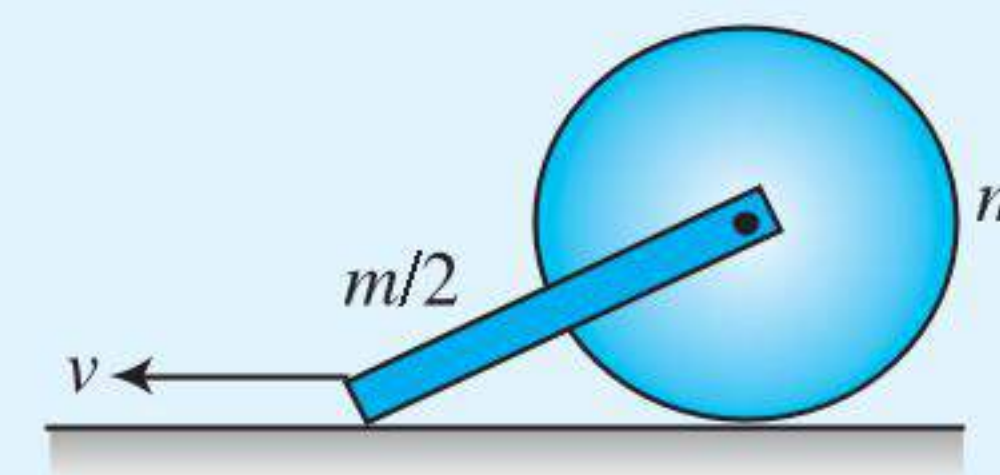
5. A uniform spherical shell of mass M and radius R rotates about a vertical axis on frictionless bearing. A massless cord passes around the equator of the shell, over a pulley of rotational inertia I and radius r and is attached to a small object of mass m that is otherwise free to fall under the influence of gravity. There is no friction of pulley's axle; the cord does not slip on the pulley. What is the speed of the object after it has fallen a distance h from rest? Use work-energy considerations.



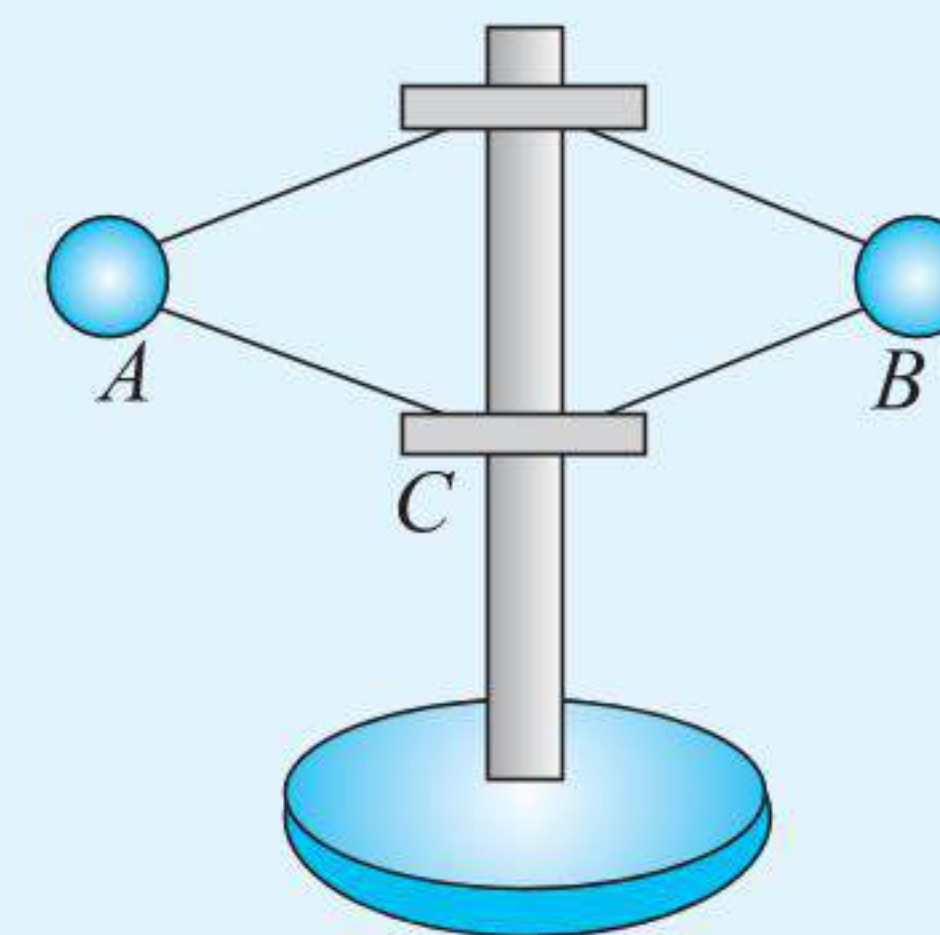
6. A uniform rod of mass m and length l is kept vertical with the lower end clamped. It is slightly pushed to let it fall down under gravity. Find its angular speed when the rod is passing through its lowest position. Neglect any friction at the clamp. What will be the linear speed of the free end at this instant?
7. A uniform rod of length l is released from rest such that it rotates about a smooth pivot. Find the angular speed of the rod when it becomes vertical.



8. A solid uniform disk of mass m and radius R is pivoted about a horizontal axis through its centre and a small body of mass m is attached to the rim of the disk. If the disk is released from rest with the small body, initially at the same level as the centre, find the angular velocity when the small body reaches the lowest point of the disk.
9. A copper ball of mass $m = 1$ kg with a radius of $r = 10$ cm rotates with angular velocity $\omega = 2$ rad/s about an axis passing through its centre. What work should be performed to increase the angular velocity of rotation of the ball two fold?
10. A uniform disc of mass m is fitted (pivoted smoothly) with a rod of mass $m/2$. If the bottom of the rod is pulled with a velocity v , it moves without changing its angle of orientation and the disc rolls without sliding. Find the kinetic energy of the system (rod + disc).



11. The steel balls A and B have a mass of 500 g each and are rotating about the vertical axis with an angular velocity of 4 rad/s at a distance of 15 cm from the axis. Collar C is now forced down until the balls are at a distance of 5 cm from the axis. How much work must be done to move the collar down?

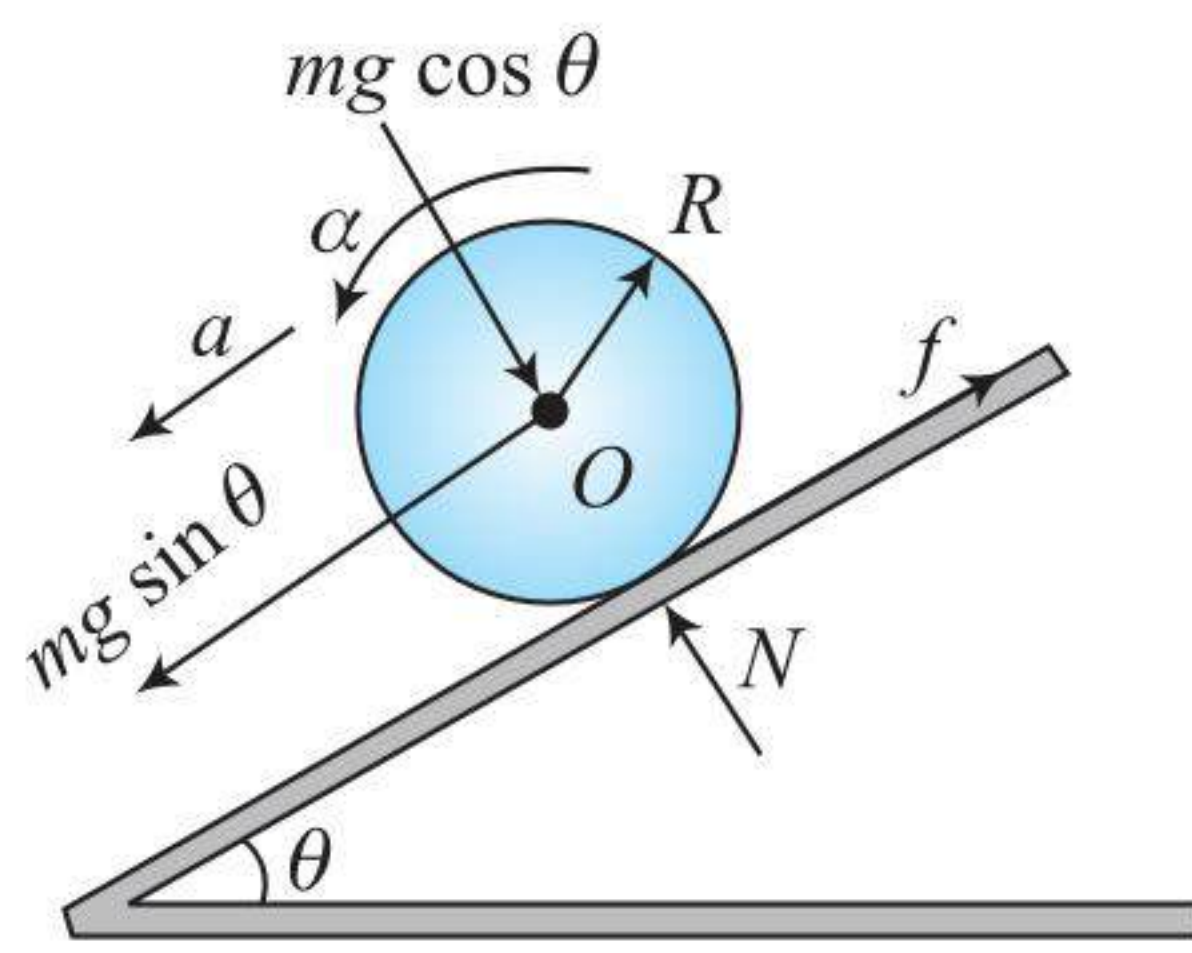
**ANSWERS**

1. (a) $\frac{mgl}{24}$ (b) $\frac{mgl}{8}$ (c) $\frac{mgl}{6}$
2. 250 J
3. $\sqrt{3gl}$
4. $\sqrt{\frac{9g}{4l}}$
5. $\left(\frac{mgh}{\frac{m}{2} + \frac{I}{2r^2} + \frac{M}{3}}\right)^{1/2}$
6. $\sqrt{\frac{6g}{l}}, \sqrt{6gl}$
7. $2\sqrt{\frac{6g}{7l}}$
8. $\sqrt{\frac{12g}{11r}}$
9. 2.4×10^{-2} J
10. mv^2
11. 1.44 J

ROLLING OF A BODY ON AN INCLINED PLANE

A rigid body of radius of gyration k and radius R rolls (without slipping) down a plane inclined at an angle θ with the horizontal.

When a body is placed on an inclined plane, it tries to slip down and hence a static friction f acts upwards. This friction provides a torque which causes the body to rotate. Let a be the linear acceleration of centre of mass and α be the angular acceleration of the body.



Acceleration of the rolling body can be determined by considering its pure rolling motion as superposition of rotational (about the centre of mass) and translational motion.

From force diagram: for linear motion parallel to the plane,

$$mg \sin \theta - f = ma \quad \dots(i)$$

For rotation around the axis through centre of mass, net torque

$$\tau = I\alpha$$

$$\Rightarrow fR = (mk^2)\alpha \quad \dots(ii)$$

As there is no slipping, the point of contact of the body with the plane is instantaneously at rest.

$$\Rightarrow v = R\omega \text{ and } a = R\alpha \quad \dots(iii)$$

Solving Eqs. (i), (ii) and (iii) for a and f ,

$$a = \frac{g \sin \theta}{\left(1 + \frac{k^2}{R^2}\right)} = \frac{g \sin \theta}{(1+c)}$$

$$\text{and } f = \frac{mg \sin \theta}{\left(1 + \frac{R^2}{k^2}\right)} = \frac{mg \sin \theta}{\left(1 + \frac{1}{c}\right)} = \frac{c}{(1+c)} mg \sin \theta$$

where c is a constant depends upon the shape of the rolling object.

We can also derive the condition for pure rolling (rolling without slipping), to avoid slipping, $f \leq \mu_s N$.

$$\left(\frac{c}{1+c}\right) mg \sin \theta \leq \mu_s mg \cos \theta \Rightarrow \mu_s \geq \left(\frac{c}{1+c}\right) \tan \theta$$

This is the condition of μ_s so that the body rolls without slipping.

Important Points:

- If $\mu_s = 0$, then the body will perform pure sliding and its acceleration will be maximum.

$$a_{\max} = g \sin \theta$$

- If $0 < \mu_s < \left(\frac{c}{1+c}\right) \tan \theta$, then the motion will be rolling

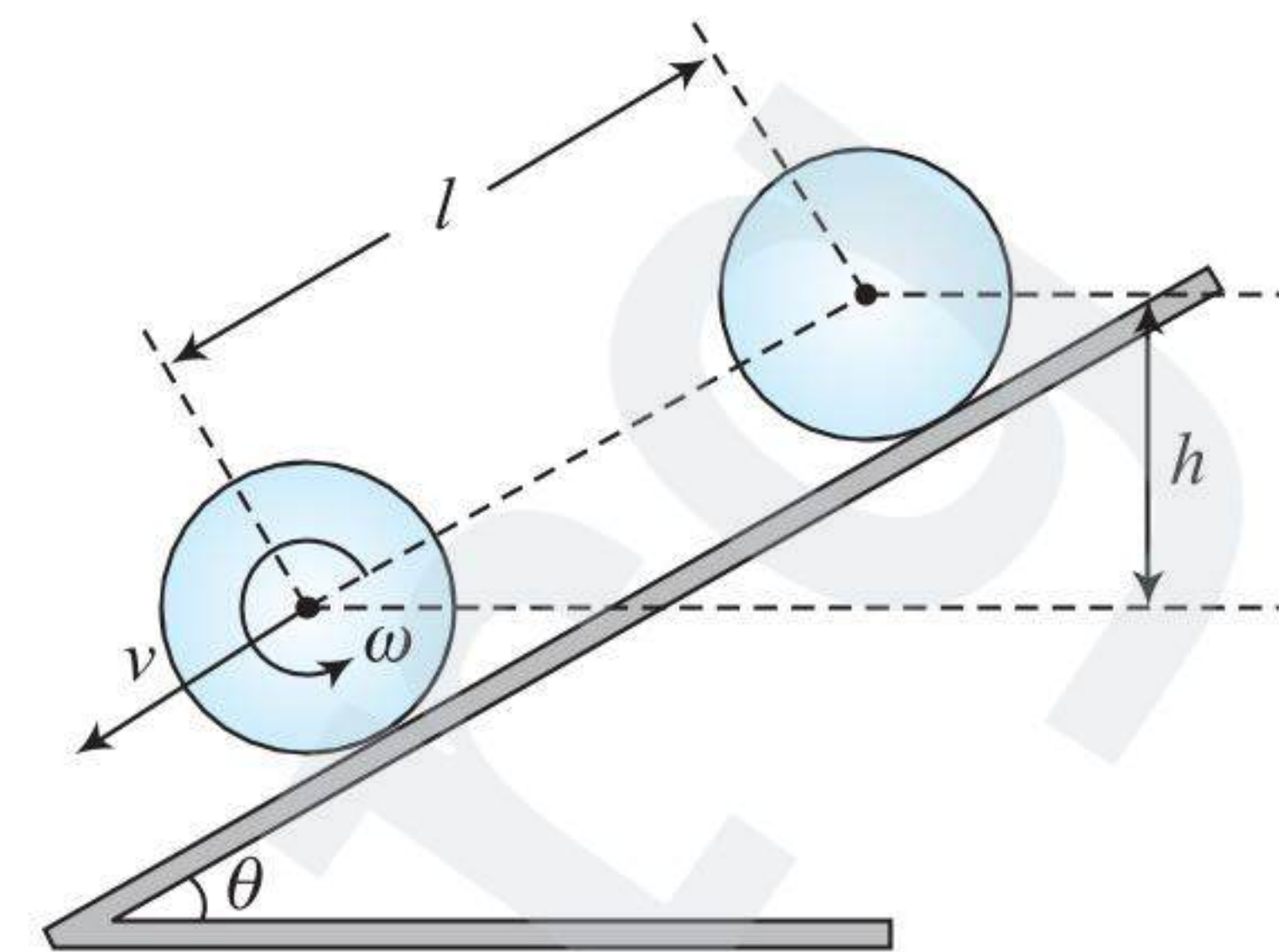
with sliding and then $\frac{g \sin \theta}{c+1} < a < g \sin \theta$

- If $\mu_s \geq \left(\frac{c}{1+c}\right) \tan \theta$, the motion will be pure rolling and the acceleration will be minimum.

$$a_{\min} = \frac{g \sin \theta}{c+1}$$

APPLICATION OF CONSERVATION OF MECHANICAL ENERGY FOR FINDING THE ACCELERATION OF ROLLING OBJECT

Let the rolling object is released from rest, after moving down the height h its velocity be v .



As the object is rolling, mechanical energy will remain conserved applying conservation of mechanical energy principle,

$$\Delta K + \Delta U = 0$$

$$\Rightarrow (K_f - K_i) + \Delta U = 0$$

$$\Rightarrow \left((1+c) \frac{1}{2} mv^2 - 0 \right) + (-mgh) = 0$$

$$\text{or } v^2 = \frac{2gh}{1+c} \Rightarrow v = \sqrt{\frac{2gh}{1+c}} \quad \dots(i)$$

Equation (i) gives the velocity of the rolling object when its CM moves down the height h .

$$\text{As } h = l \sin \theta \Rightarrow v^2 = \frac{2gl \sin \theta}{1+c} \quad \dots(ii)$$

Differentiating Eq. (i) w.r.t. time

$$2v \left(\frac{dv}{dt} \right) = \frac{2g \sin \theta}{1+c} \left(\frac{dl}{dt} \right) \Rightarrow 2va = \frac{2g \sin \theta}{1+c} v$$

$$\text{or } a = \frac{g \sin \theta}{1+c} \quad \left(\text{As } \frac{dv}{dt} = a \text{ and } \frac{dl}{dt} = v \right)$$

APPLICATION OF ANGULAR MOMENTUM PRINCIPLE FOR FINDING THE ACCELERATION OF ROLLING OBJECT

Let a rolling object with radius of gyration is rolling on an inclined plane. Let P is a fixed point on inclined plane. The angular momentum of the rolling body at any time about P is

$$\begin{aligned} \vec{L}_P &= \vec{r} \times m\vec{v} + mk^2 \vec{\omega} \\ &= -mvR\hat{k} - mk^2 \omega \hat{k} \\ &= -(mvR + mk^2 \omega) \hat{k}, \end{aligned}$$

where $v = R\omega$ for rolling,

$$\text{or } (\vec{L}_P)_t = -m(R^2 + k^2) \omega \hat{k}$$

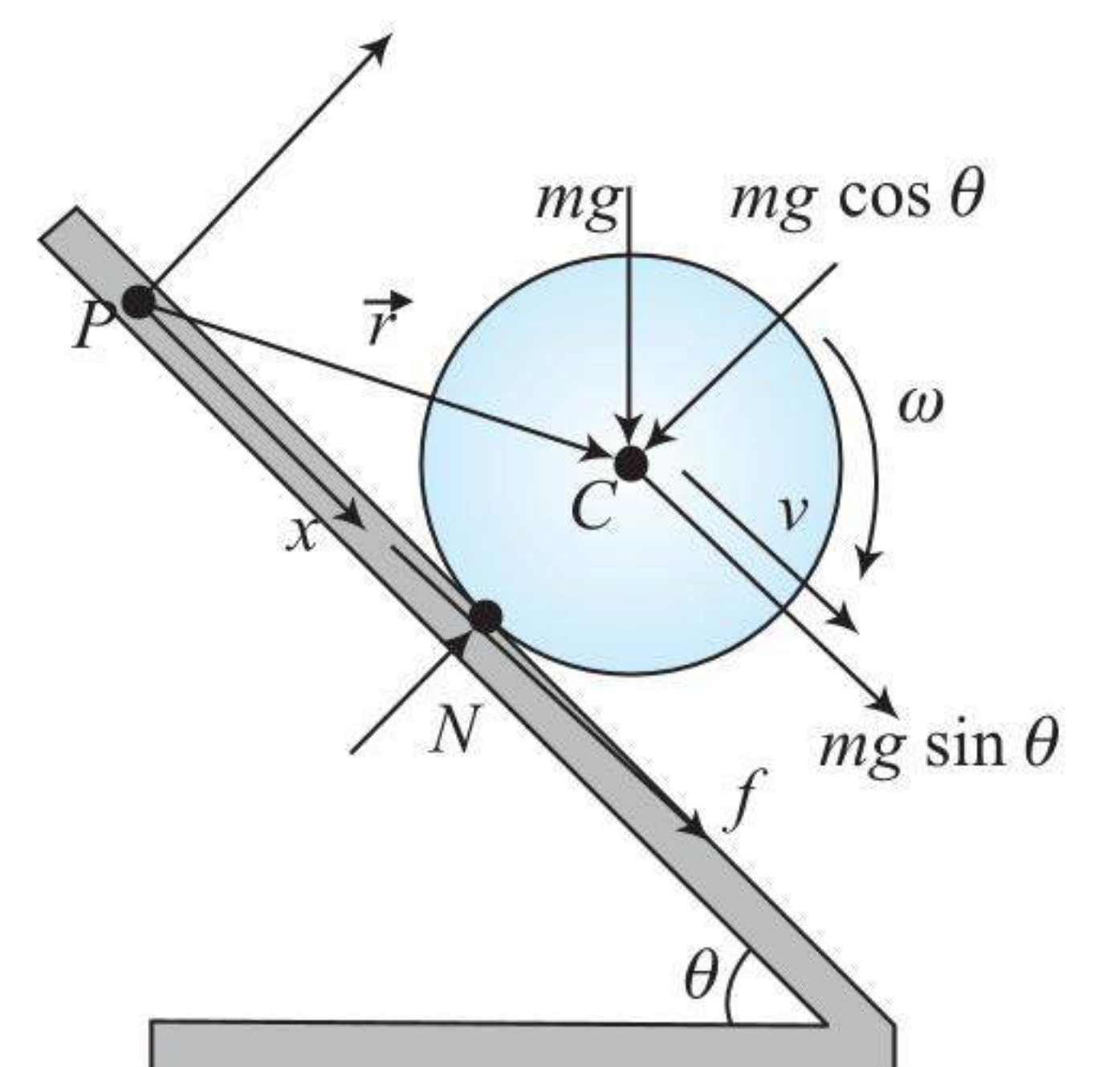
Since $(\vec{L}_P)_{\text{initial}} = 0$,

Change in angular momentum in time t

$$\Delta \vec{L}_P = (\vec{L}_P)_t - 0 = -m(R^2 + k^2) \omega \hat{k} \quad \dots(i)$$

The net torque about P is

$$\vec{\tau}_P = Rmg \sin \theta (-\hat{k}) \quad \dots(ii)$$



Substituting, $\Delta \vec{L}_p$ from Eq. (i) and τ_p from Eq. (ii) in

$$\int \vec{\tau}_p dt = \Delta \vec{L}_p, \text{ we have}$$

$$-\int mgR \sin \theta \hat{k} dt = -m(R^2 + k^2) \omega \hat{k}$$

Differentiating both sides, we have

$$mgR \sin \theta = m(R^2 + k^2) \alpha \hat{k}$$

$$\text{This gives } \alpha = \frac{gR \sin \theta}{R^2 + k^2}$$

But acceleration of CM of the rolling object

$$a = \alpha R = \left(\frac{gR \sin \theta}{R^2 + k^2} \right) R = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}} = \frac{g \sin \theta}{(1 + c)}$$

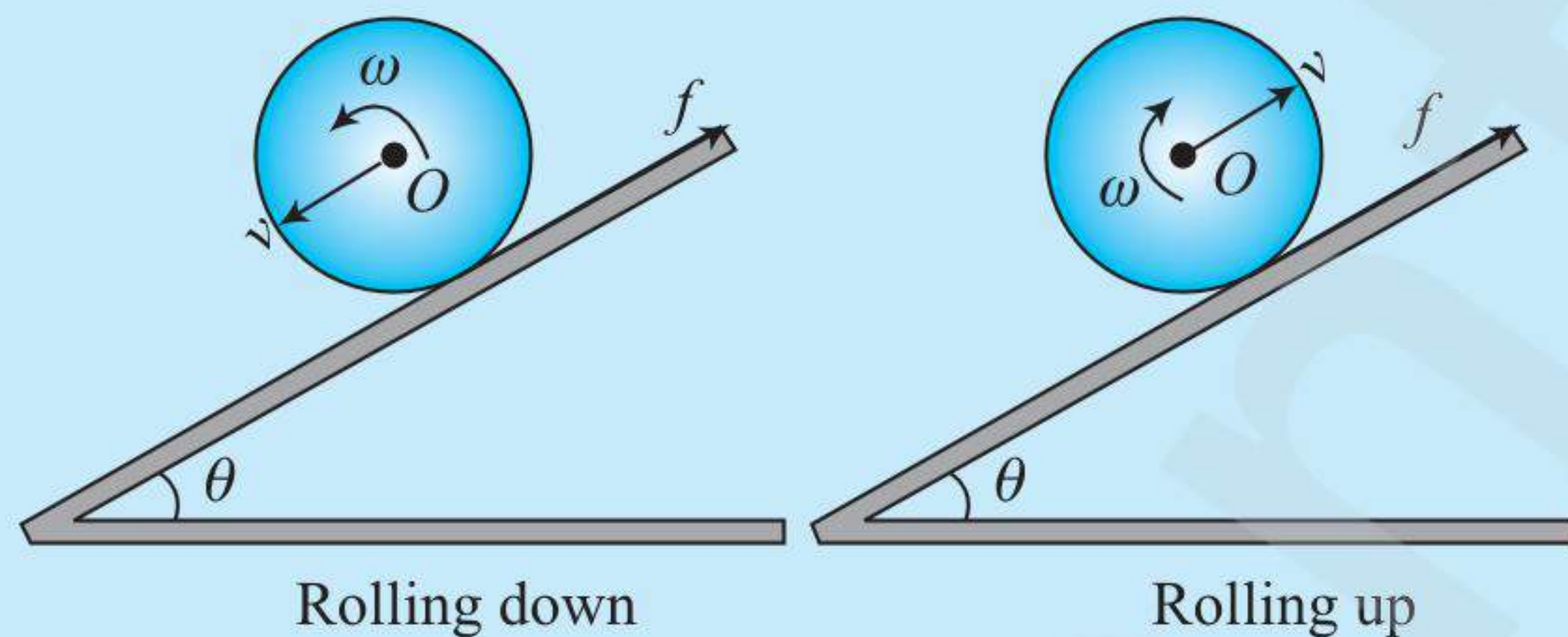
Comparison of pure rolling and pure sliding

	Pure Rolling	Pure Sliding
Acceleration	$a = \frac{g \sin \theta}{1 + c}$	$a = g \sin \theta$
Velocity	$v = \sqrt{\frac{2gh}{1 + c}}$	$v = \sqrt{2gh}$

The comparison reveals that $a_{\text{rolling}} < a_{\text{sliding}}$ and $v_{\text{rolling}} < v_{\text{sliding}}$

Important Points:

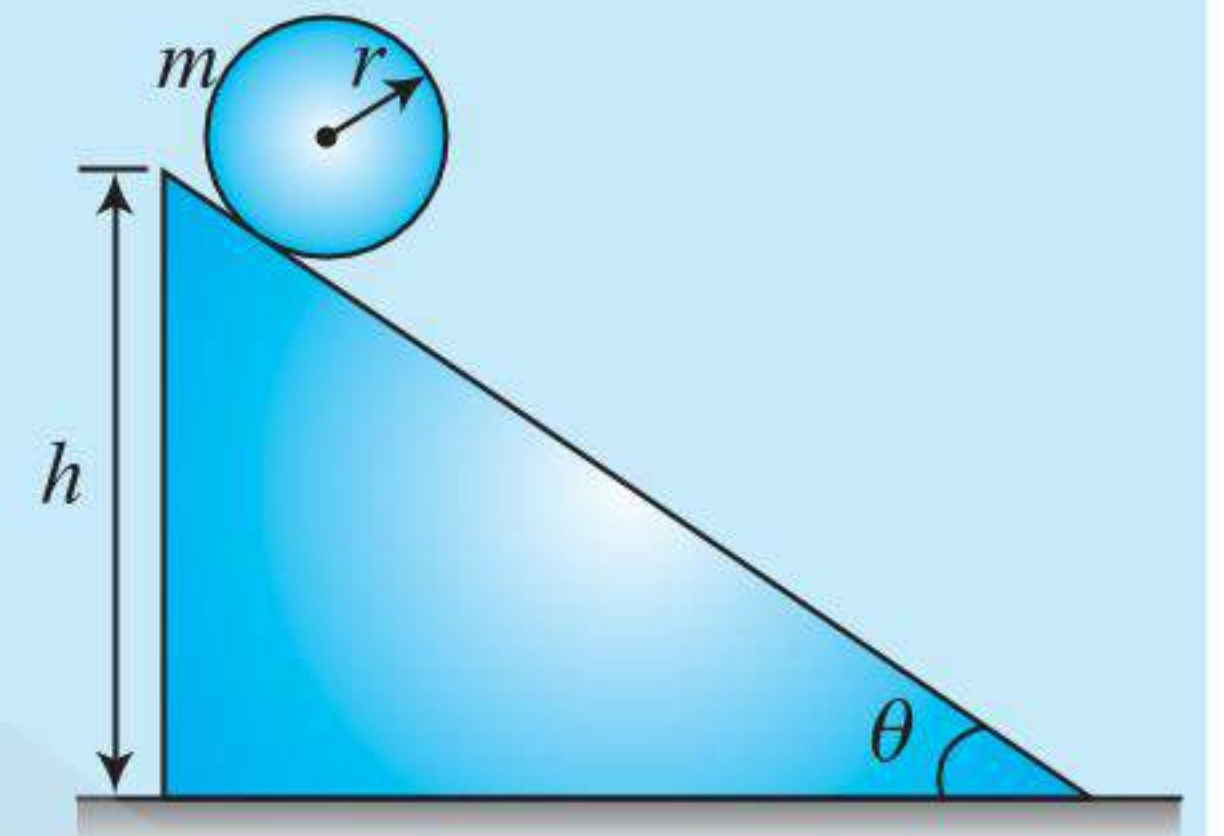
- If a body rolls up an inclined plane the direction of friction force in this case also in upward direction. As body rolls up velocity of centre of mass decreases, hence torque of friction should decrease angular velocity.



- The acceleration of the bodies rolling on inclined plane depends on radius of gyration (k). We can say the bodies with same value of k will have the same acceleration.
- For example, two uniform solid spheres made of different material, unequal mass and unequal radii are released from the same inclined plane if they roll, they will have same accelerations.
- If a solid sphere, a hollow sphere, a disc and a ring of same mass and radius are released from top of incline and they roll. The solid sphere takes the least time while the ring will take maximum time to reach the bottom.
- If all the four objects are released from the top of a smooth incline. All the four bodies will reach the bottom at the same time.
- If all the rolling objects are released on a rough incline, such that friction is not sufficient for pure rolling, still the objects reach the bottom of incline at the same time because acceleration of centre of mass is the same in each case. Each object will have the same velocity of centre of mass but different ω .

ILLUSTRATION 4.53

Consider a uniform solid cylinder of mass m and radius r rolling (without slipping) down at an angle θ with the horizontal, as shown in figure. Find the time taken and speed of its centre of mass when the cylinder reaches the bottom.



Sol. We can use the formula which we derived

$$a = \frac{g \sin \theta}{(1 + c)}; \text{ for cylinder, } c = \frac{k^2}{R^2} = \frac{R^2/2}{R^2} = \frac{1}{2}$$

$$\text{Hence, acceleration of the cylinder } a = \frac{2}{3} g \sin \theta$$

Let v_{CM} be the velocity of the centre of mass at the bottom. From equation $v^2 = u^2 + 2as$.

$$v_{\text{CM}}^2 = 0^2 + 2 \left(\frac{2}{3} g \sin \theta \right) \left(\frac{h}{\sin \theta} \right) = \frac{4}{3} gh$$

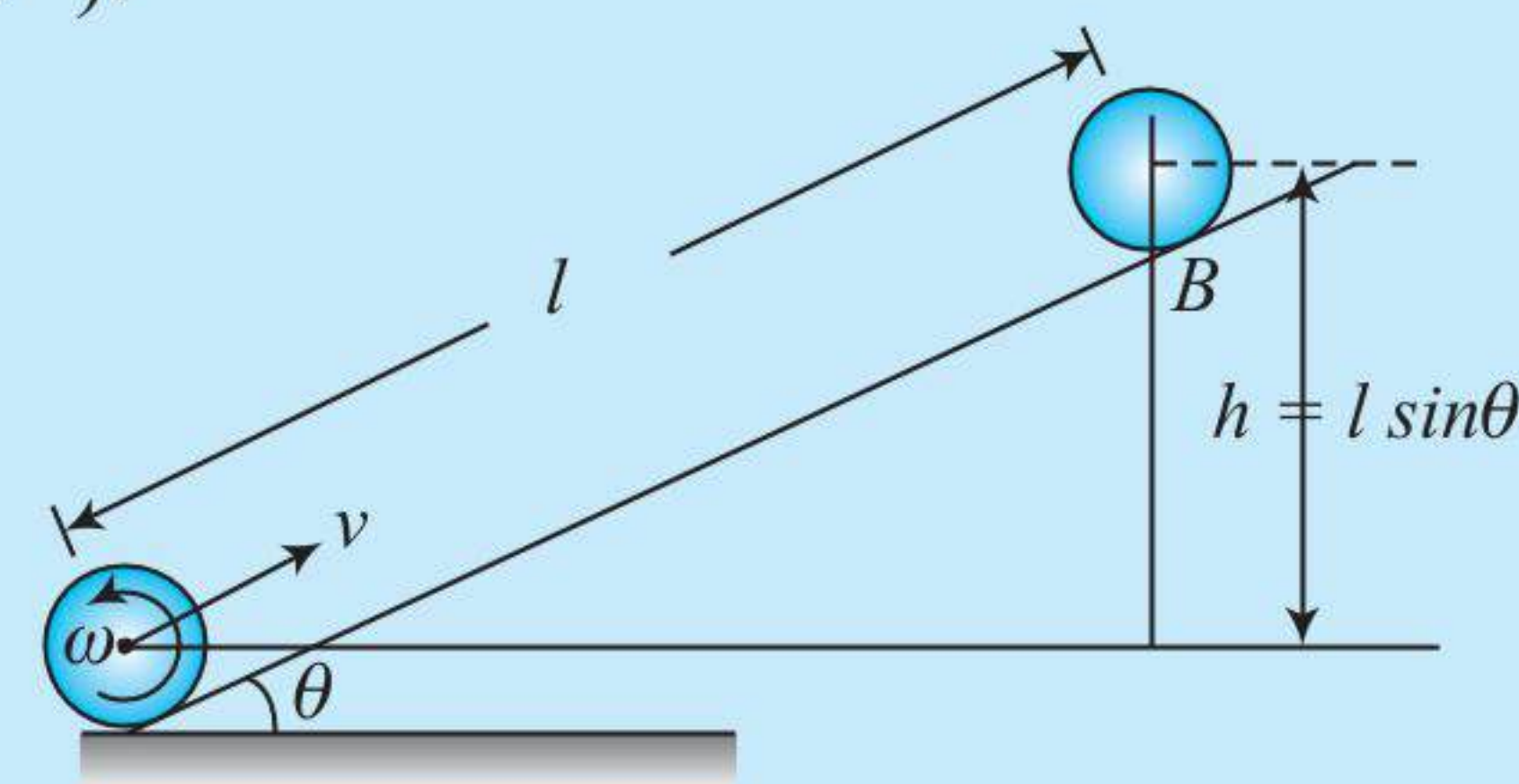
$$\Rightarrow v_{\text{CM}} = \sqrt{\frac{4gh}{3}}$$

Let time to reach bottom by cylinder is t , using $v = u + at$

$$\Rightarrow t = \frac{v - u}{a} = \frac{\sqrt{\frac{4gh}{3}} - 0}{\frac{2}{3} g \sin \theta} = \sqrt{\frac{3h}{g}} \frac{1}{\sin \theta}$$

ILLUSTRATION 4.54

A solid sphere is projected up along an inclined plane of inclination $\theta = 30^\circ$ with a speed $v = 2 \text{ m s}^{-1}$. If it rolls without slipping, find the maximum distance traversed by it. ($g = 10 \text{ m s}^{-2}$).



Sol. Let the solid sphere of mass m and radius r traverse a distance l along the inclined plane. Since it rolls without sliding its initial KE is given as

$$KE_i = (1 + c) \frac{1}{2} mv^2$$

where for solid sphere $c = \frac{2}{5}$

$$KE_i = \left(1 + \frac{2}{5} \right) \frac{1}{2} mv^2 = \frac{7}{10} mv^2$$

As it comes to rest attaining a height $h = l \sin \theta$, its final $KE = 0$. Therefore,

$$\Delta KE = -\frac{7}{10} mv^2 \quad \dots(i)$$

Change in gravitational potential energy, $\Delta U = mgh \quad \dots(ii)$
Conservation of energy yields $\Delta K + \Delta U = 0$

$$\Rightarrow \left(-\frac{7}{10}mv^2 \right) + (mgh) = 0$$

$$\Rightarrow h = \frac{7v^2}{10g} = \frac{7 \times (2)^2}{10 \times 10} = 0.28 \text{ m}$$

$$\therefore l = h \operatorname{cosec} \theta = 0.28 \operatorname{cosec} 30^\circ = 0.56 \text{ m}$$

ILLUSTRATION 4.55

A ring of mass m and radius R is released on a rough inclined plane inclined at angle θ with horizontal. The ring rolls with sliding on the inclined plane. The coefficient of the friction between ring and plane is $\mu = \frac{\tan \theta}{4}$. Find out work done by kinetic friction if centre of mass of the ring has displaced ' l ' length along the inclined plane.

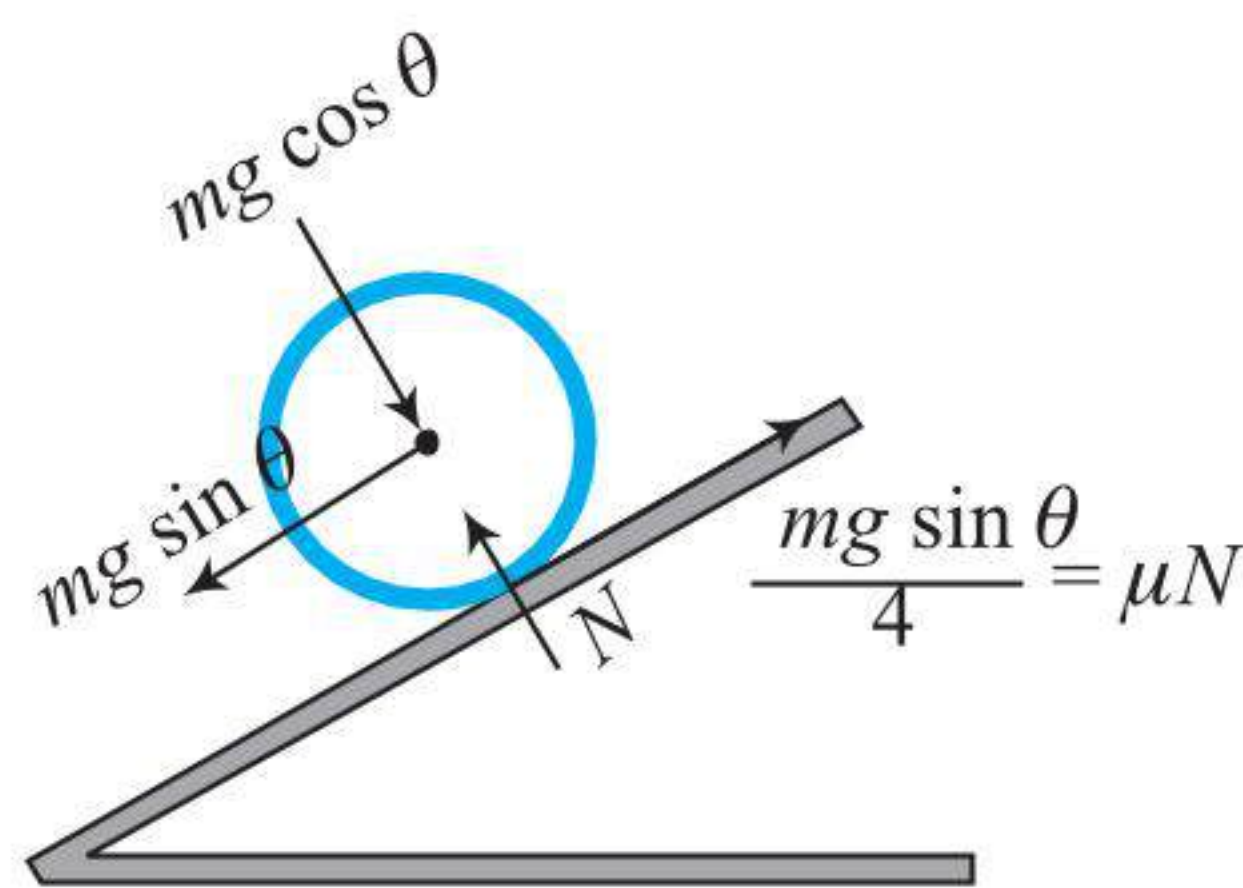
Sol. As the ring rolls with sliding it means the friction is not sufficient to provide rolling. In this case kinetic friction will act on it in upward direction.

For translatory motion:

$$mg \sin \theta - \mu N = ma \quad [N = mg \cos \theta]$$

$$mg \sin \theta - \left(\frac{\tan \theta}{4} \right) mg \cos \theta = ma$$

$$\Rightarrow a = \frac{3g \sin \theta}{4} \quad \dots(i)$$



For rotatory motion, applying torque equation about centroid axis: $\tau_{\text{cm}} = I_{\text{cm}} \alpha$

$$\left[\frac{mg \sin \theta}{4} \right] R = mR^2 \alpha$$

$$\alpha = \frac{g \sin \theta}{4R} \quad \dots(ii)$$

Time required to reach bottom of incline

$$s = ut + \frac{1}{2}at^2 \Rightarrow l = \frac{1}{2} \left[\frac{3g \sin \theta}{4} \right] t^2 \Rightarrow t = \sqrt{\frac{8l}{3g \sin \theta}}$$

At this time velocity and angular velocity of ring are

$$v = u + at = \sqrt{\frac{3gl \sin \theta}{2}}$$

$$\text{and } \omega = \omega_i + \alpha t = \sqrt{\frac{gl \sin \theta}{6R^2}}$$

By work energy theorem,

$$K_f - K_i = \text{work done by friction} + \text{work done by gravity}$$

$$\frac{1}{2}mv^2 + \frac{1}{2}mR^2\omega^2 = W_{fr} + W_g$$

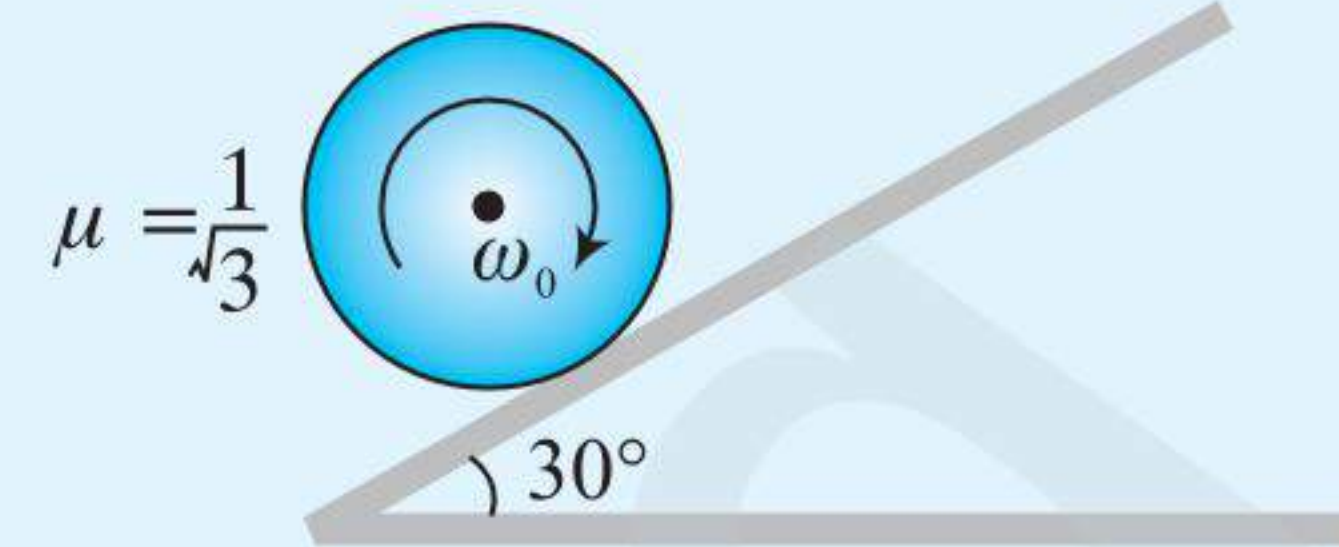
$$\Rightarrow \frac{1}{2}m \frac{3gl \sin \theta}{2} + \frac{1}{2}mR^2 \frac{gl \sin \theta}{6R^2} = W_{fr} + W_g$$

$$\frac{1}{2}mgl \sin \theta \left[\frac{3}{2} + \frac{1}{6} \right] = W_{fr} + mgl \sin \theta$$

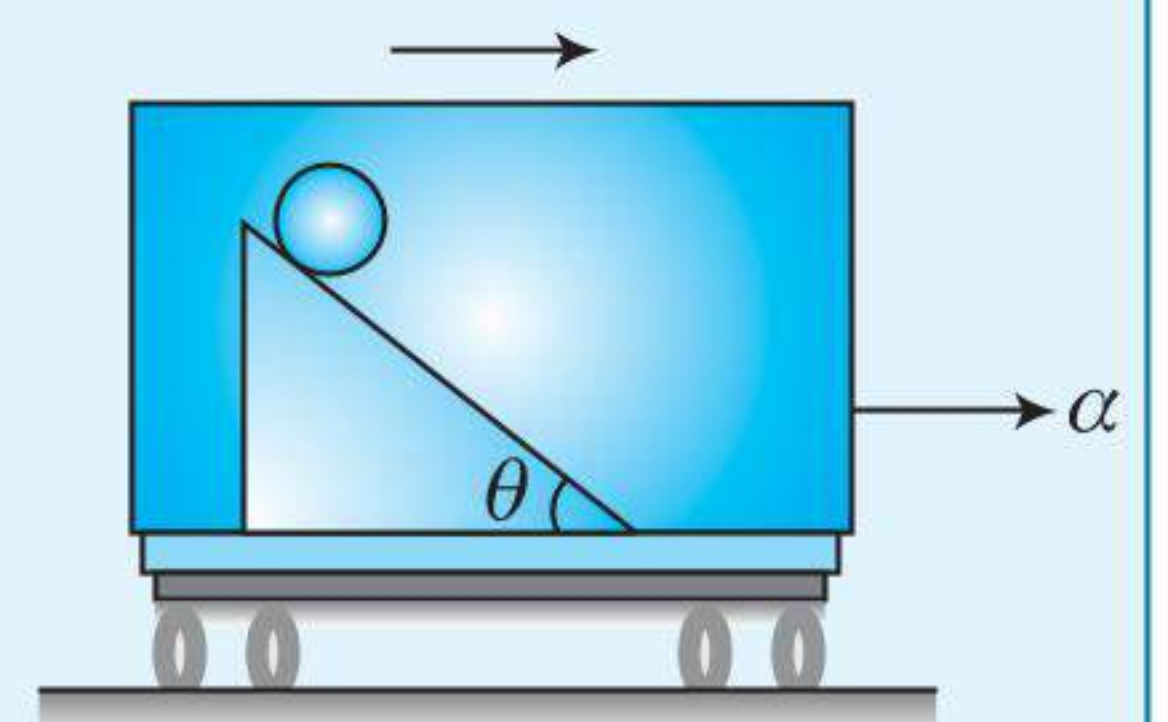
$$\Rightarrow W_{fr} = -\frac{1}{6}mgl \sin \theta$$

CONCEPT APPLICATION EXERCISE 4.6

1. A cylinder of mass m is rotated about its axis by an angular velocity ω and lowered gently on an inclined plane as shown in figure. Then:

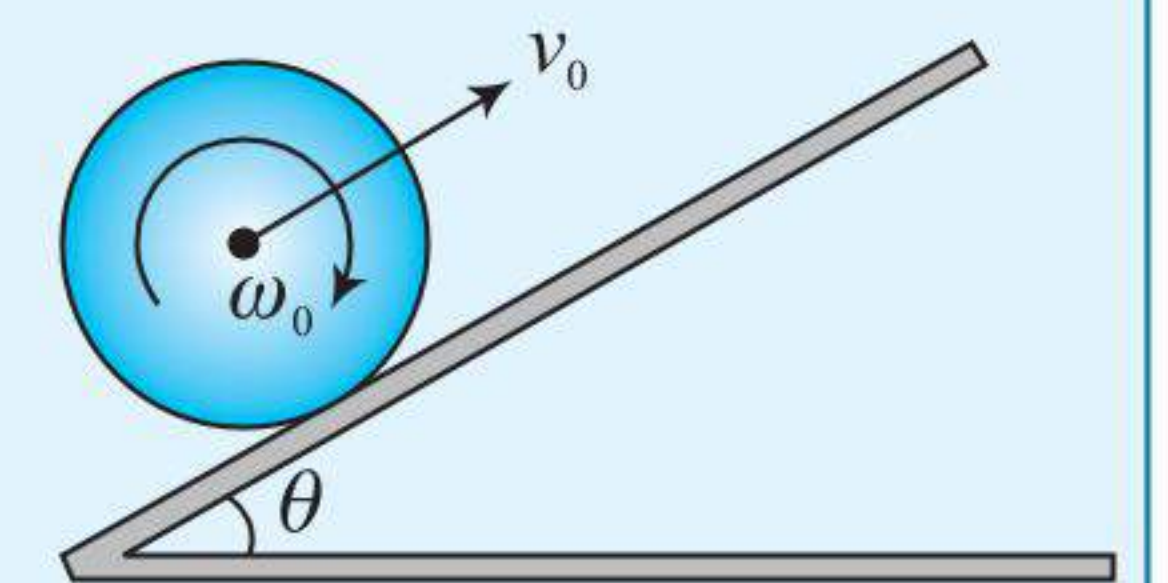


- (a) it will start going upward
 - (b) it will first go upward and then downward
 - (c) it will go downward just after it is lowered
 - (d) it can never go upward
2. Figure shows a smooth inclined plane fixed in a car accelerating on a horizontal road. The angle of incline θ is released to the acceleration ' a ' of the car as $a = g \tan \theta$. If the sphere is set in pure rolling on the incline:



- (a) it will continue pure rolling
- (b) it will slip down the plane
- (c) its linear velocity will increase
- (d) its linear velocity will decrease

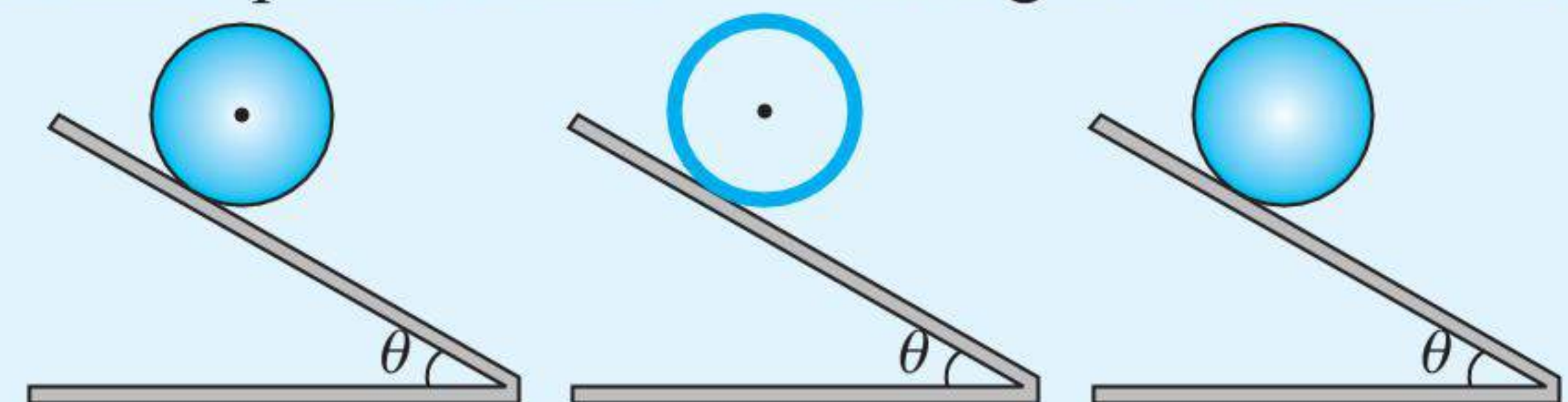
3. A sphere is moving on a smooth surface with linear speed v_0 and angular velocity ω_0 . It finds a rough inclined surface and it starts climbing up:



- (a) if $v_0 > R\omega_0$, friction force will act downwards
- (b) if $v_0 < R\omega_0$, friction force will act downwards
- (c) if $v_0 = R\omega_0$, no friction force will act
- (d) in all above case friction will act upwards

PASSAGE

A sphere, a ring and a disc of same mass and radius are allowed to roll down three similar sufficiently rough inclined planes as shown in the figure from same height.



4. Which of the following order is true for final KE of the bodies?
 - (a) sphere > disc > ring
 - (b) ring > disc > sphere
 - (c) disc > ring > sphere
 - (d) disc = ring = sphere
5. Which of the following order is true for final linear velocity of the bodies?
 - (a) sphere > disc > ring
 - (b) ring > disc > sphere
 - (c) disc > ring > sphere
 - (d) disc = ring = sphere
6. Which of the following order is true for time taken by the bodies to reach the bottom of incline?
 - (a) sphere > disc > ring
 - (b) ring > disc > sphere
 - (c) disc > ring > sphere
 - (d) disc = ring = sphere

ANSWERS

1. (d)
2. (a)
3. (a)
4. (d)
5. (a)
6. (b)

ANGULAR IMPULSE

The sum of torques of all external forces acting on a system of particles (or rigid body) about any fixed point is equal to the rate of change in angular momentum about that point

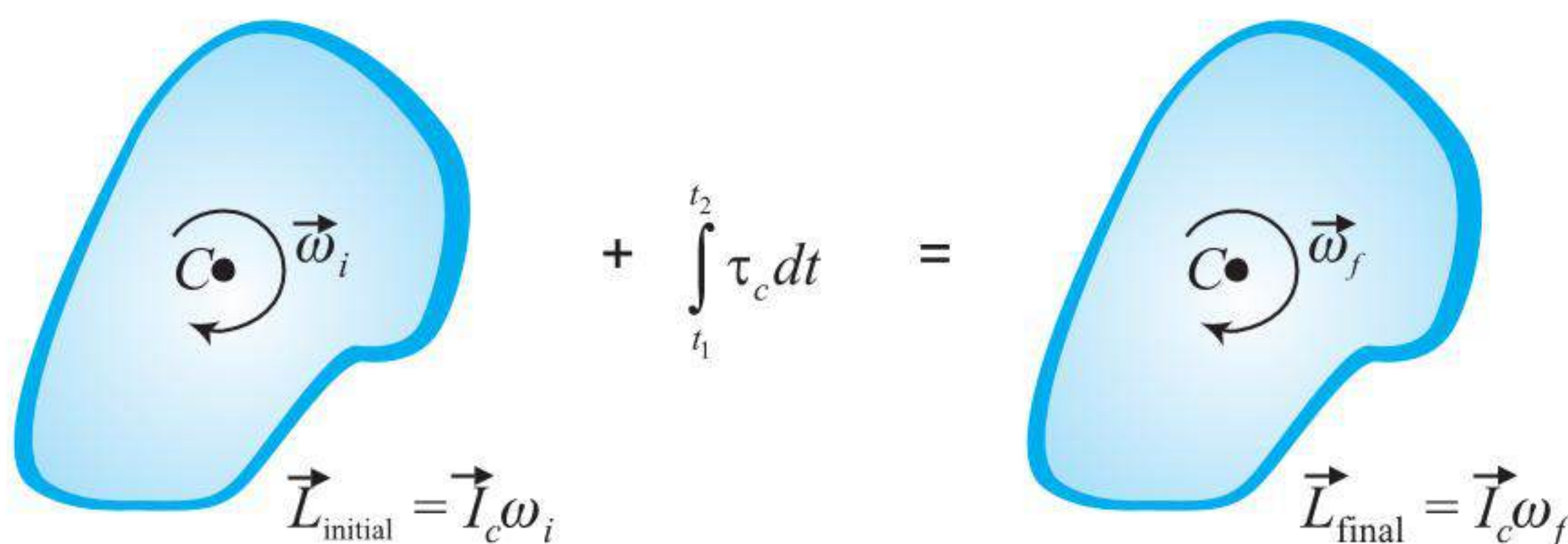
$$\text{As } \vec{\tau} = \frac{d\vec{L}}{dt}$$

Hence the change in angular momentum during time dt is given by

$$d\vec{L} = \vec{\tau} \cdot dt$$

It means the change in angular momentum of the rigid body during a time interval $\Delta t(t_2 - t_1)$ is given by

$$\Delta\vec{L} = \int d\vec{L} = \int_{t_1}^{t_2} \vec{\tau} \cdot dt = \text{Angular impulse}$$

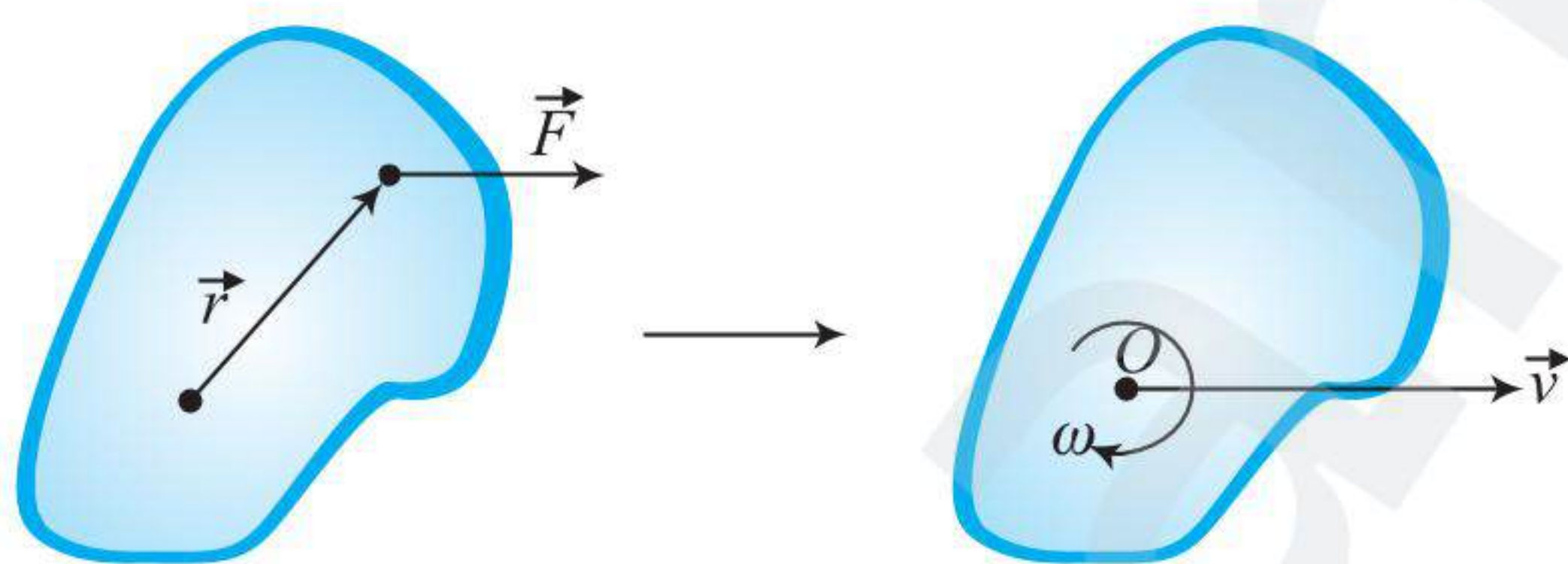


Initial angular momentum about CM + Net angular impulse about CM = final angular momentum about CM

Hence, “the net angular impulse about any fixed point or CM of the rigid body is equal to the change in angular momentum measured about that reference point.” This statement is called **angular impulse–momentum theorem**.

CALCULATION OF ANGULAR IMPULSE

Let us assume a force $\vec{F}$ is acting at position P in a rigid body. Apart from translational effect, this force also produces rotational effect, that is, torque of the force.



The angular impulse of the force $\vec{F}$ related to the centre of mass can be given as the time integral of the torque taken relative to centre of mass (CM).

Then, angular impulse of the force $\vec{F}$ about C is

$$\Delta\vec{L} = \int \vec{\tau}_{P,C} \cdot dt = \int (\vec{r}_{P,C} \times \vec{F}) dt$$

$$\Rightarrow \Delta\vec{L} = \vec{r}_{P,C} \times \int \vec{F} \cdot dt \quad \dots(i)$$

The term $\int \vec{F} \cdot dt$ is the linear impulse of the force $\vec{F}$. Now Eq. (i) can be write as

$$\Delta\vec{L} = \vec{r}_{P,C} \times \vec{J} \quad \dots(ii)$$

From Eq. (ii) we can say that angular impulse is the moment of linear impulse. It means we can calculate the angular impulse in the same way as we calculate the torque

Hence, we can write the magnitude of angular impulse as

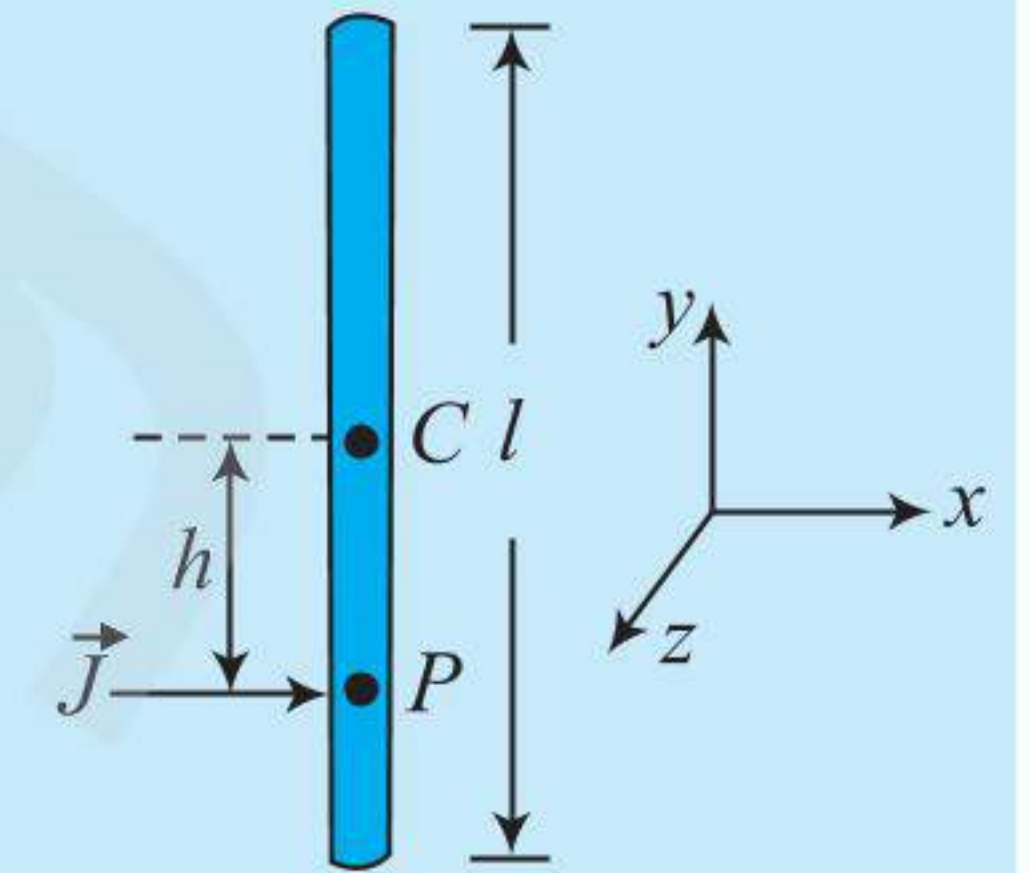
$$|\Delta\vec{L}| = \text{Linear impulse} \times \text{lever arm}$$

Hence lever arm is the perpendicular distance from the reference point and line of action of the linear impulse.

ILLUSTRATION 4.56

A uniform rigid rod of mass m , length l is placed as a smooth horizontal surface. An impulse $\vec{J}$ is give at point P which is at a distance h from the CM of the rod. Find:

- velocity of CM
- angular velocity of the rod.



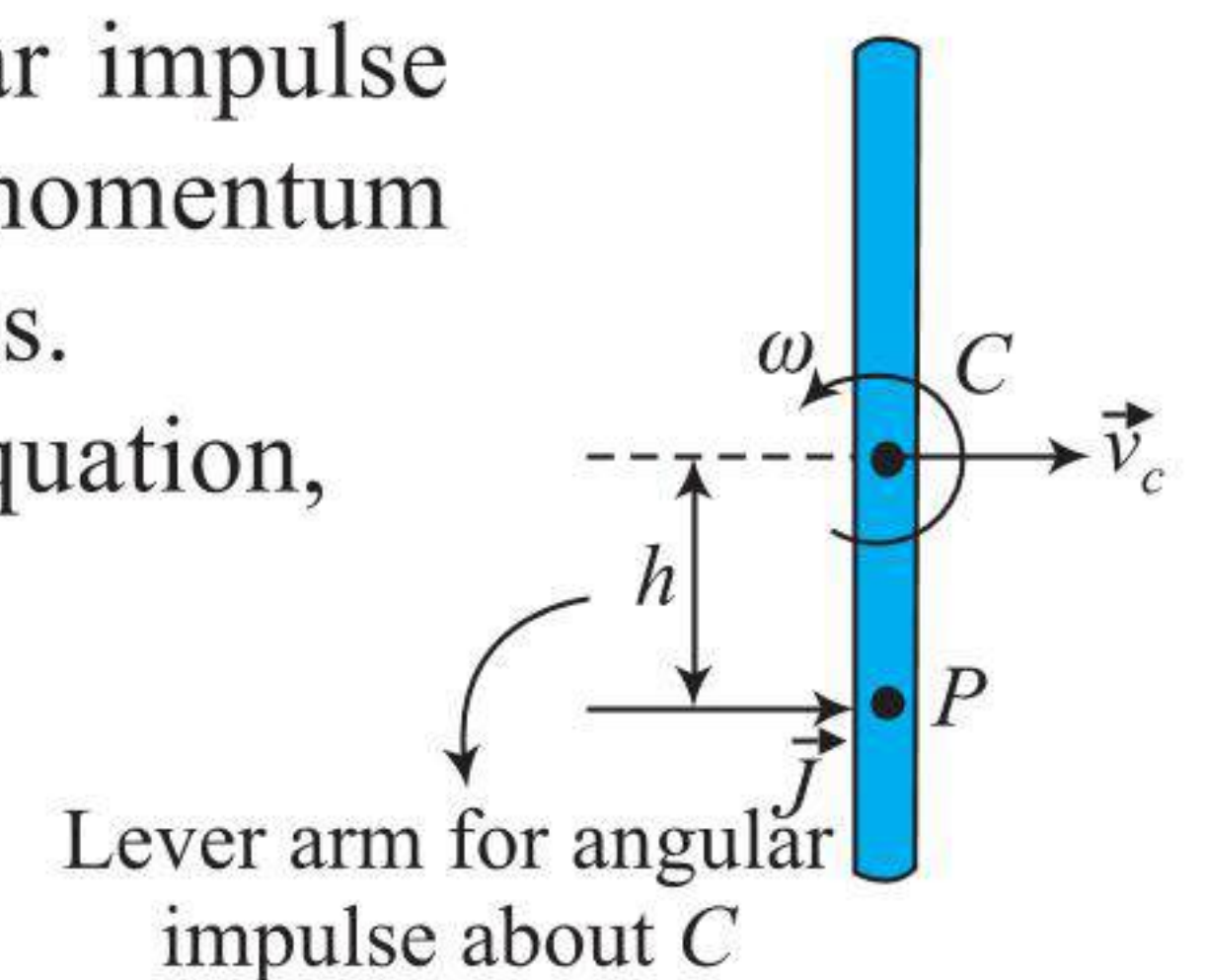
Sol.

- In case of rigid body, the linear impulse acting on it changes the linear momentum associated with its centre of mass.

Applying impulse momentum equation,

$$\vec{J} = \Delta\vec{p} \Rightarrow J(\hat{i}) = m\vec{v}_C - 0$$

$$\text{or } \vec{v}_C = \frac{J}{m}(\hat{i})$$



- For finding angular velocity of the rod just after applying the impulse, we use angular impulse momentum equation

$$\text{Angular impulse } \Delta\vec{L} = I_C \vec{\omega}_{\text{final}} - I_C \vec{\omega}_{\text{initial}} \quad \dots(i)$$

The magnitude of angular impulse

$$|\Delta\vec{L}| = \text{Linear impulse} \times \text{lever arm} = J \times h$$

For finding direction of angular impulse we use right hand rule

$$\Rightarrow \Delta\vec{L} = Jh(\hat{k}) \quad \dots(ii)$$

From (i) and (ii), we get

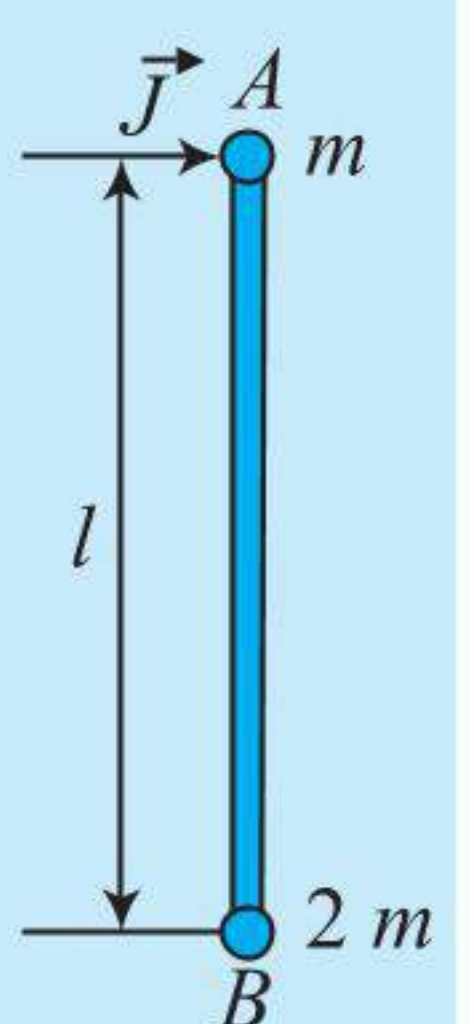
$$Jh(\hat{k}) = \frac{ml^2}{12} \vec{\omega} - 0 \text{ or } \vec{\omega} = \frac{12Jh}{ml^2}(\hat{k})$$

Hence the angular velocity of the rod will be $\frac{12Jh}{ml^2}$ in counter clockwise sense.

ILLUSTRATION 4.57

Two particles of masses m and $2m$ are attached at the ends A and B of a light rigid rod of length l . The system ($m + \text{rod} + 2m$) is placed on a smooth horizontal surface. An impulse $\vec{J}$ is given at the end A of the rod perpendicular to its length as shown in figure. Find the

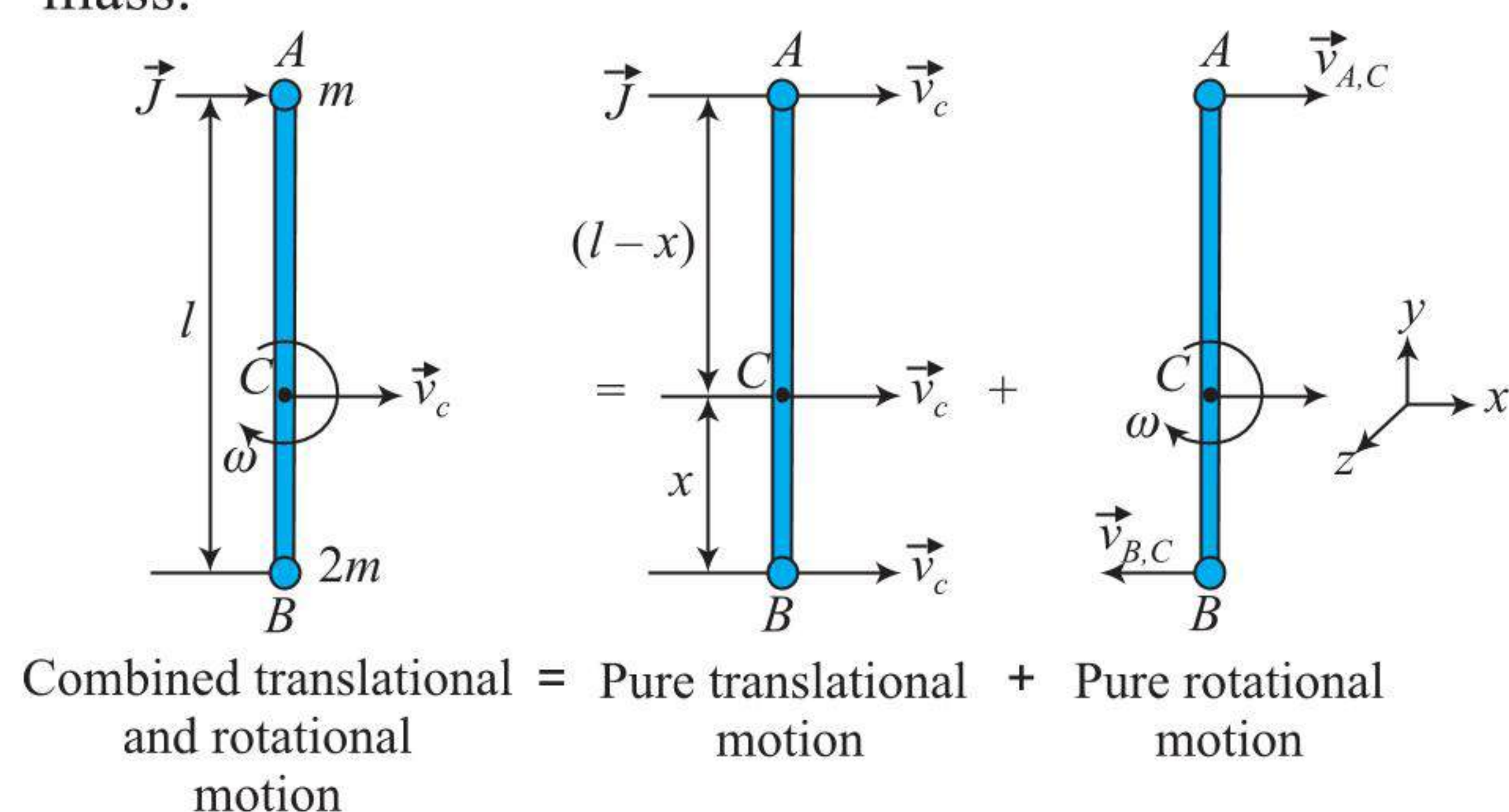
- velocity of centre of mass of the system
- angular velocity ($\vec{\omega}$) of the angular velocity of the rod
- velocity of end A and B of the rod



Sol.

- Since, line of action of impulse does not pass through centre of mass of the system, therefore, just after application of impulse, the system starts to move, both translationally

and rotationally. The linear impulse acting on the system changes the linear momentum associated with its centre of mass.



- (ii) Let us calculate the angular velocity of the rod, for this we need to calculate angular impulse on the system about CM:

$$\Delta \vec{L} = J(l-x)(-\hat{k}) \quad \dots(i)$$

x is the distance of centre of mass of the system from end B

$$x = \frac{2m(0) + m(l)}{2m + m} = \frac{l}{3}$$

$$\text{Hence, } \Delta \vec{L} = J\left(l - \frac{l}{3}\right)(-\hat{k}) = \frac{2}{3}Jl(-\hat{k})$$

$$\text{But } \Delta \vec{L} = I_{CM}\vec{\omega}_{\text{final}} - I_{CM}\vec{\omega}_{\text{initial}}$$

I_{CM} is the moment of inertia of the system about CM

$$I_{CM} = m(l-x)^2 + 2mx^2$$

$$= m\left(\frac{2}{3}l\right)^2 + 2m\left(\frac{l}{3}\right)^2 = \frac{2}{3}ml^2$$

$$\text{Hence, } \frac{2}{3}Jl(-\hat{k}) = \frac{2}{3}ml^2\vec{\omega} - 0 \Rightarrow \vec{\omega} = \frac{J}{ml}(-\hat{k})$$

It means the system will start rotating with angular velocity J/ml is clockwise sense.

- (iii) Velocity of end A; $\vec{v}_A = \vec{v}_{A,C} + \vec{v}_C$

$$|\vec{v}_{A,C}| = \omega(l-x) = \frac{J}{ml} \times \frac{2}{3}l$$

$$\Rightarrow \vec{v}_{A,C} = \frac{2J}{3m}(\hat{i})$$

$$\vec{v}_A = \frac{2J}{3m}(\hat{i}) + \frac{J}{3m}(\hat{i}) = \frac{J}{m}\hat{i}$$

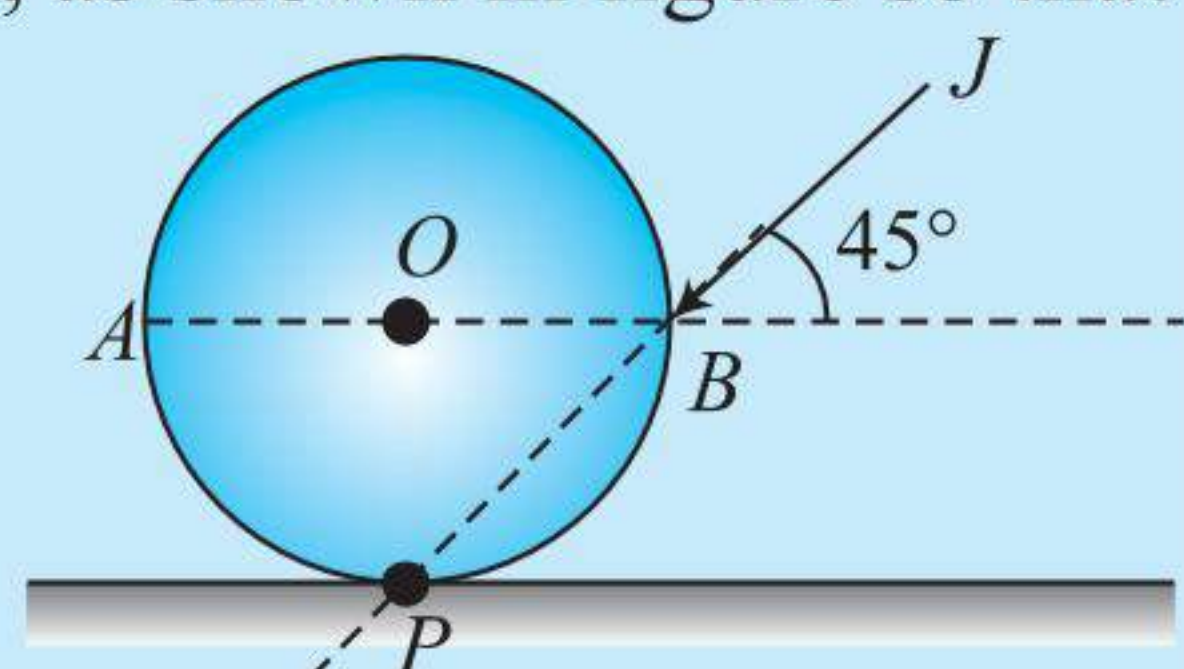
Similarly, velocity of end B; $\vec{v}_B = \vec{v}_{B,C} + \vec{v}_C$

$$\text{Hence, } \vec{v}_B = \omega x(-\hat{i}) + \frac{J}{3m}(\hat{i})$$

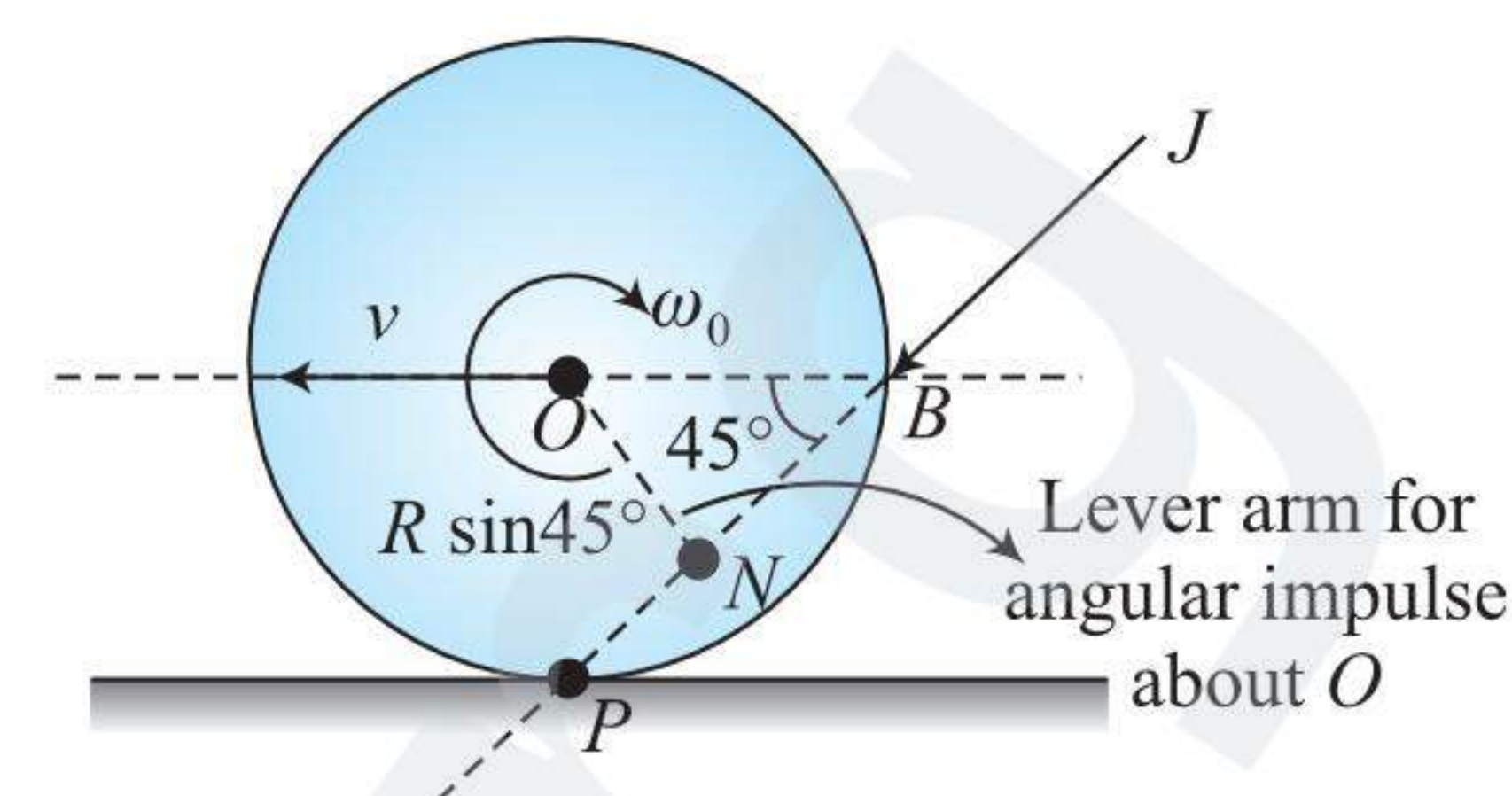
$$\Rightarrow \vec{v}_B = \frac{J}{ml} \times \frac{l}{3}(-\hat{i}) + \frac{J}{3m}(\hat{i}) = 0$$

ILLUSTRATION 4.58

AB is a horizontal diameter of a ball of mass $m = 0.4$ kg and radius $R = 0.10$ m. At time $t = 0$, a sharp impulse is applied at B at angle of 45° with the horizontal, as shown in figure so that the ball immediately starts to move with velocity $v_0 = 10$ m s⁻¹. Calculate the impulse and angular velocity of ball just after impulse provided.



Sol. Since, line of action of impulse does not pass through centre of mass of the sphere, therefore, just after application of impulse, the sphere starts to move, both translationally and rotationally. Translation motion is produced by horizontal component of the impulse, while rotational motion is produced by moment of the impulse. Let the impulse applied be J .



Then its horizontal component provides horizontal motion.

$$J \cos 45^\circ \text{ where gives } J = 4\sqrt{2} \text{ kg ms}^{-1}$$

Moment of inertia of ball about centroidal axis is

$$I = \frac{2}{5}mR^2 = 1.6 \times 10^{-3} \text{ kg m}^2$$

Initial angular momentum of ball (about centre) = $J(R \sin 45^\circ)$

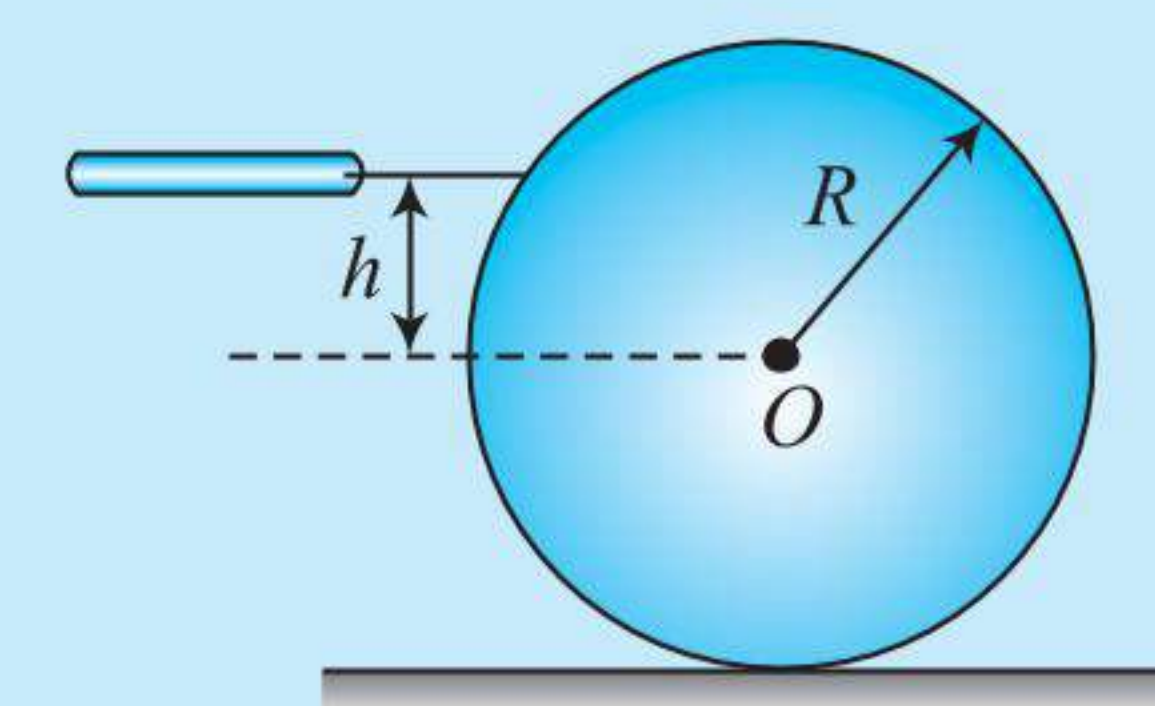
Angular impulse will change angular momentum of the ball.

$$JR \sin 45^\circ = \Delta L = (I\omega_0 - 0)$$

which gives $\omega_0 = 250$ rad s⁻¹ (clockwise)

ILLUSTRATION 4.59

A billiard ball (of radius R), initially at rest is given a sharp impulse by a cue. The cue is held horizontally at a distance h above the central line as shown figure. The ball leaves the cue with a speed v_0 . It rolls and slides while moving forward and eventually acquires a final speed of $\frac{9}{7}v_0$.



Show that $h = \frac{4}{5}R$.

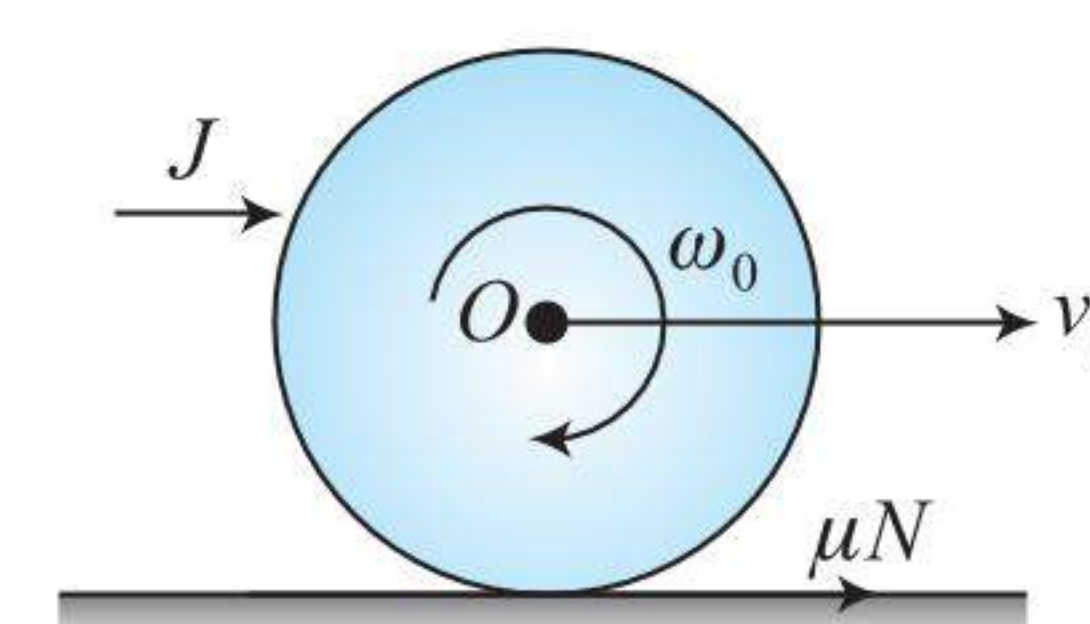
Sol. Let v_0 be the linear velocity and ω_0 be the angular velocity imparted.

$$\text{Linear impulse} = J = mv_0$$

$$\text{Angular impulse} = Jh = I\omega_0$$

$$h = \frac{(2/5 mR^2)\omega_0}{mv_0} \Rightarrow v_0 = \frac{2R^2\omega_0}{5h} \quad \dots(i)$$

As linear velocity increases to $9/7v_0$, friction must be in forward direction and hence it opposes angular motion.



For linear motion: friction (μmg) increases the velocity. Hence,

$$\mu mg = ma \Rightarrow a = \mu g \quad \dots(ii)$$

Let t be the time when rolling starts.

For rotation: frictional torque (μmgR) opposes rotation and hence decreases ω .

$$\alpha = -\frac{\mu mgR}{I} \Rightarrow \omega = \omega_0 - \frac{\mu mgR}{\left(\frac{2}{5}mR^2\right)}t \quad \dots(iii)$$

$$\text{After time } t, v = \frac{9}{7}v_0 = R\omega \text{ and } \frac{9}{7}v_0 = v_0 + \mu gt \quad \dots(iv)$$

Combining all the equations, we get

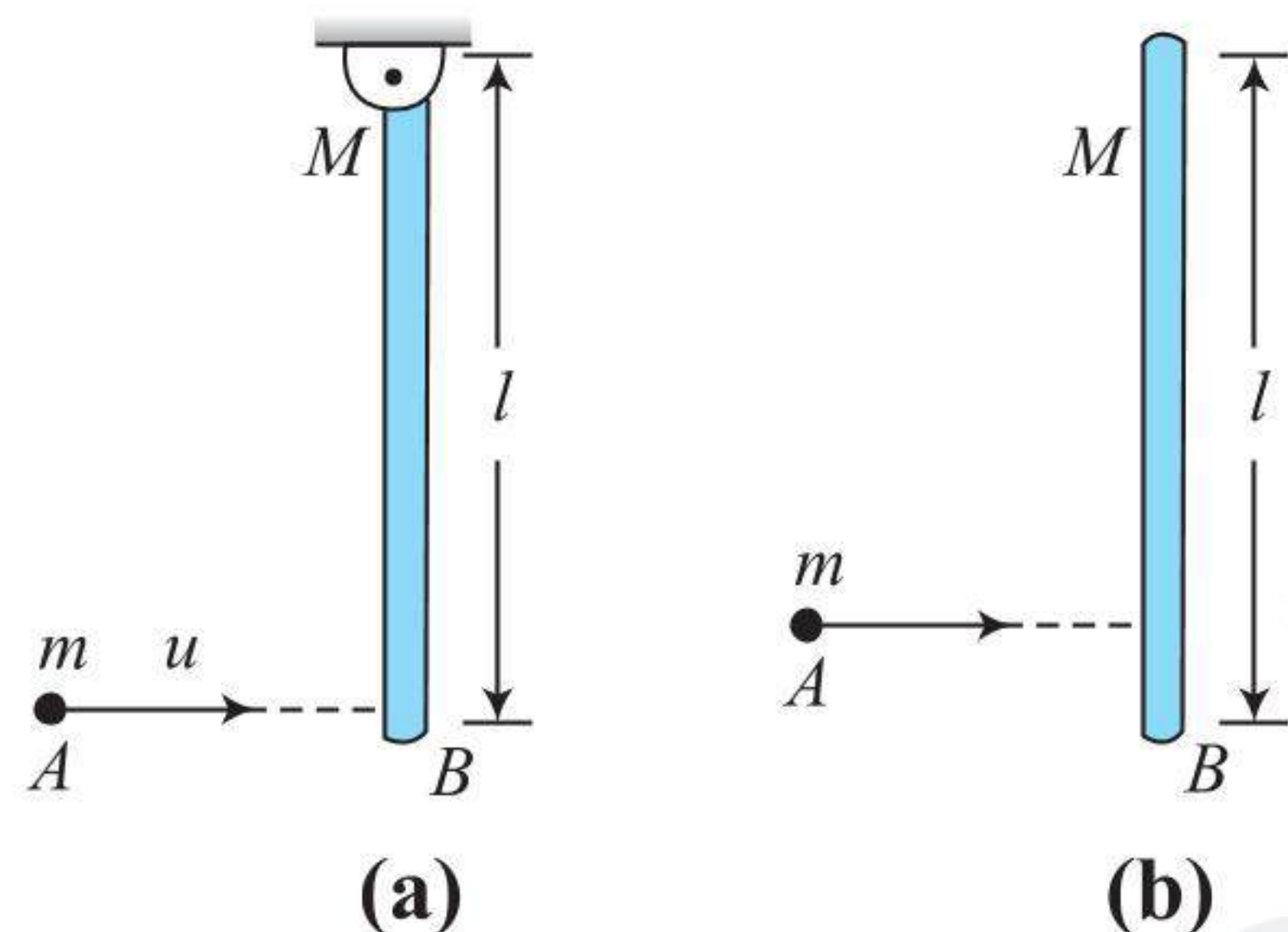
$$\frac{9}{7}v_0 - v_0 = -\left(\frac{9v_0}{7R} - \omega_0\right)\frac{2R}{5} \quad (\text{eliminating } \mu gt)$$

Substituting for v_0 from Eq. (i), we get

$$\frac{2}{7}\left(\frac{2}{5}\frac{R^2\omega_0}{h}\right) = \left(\omega_0 - \frac{9}{7}\frac{2R\omega_0}{5h}\right)\frac{2R}{5} \Rightarrow h = \frac{4R}{5}$$

CONSERVATION OF ANGULAR MOMENTUM IN CASE OF COLLISION OF RIGID BODIES

In chapter “Impulse and Collision”, we have discussed head on and oblique collision. Our analysis was limited to find the velocities of the particles or bodies just after collision. In this section we will discuss the different cases of collision of two rigid bodies in which during or after collision rotational motion of the bodies is also considered.



As a particle collides with a rigid body, an impulsive force acts on the body and the particle which can change the state of motion of the body and particle.

Depending on the fact that whether the body is restricted to rotate about fixed axis of rotation or is free to rotate in any way, we are here classifying this topic into two ways:

- (a) When the axis of rotation is fixed [Fig. (a)].
 - (b) When the body is free to rotate as well as translate (Fig. (b)).
- We perform following steps while analyzing the collision in case of rigid bodies.

Step 1: Application of linear momentum. If there is no external impulse acting on the system, we can conserve linear momentum of the system just before and just after collision.

In the figures shown, if we take the particle A and rod B as system we cannot use conservation of linear momentum in case of Fig. (a) as there will be external impulse from hinge connected. But we can use conservation of linear momentum in the case of Fig. (b).

Step 2: Application of angular momentum. We can apply conservation of angular momentum if no external torque or angular impulse acts on the system about certain point.

In Fig. (a), there is impulse at hinge during collision. We can conserve angular momentum of the system (rod and particle) about hinge O. In Fig. (b) there is no external impulse, so we can

conserve angular momentum of the system just before and just after collision.

Step 3: Application of impulse momentum. In the situations where we cannot use conservation of angular momentum, we can use impulse momentum equation

$$\vec{L}_{\text{initial}} + \int \vec{\tau} dt = \vec{L}_{\text{final}}$$

$$\int \vec{\tau} dt = \text{angular impulse}$$

Step 4: Application of coefficient of restitution equation. The ratio of velocity of separation after collision to the velocity of approach before collision is equal to coefficient of restitution.

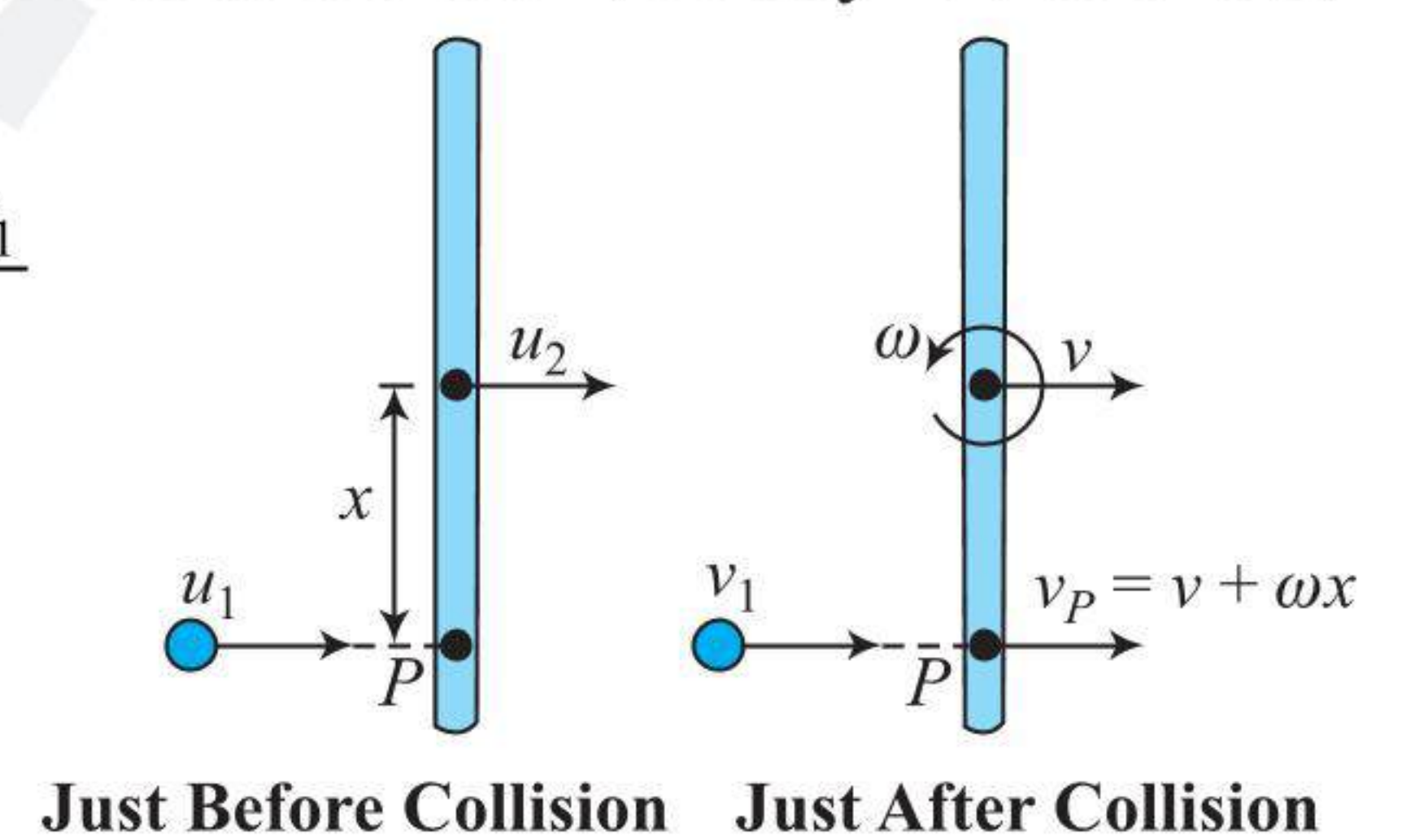
$$\vec{v}_{1,2} = -e\vec{u}_{1,2}$$

$\vec{u}_{1,2}$ and $\vec{v}_{1,2}$ are the relative velocities of the colliding points along the line of impact.

For example, in the figure a particle collides with a rod as shown in figure. Just after collision the velocity of rod and particle are given as

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{(v + \omega x) - v_1}{u_1 - u_2}$$

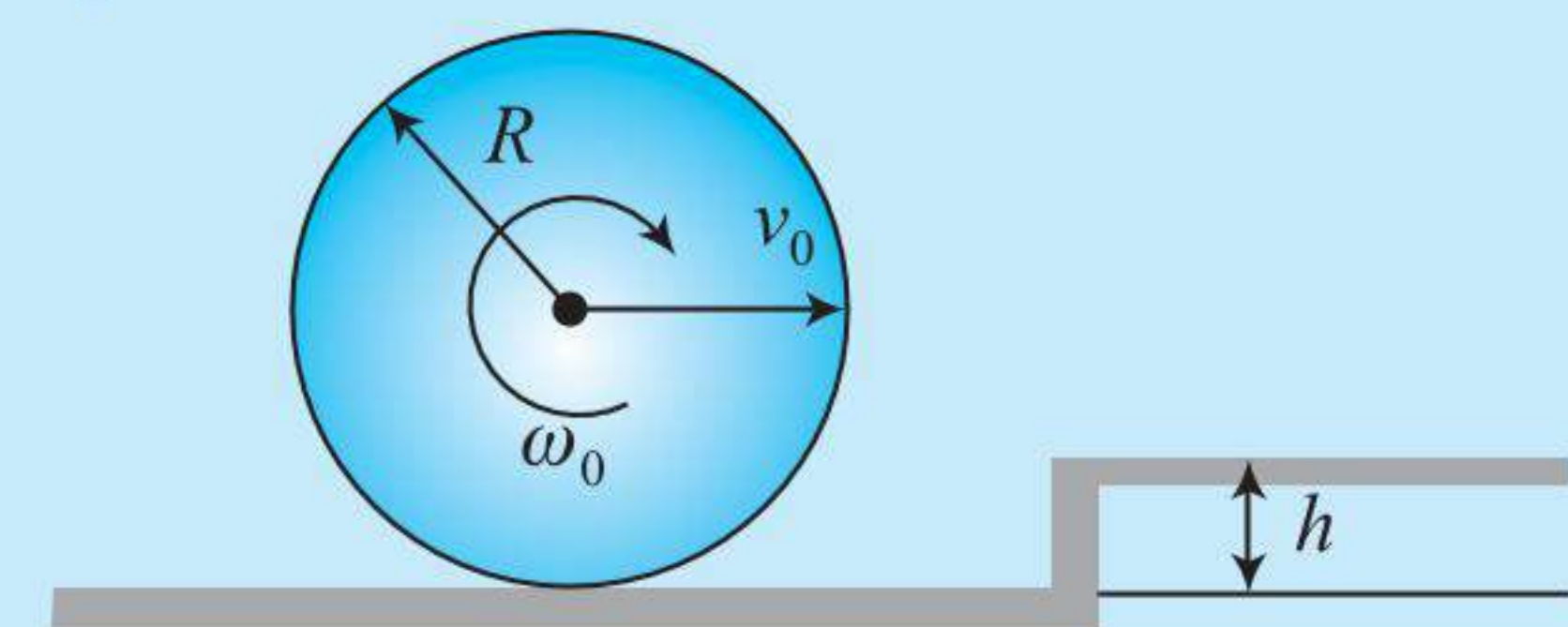
Here we are going to discuss different categories of rigid body collision through illustrations.



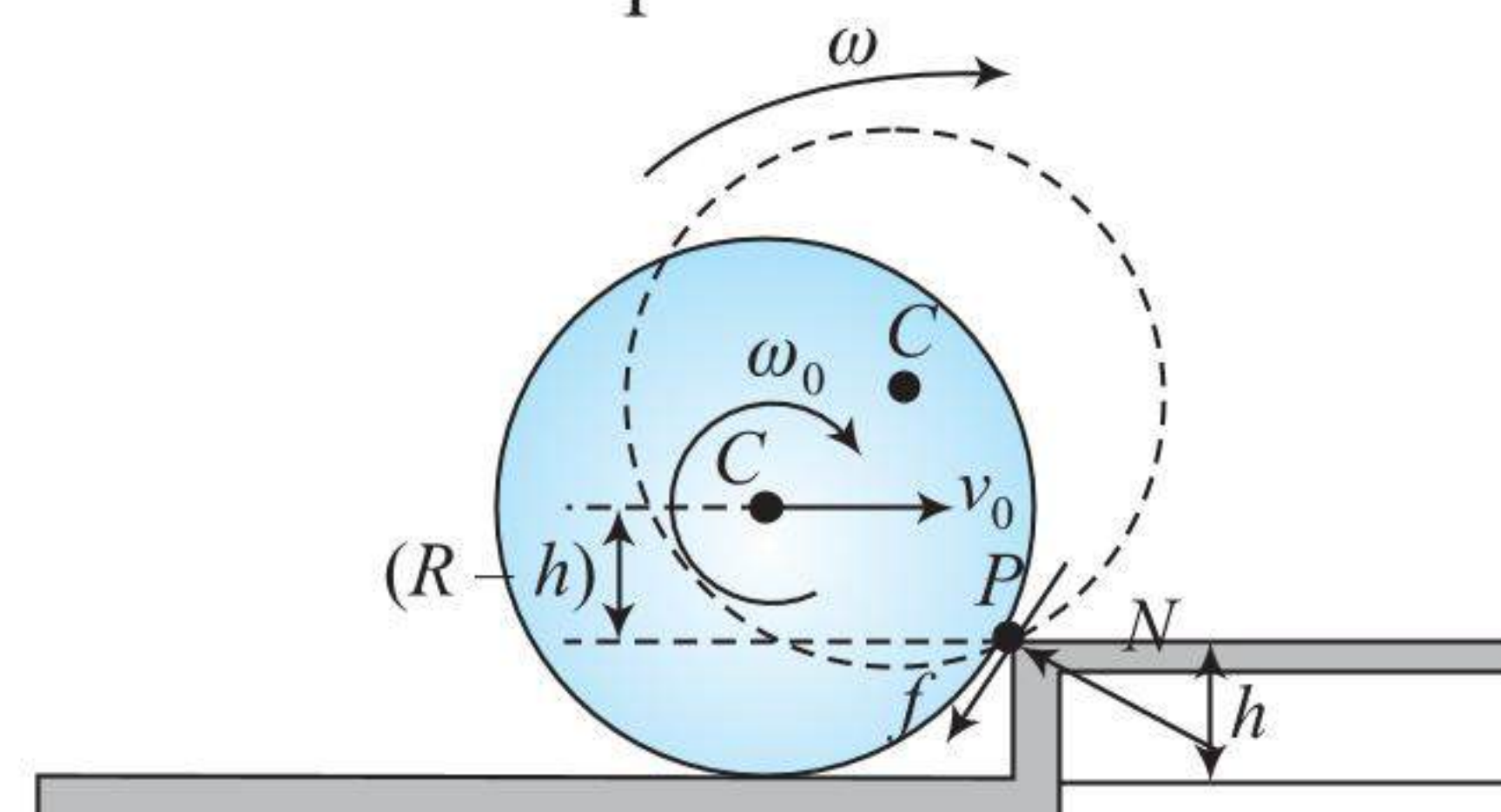
CASE 1: COLLISION OF A RIGID BODY WITH A FIXED SURFACE

ILLUSTRATION 4.60

A uniform solid sphere of radius R is rolling without sliding on a horizontal surface with a velocity v_0 . It collides with an obstacle of height h ($< R$) inelastically. Find the angular speed of the sphere just after the collision.



Sol. During collision normal contact force N acting at the point of impact P is impulsive in nature hence friction will also be impulsive. The gravitational force (weight of the sphere) is ignored because it is non-impulsive.



If we consider point P , as the impulsive forces f and N are passing through this point, they cannot produce torques about P . Then the angular impulses of both forces f and N are zero about P . Hence, we can conserve the angular momentum of the body about P just before and just after the impact.

$$\vec{L}_{1p} = \vec{L}_{2p}$$

$$\text{where } \vec{L}_{1p} = mv_0(R-h) + \frac{2}{5}mR^2\omega_0$$

$$\text{and } \vec{L}_{2P} = \left(\frac{2}{5}mR^2 + mR^2 \right) \omega$$

because the body rotates perfectly about P just after collision as it does not slide and bounce.

Then, we have

$$v_0(R-h) + \frac{2}{5}R^2\omega_0 = \frac{7}{5}R^2\omega$$

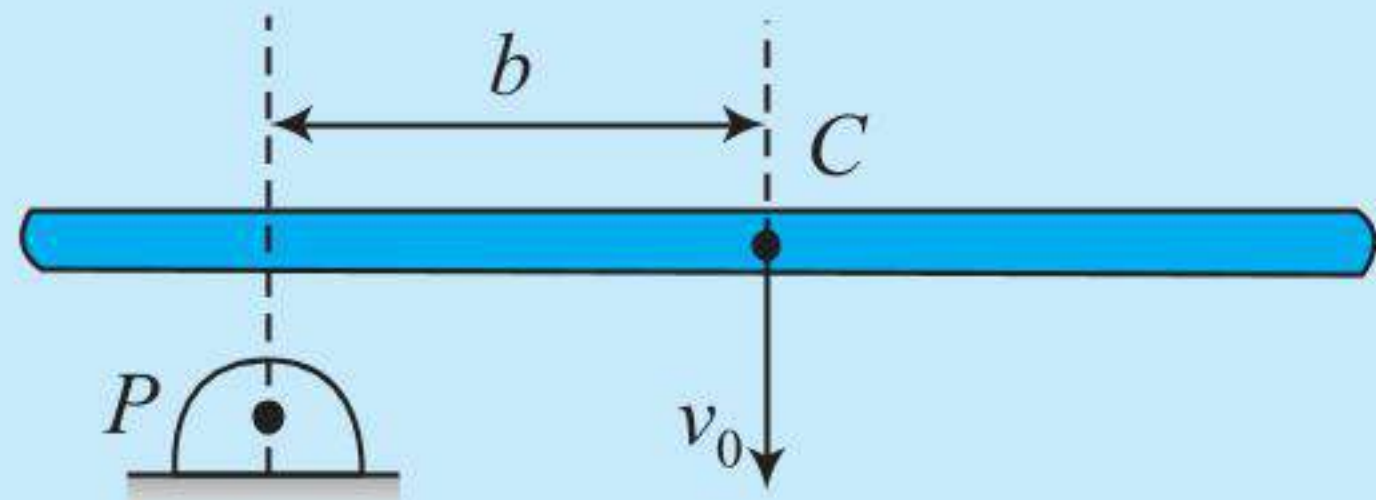
As the body was rolling before the impact, $v_0 = R\omega_0$

Putting $v_0 = R\omega_0$ in the above expression.

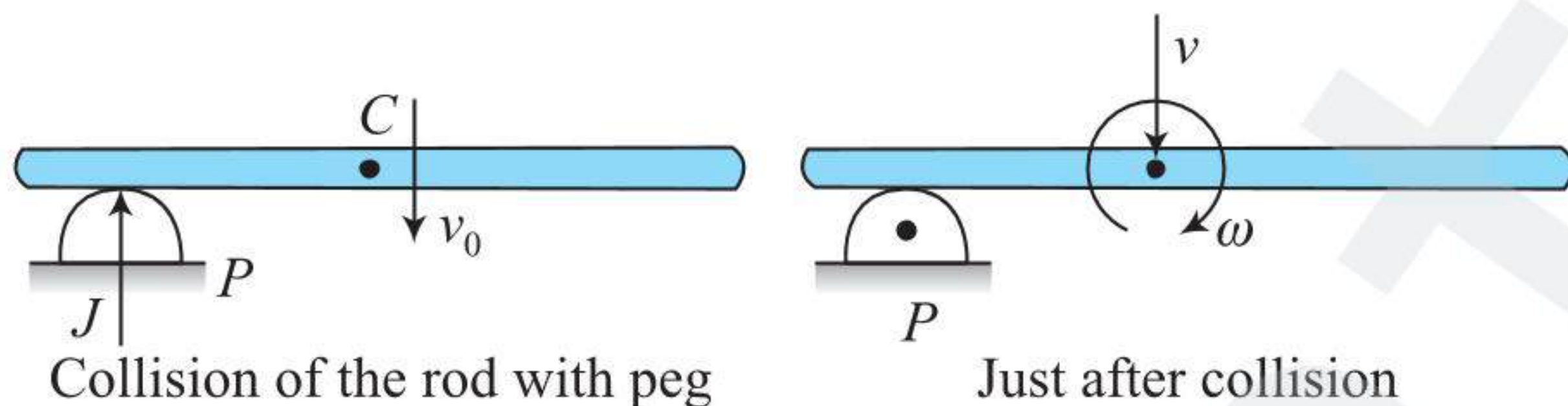
$$\text{we have } \omega = \left(1 - \frac{5h}{7R} \right) \frac{v_0}{R}$$

ILLUSTRATION 4.61

A uniform rod of length l moving with a downward velocity v_0 strikes a peg P . If the coefficient of restitution of impact is e , find the velocity of CM of the rod just after impact.



Sol. Method 1: The forces acting on the rod at the time of collision are N (upward) and mg (downward). The normal contact force N acting at the point of impact P is impulsive in nature. The gravitational force (weight of the sphere) is ignored because it is non-impulsive. Then, we disregard the effect of gravity during the impact.



From impulse momentum equation

$$mv_0 - J = mv \quad \dots(i)$$

The angular impulse of N about centre of mass of the rod

$$J \cdot b = \frac{ml^2}{12} \omega \quad \text{or} \quad J = \frac{ml^2}{12b} \omega \quad \dots(ii)$$

Adding Eqs. (i) and (ii), we have

$$v_0 - v = \frac{l^2 \omega}{12b} \quad \dots(iii)$$

Newton's empirical formula gives us

Velocity of separation = e (Velocity of approach)

$$v_P = -ev_0 \quad \dots(iv)$$

The velocity of P is $\vec{v}_P = \vec{v}_{PC} + \vec{v}_C$

Substituting $v_{PC} = -b\omega$ and $v_C = v$, we have

$$v_P = v - b\omega \quad \text{or} \quad v - b\omega = -ev_0 \quad \dots(v)$$

Now we have two Eqs. (iii) and (v) for two unknowns v and ω .

Solving these two equations, we have

$$v = \frac{12b^2 - el^2}{12b^2 + l^2} v_0$$

Method 2: If we choose a fixed point at P , the angular impulse about P is zero because the impulse J passes through P during impact.

Hence the angular momentum of the rod just before and just after the collision about P remains constant.

$$\vec{L}_1 = \vec{L}_2, \quad \text{where } \vec{L}_1 = mv_0 b (-\hat{k})$$

$$\text{and } \vec{L}_2 = \frac{ml^2}{12} \omega (-\hat{k}) + mvb (-\hat{k})$$

$$\text{Then, } mv_0 b = mvb + \frac{ml^2}{12} \omega$$

$$\text{This gives } v_0 - v = \frac{l^2 \omega}{12b}$$

The other equations remain unchanged.

Solving the above equations, we will obtain the same value of v .

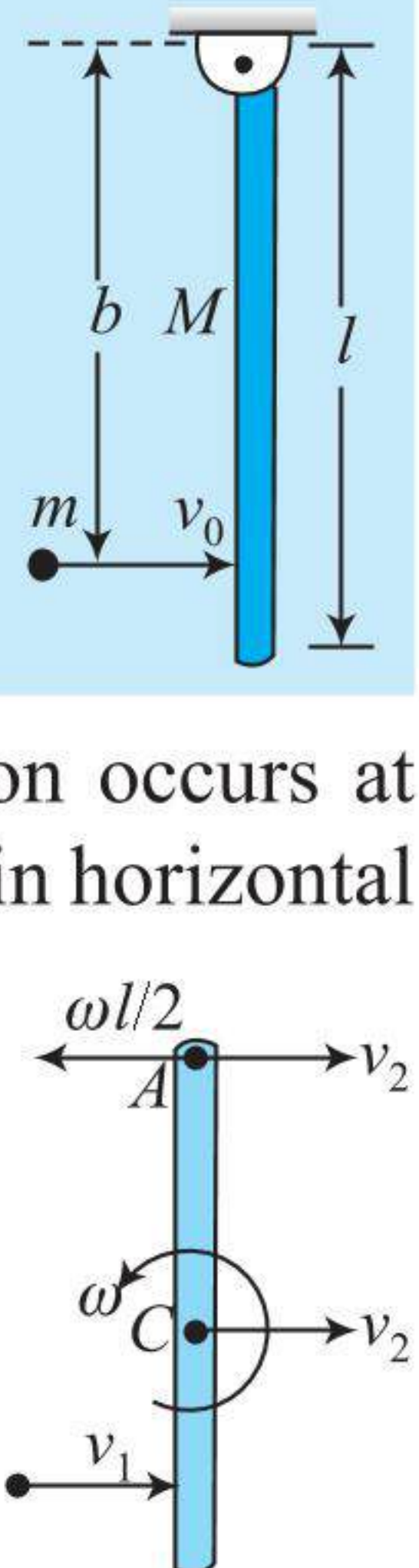
CASE 2: COLLISION OF A PARTICLE WITH RIGID BODY WITH THE AXIS OF ROTATION IS FIXED

ILLUSTRATION 4.62

A ball of mass m collides elastically with a smooth hanging rod of mass M and length l .

(a) If $M = 3m$, find the value of b for which no horizontal reaction occurs at the pivot.

(b) For what value of m/M , the ball falls dead just after the collision assuming $e = (1/2)$?



Sol. According to problem no horizontal reaction occurs at pivot end it means there is no impulse on the system in horizontal direction. We can use conservation of linear momentum just before collision and just after collision in horizontal direction.

(a) Let the velocities of particle and rod just after collision is v_1 and v_2 respectively.

$$mv_0 = mv_1 + 3mv_2$$

$$\text{or } v_0 = v_1 + 3v_2 \quad \dots(i)$$

Now applying conservation of angular momentum about pivot end just before collision and just after collision. Let angular velocity of the rod about pivot end is ω .

$$mv_0 \cdot b = mv_1 \cdot b + \frac{3ml^2}{3} \omega$$

$$\text{or } v_0 = v_1 + \frac{\omega l^2}{b} \quad \dots(ii)$$

From (i) and (ii),

$$3v_2 = \frac{\omega l^2}{b} \quad \dots(iii)$$

For no impulse at end, there should not be movement of end A .

$$\vec{v}_A = \vec{v}_{A,C} + \vec{v}_C$$

$$v_A = 0 = -\frac{\omega l}{2} + v_2 \Rightarrow v_2 = \frac{\omega l}{2} \quad \dots(iv)$$

$$\text{From (iii) and (iv), } b = \frac{2l}{3}$$

- (b) If the ball falls dead just after the collision $v_1 = 0$
Hence, from conservation of angular momentum about pivot gives

$$mv_0 b = 0 + \frac{Ml^2}{3} \omega \Rightarrow mv_0 \cdot b = \frac{Ml^2}{3} \omega \quad \dots(i)$$

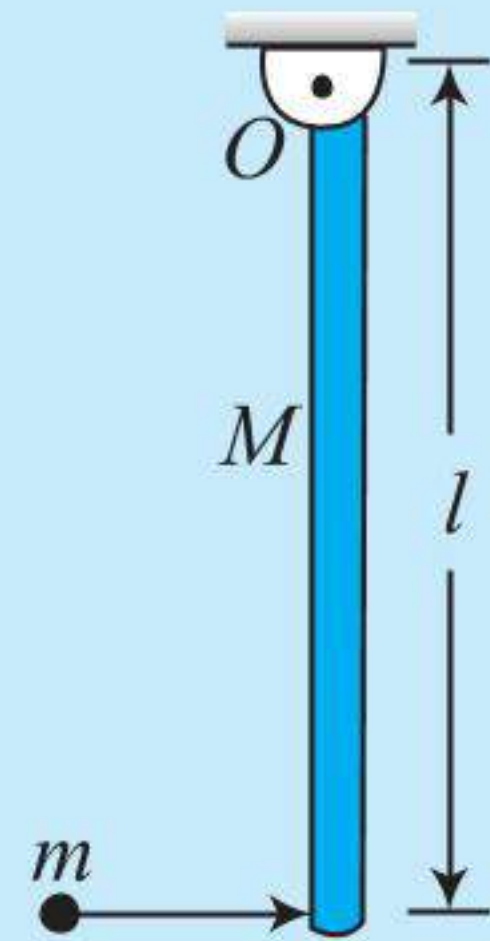
Again, applying Newton's restitution equation at the point of collision.

$$e = \frac{1}{2} = \frac{\omega b - 0}{v_0 - 0} \Rightarrow \omega = \frac{v_0}{2b} \quad \dots(ii)$$

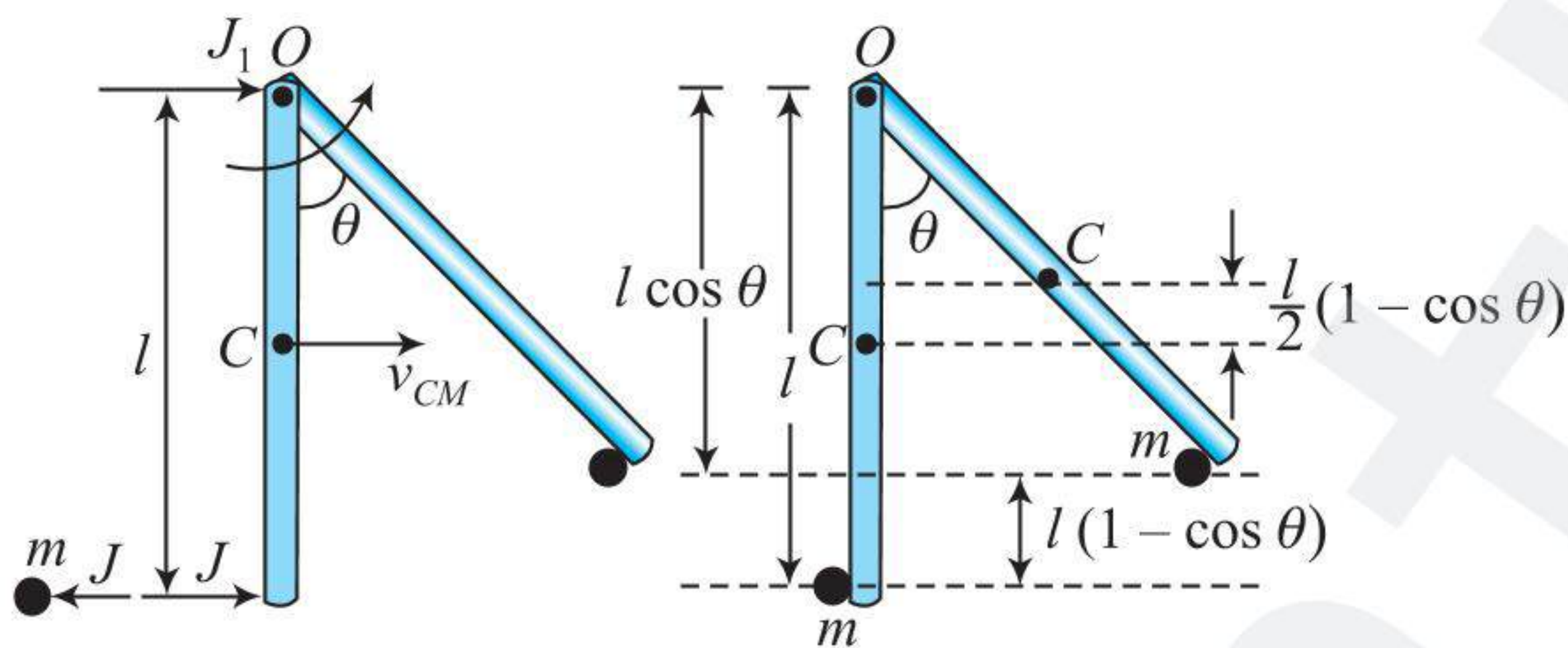
From Eqs. (i) and (ii), $\frac{m}{M} = \frac{l^2}{6b^2}$

ILLUSTRATION 4.63

A vertically oriented uniform rod of mass M and length l can rotate about its upper end. A horizontally flying bullet of mass m strikes the lower end of the rod and gets stuck in it; as a result, the rod swings through an angle θ . Assuming $m \ll M$, find the velocity of the flying bullet.



Sol. The impulse acting on the rod-bullet system during the impact is shown in figure. J_1 is the impulse acting on the rod by the hinge. Hence, we cannot apply conservation of linear momentum.



During the impact, the moment of impulse acting on the rod-bullet system about the hinge is zero. Angular momentum about the hinge will remain constant. Let ω be the angular velocity of the system about the hinge.

[Angular momentum of the system]_{initial} = [Angular momentum of the system]_{final}

$$mv \times l = I_0 \times \omega \quad \dots(i)$$

where I_0 is the moment of inertia of the rod-bullet system after collision about O

$$I_0 = I_{\text{rod}} + I_{\text{bullet}} = \frac{1}{3} Ml^2 + ml^2 \quad \dots(ii)$$

$$mvl = \left(\frac{1}{3} Ml^2 + ml^2 \right) \omega \Rightarrow \omega = \frac{3mv}{l(M + 3m)} \quad \dots(iii)$$

As the system swings about hinge, the mechanical energy of the system will remain constant, $\Delta K + \Delta U = 0$.

$$\left[0 - \frac{1}{2} \left(\frac{1}{3} Ml^2 + ml^2 \right) \omega^2 \right] + \left(Mg \frac{l}{2} (1 - \cos \theta) + mgl(1 - \cos \theta) \right) = 0 \quad \dots(iv)$$

Substituting this value of ω in Eq. (iv), we get

$$\frac{1}{2} \left(\frac{1}{3} Ml^2 + ml^2 \right) \left(\frac{3mv}{l(M + 3m)} \right)^2 = gl \frac{(1 - \cos \theta)}{2} (M + 2m)$$

$$\Rightarrow v^2 = \frac{gl}{3} \frac{(M + 2m)(M + 3m)}{m^2} (1 - \cos \theta)$$

Now for $m \ll M$, $(M + 3m)(M + 2m) \approx M^2$

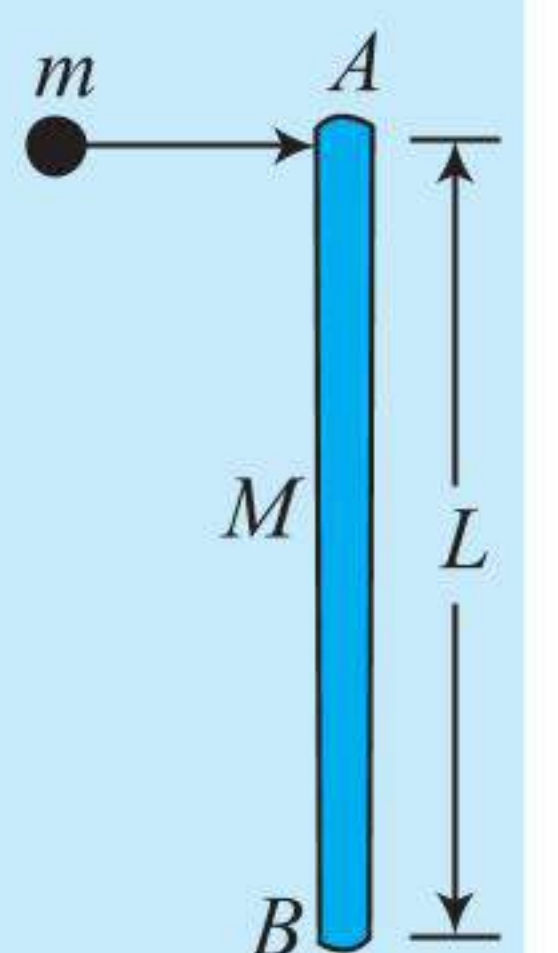
$$\text{and } (1 - \cos \theta) = 2 \sin^2 \frac{\theta}{2}$$

$$\text{Thus, } v^2 = \frac{2}{3} gl \frac{M^2}{m^2} \sin^2 \frac{\theta}{2} \Rightarrow v = \frac{M}{m} \sqrt{\frac{2}{3} gl} \sin \frac{\theta}{2}$$

CASE 3: COLLISION OF A PARTICLE WITH RIGID BODY WHEN THE BODY IS FREE TO ROTATE AS WELL AS TRANSLATE

ILLUSTRATION 4.64

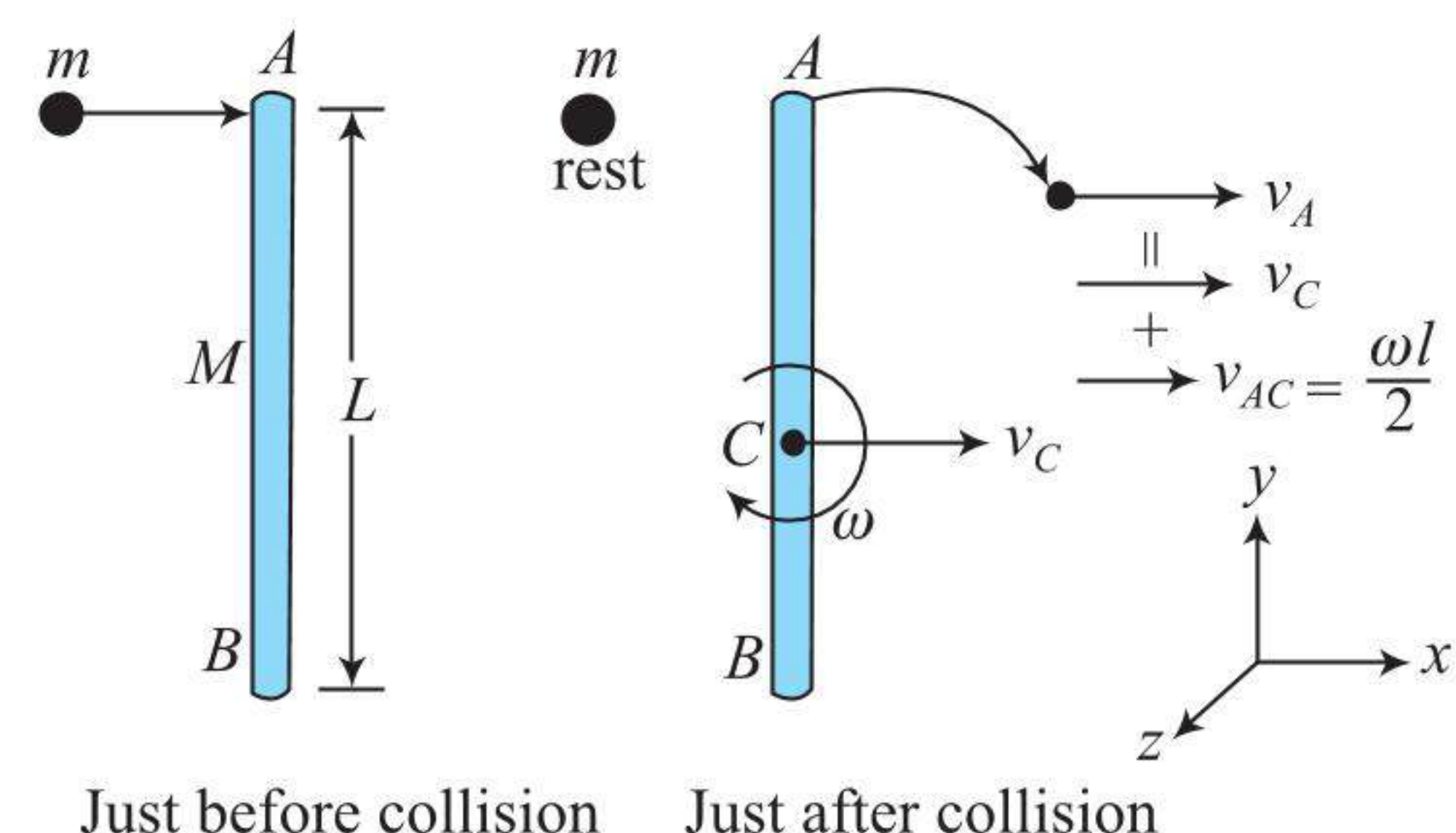
A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end A of the rod with a velocity v_0 in a direction perpendicular to AB . The collision is completely elastic. After the collision the particle comes to rest.



- Find the ratio m/M .
- A point P on the rod is at rest immediately after the collision. Find the distance AP .
- Find the linear speed of a point P at time $\pi L/3v_0$ after the collision.

Sol.

- If we take rod and particle as a system, the impulse between them will be internal. We can conserve the linear momentum of rod + particle's system. Let v_c and ω be the velocity and angular velocity of the CM just after collision.



Using conservation of linear momentum, we have

$$mv_0 = 0 + Mv_c \quad \dots(i)$$

Here in this case we can conserve angular momentum about any point on the rod.

And by conservation of angular momentum about the entire mass of the rod, we get

$$mv_0 \frac{L}{2} = I\omega \text{ or } mv_0 \frac{L}{2} = \left(\frac{ML^2}{12} \right) \omega \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$v_c = \frac{mv_0}{M} \text{ and } \omega = \frac{6mv_0}{ML}$$

We can apply conservation of angular momentum about point of collision.

$$\vec{L}_{\text{initial}} = \vec{L}_{\text{final}}$$

$$0 = I_0 \omega (-\hat{k}) + M v_c \frac{L}{2} (\hat{k})$$

$$\frac{ML^2}{12} \omega = \frac{M v_c L}{2}$$

which gives $\omega L = 6 v_c$... (iii)

Solving Eqs. (i) and (iii), we get the same results.

Since collision is completely elastic, therefore KE before collision is equal to KE after collision

$$\text{or } \frac{1}{2} m v_0^2 + 0 = \frac{1}{2} M v_c^2 + \frac{1}{2} I \omega^2$$

$$\text{or } \frac{1}{2} m v_0^2 = \frac{1}{2} M \left(\frac{m v_0}{M} \right)^2 + \frac{1}{2} \frac{ML^2}{12} \left(\frac{6 m v_0}{ML} \right)^2$$

$$\text{which gives } \frac{m}{M} = \frac{1}{4} \text{ and } v_c = \frac{v_0}{4}$$

Alternative method: We can apply coefficient of restitution equation at the point of collision, we have

$$e = \left(\frac{v_2 - v_1}{u_1 - u_2} \right)$$

Here u_1, u_2, v_1 and v_2 are the velocities of particles and point on the rod at collision point.

$$u_1 = v_0, u_2 = 0$$

$$v_1 = 0 \text{ and } v_2 = v_c + \frac{\omega l}{2}$$

As collision is elastic $e = 1$.

Substituting the values in restitution equation, we get

$$1 = \frac{\left(v_c + \frac{\omega l}{2} \right) - 0}{v_0 - 0} \Rightarrow v_0 = \left(\frac{m v_0}{M} \right) + \left(\frac{6 m v_0}{ML} \right) \frac{L}{2}$$

$$\text{which gives } \frac{m}{M} = \frac{1}{4}$$

(b) Velocity of point P immediately after collision is zero, let it be at a distance y from CM

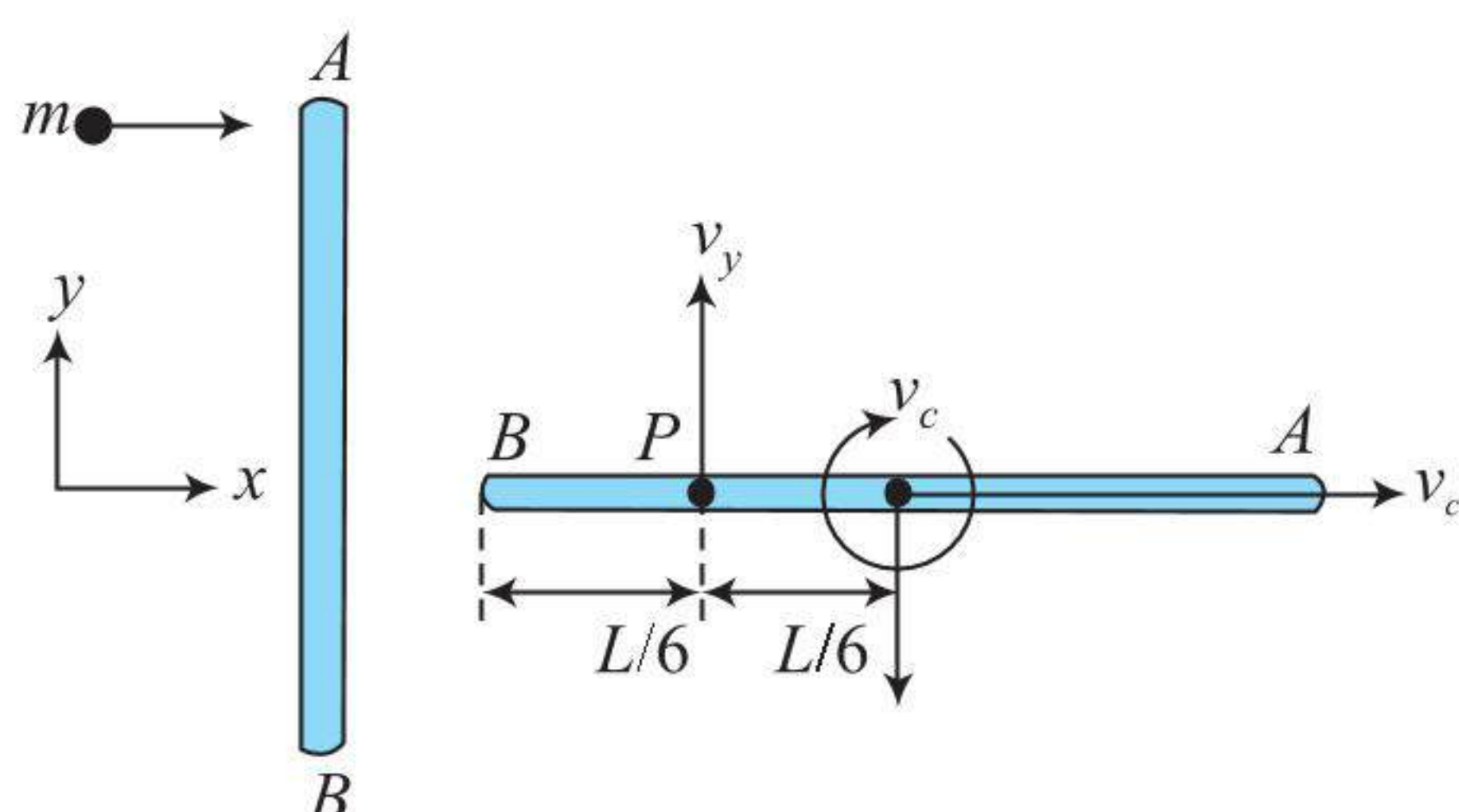
$$\therefore v_c - \omega y = 0 \text{ or } \frac{m v_0}{M} - \left(\frac{6 m v_0}{ML} \right) y = 0$$

$$\text{which gives } y = \frac{L}{6}$$

$$\therefore AP = \frac{L}{2} + \frac{L}{6} = \frac{2L}{3}$$

(c) Angle rotated by the rod at time $\frac{\pi L}{3 v_0}$,

$$\theta = \omega t = \left(\frac{6 m v_0}{ML} \right) \times \frac{\pi L}{3 v_0} = \frac{\pi}{2}$$



The rod turns through $\pi/2$ in this interval of time. The velocity of point P in y -direction will be

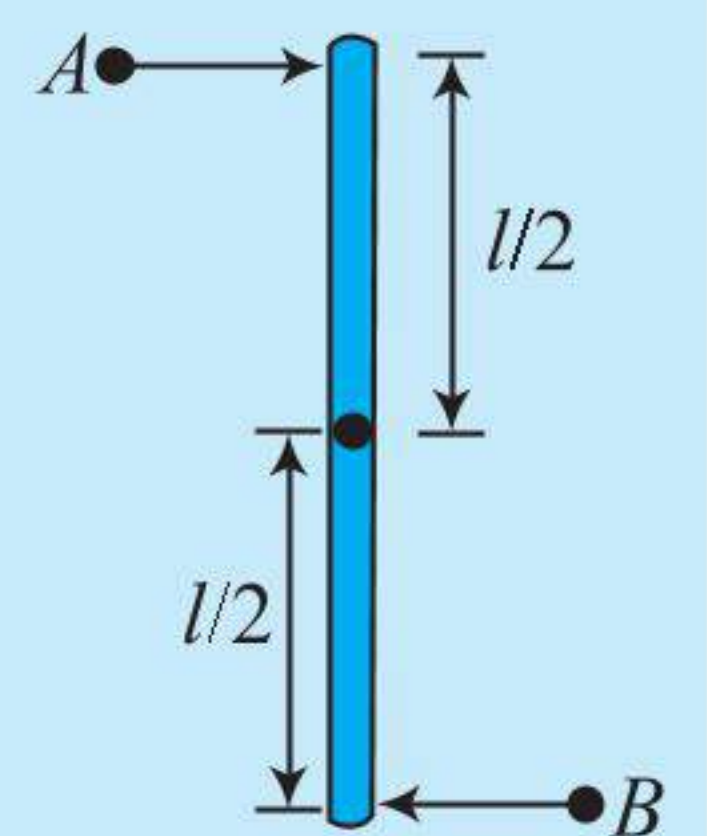
$$v_y = \omega y = \left(\frac{6 m v_0}{ML} \right) \times \frac{L}{6} = \frac{v_0}{4}$$

The resultant velocity of point P ,

$$v = \sqrt{v_c^2 + v_y^2} = \sqrt{\left(\frac{v_0}{4} \right)^2 + \left(\frac{v_0}{4} \right)^2} \Rightarrow \frac{v_0}{2\sqrt{2}}$$

ILLUSTRATION 4.65

Two particles A and B of mass m each hit the ends of a rigid bar of mass M and length l inelastically from opposite sides the speeds v and perpendicular to the rod. The bar is kept on a smooth horizontal plane (as shown). Find the linear and angular speed of the system (bar + particle).



Sol. There is no external force acting on the system along x -axis. Therefore, the linear momentum of the system before and after the system remains constant.

The momentum of the system before impact, $p_i = mv - mv = 0$

The linear momentum of the system after the impact must be zero. Therefore, the centre of mass of the system does not translate.

$$\Rightarrow v_{cm} = 0$$

Since the impact forces are internal force, the net torque about a perpendicular axis passing through CM of the system is zero. Therefore, the angular momentum of the system before and after the impact will be equal.

$$\Rightarrow L_{\text{initial}} = L_{\text{final}} \text{ where } L_{\text{initial}} = \left| \frac{ml}{2} v + \frac{ml}{2} v \right| = mvl$$

Let the system rotate about its CM ' O ' with an angular speed ω

$$\Rightarrow L_{\text{final}} = (I_{\text{system}}) \omega$$

where $I_{\text{system}} = \text{MI of the system about } O$,

$$I_{\text{system}} = (I_{\text{rod}})_0 + (I_A)_0 + (I_B)_0$$

$$\Rightarrow I_{\text{system}} = \frac{ML^2}{12} + m \left(\frac{l}{2} \right)^2 + m \left(\frac{l}{2} \right)^2 = \left(\frac{M+6m}{12} \right) l^2$$

Using the above equations, we get

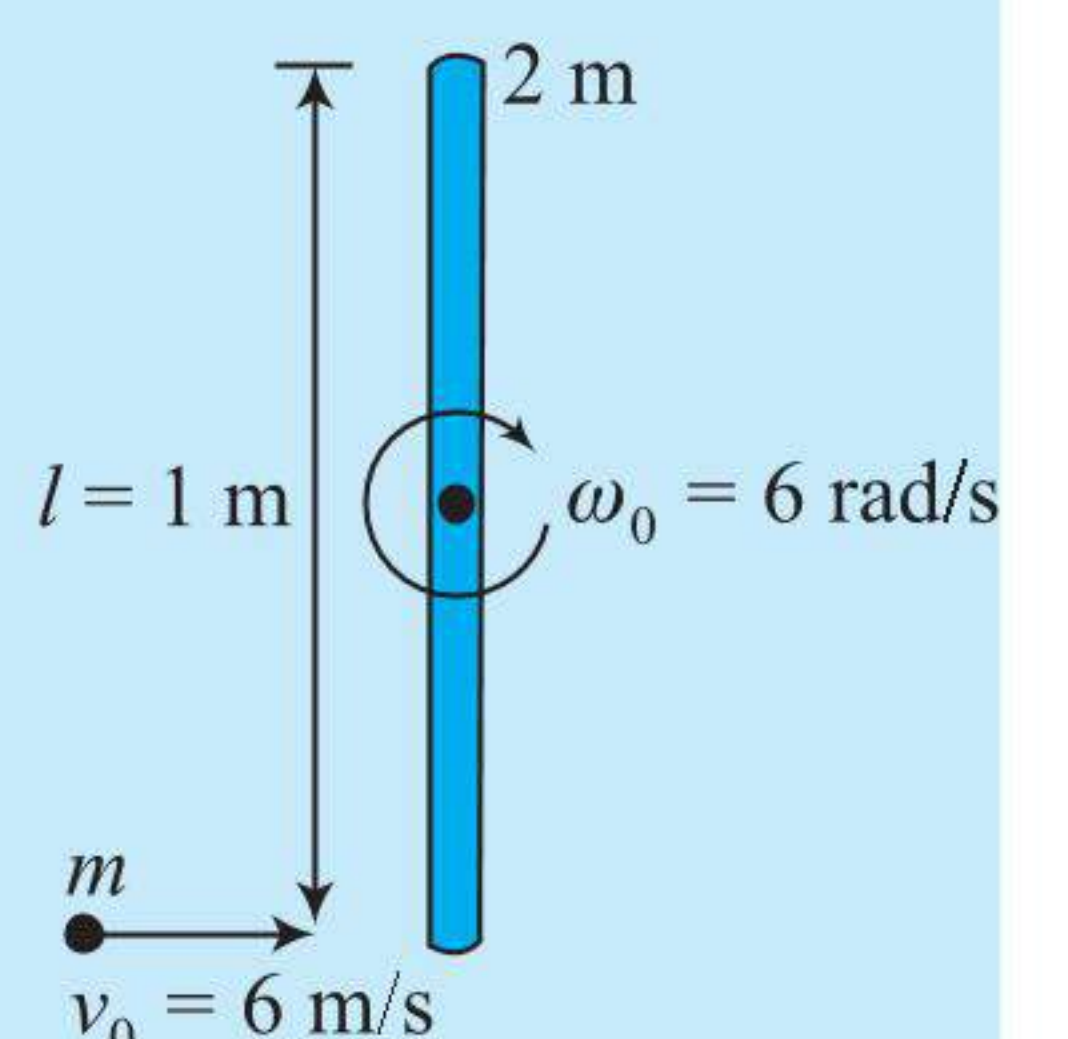
$$\left(\frac{M+6m}{12} \right) l^2 \omega = mvl \Rightarrow \omega = \frac{12mv}{(M+6m)l}$$

ILLUSTRATION 4.66

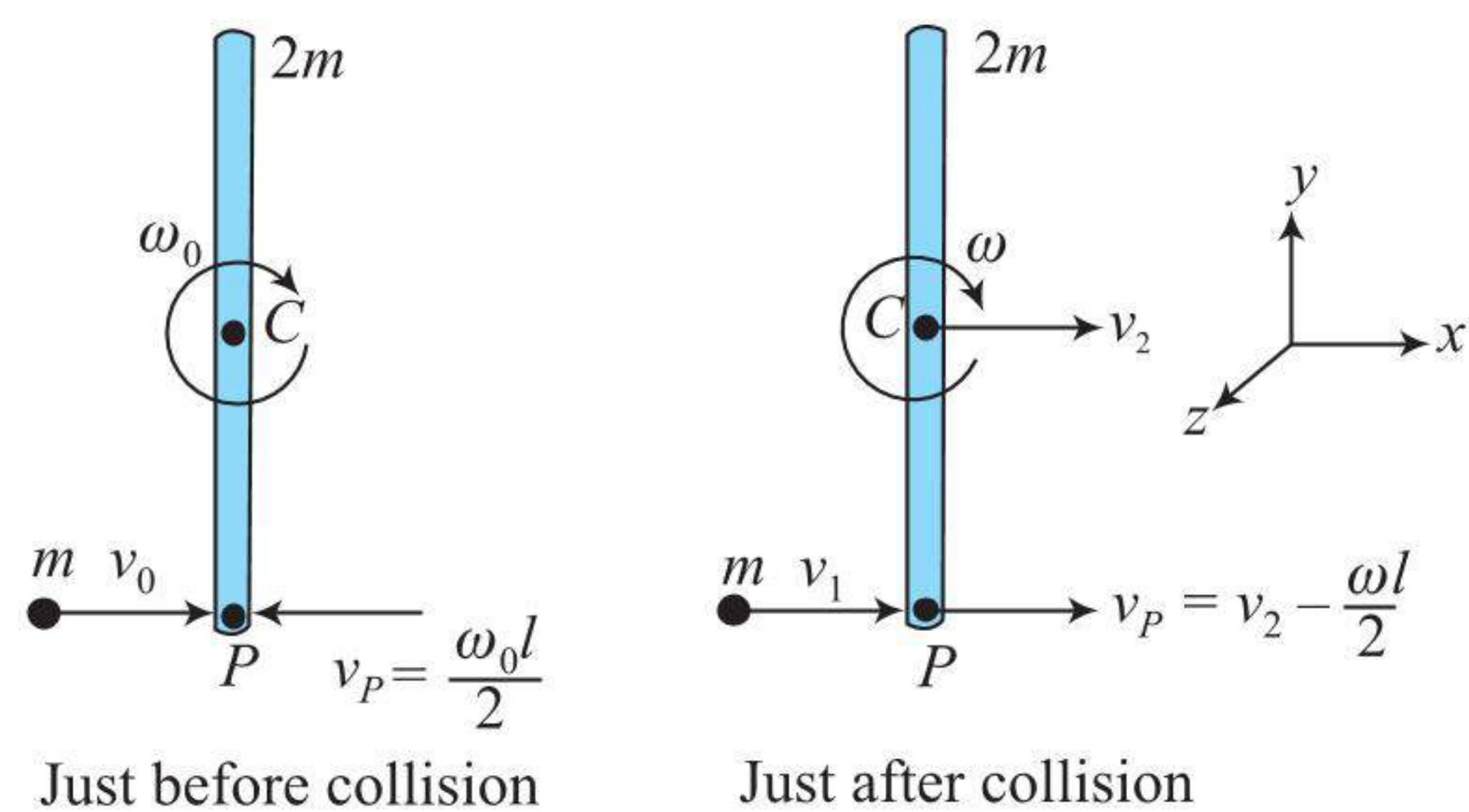
A particle of mass $m = 1 \text{ kg}$ collides with velocity $v_0 = 6 \text{ m s}^{-1}$ to a spinning rod of mass $2m$ and length $l = 1 \text{ m}$ at the end of the rod. If the coefficient of restitution $e = (2/3)$, find the:

(a) velocity of the particle.

(b) angular velocity of the rod just after the impact.



Sol. If we take rod and particle as system we find no external impulse acting on the system.



Hence, we can use conservation of linear momentum just before collision and just after collision. Let the velocities of particle and rod just after collision is v_1 and v_2 respectively.

$$mv_1 + 2mv_2 = mv_0 \text{ or } v_1 + 2v_2 = 6 \quad \dots(i)$$

Now applying Newton's restitution equation at the point of collision along the line of impulse.

$$e = \frac{2}{3} = \frac{\left(v_2 - \frac{\omega l}{2}\right) - v_1}{v_0 - \left(-\frac{\omega_0 l}{2}\right)}$$

$$\Rightarrow -2v_1 + 2v_2 = 12 + \omega \quad \dots(ii)$$

Conserving angular momentum about the point of collision.

$$\vec{L}_i = \frac{2ml^2}{12} \omega_0 (-\hat{k}) + 0$$

$$\vec{L}_f = \frac{2ml^2}{12} \omega (-\hat{k}) + 2mv_2 \times \frac{l}{2} (-\hat{k})$$

$$\text{As } \vec{L}_i = \vec{L}_f$$

$$\frac{2ml^2}{12} \omega_0 (-\hat{k}) = \frac{2ml^2}{12} \omega (-\hat{k}) + mv_2 l (-\hat{k})$$

$$\text{or } \omega_0 = \omega + \frac{6v_2}{l} \Rightarrow \omega = 6 - 6v_2 \quad \dots(iii)$$

From (ii) and (iii), we get

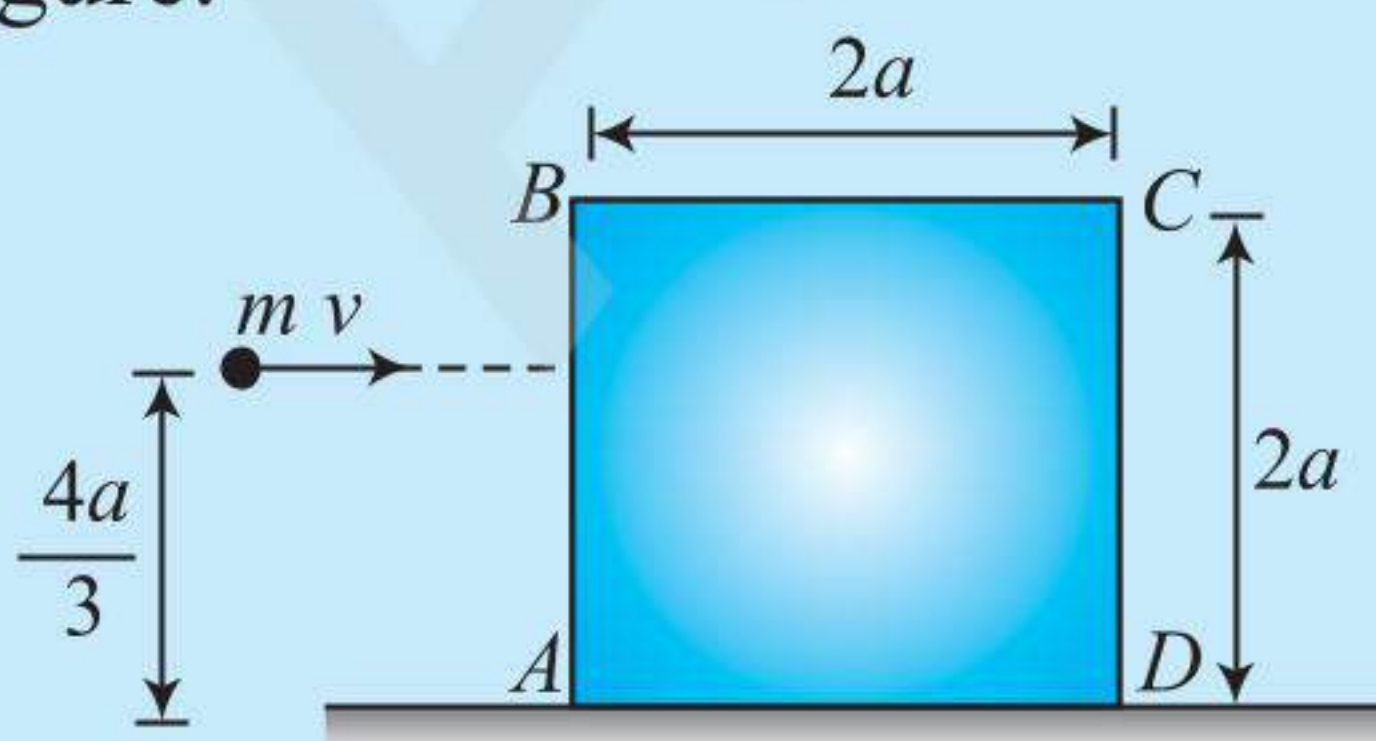
$$-v_1 + 4v_2 = 9 \quad \dots(iv)$$

From (i) and (iv), $v_1 = 1 \text{ m/s}$, $v_2 = 2.5 \text{ m/s}$ and $\omega = -9 \text{ rad/s}$

It means the just after collision the rod will start rotating with angular velocity 9 rad/s in counter clockwise sense.

ILLUSTRATION 4.67

A solid cube of wood of side $2a$ and mass M is resting on a horizontal surface. The cube is constrained to rotate about an axis passing through D and perpendicular to face $ABCD$. A bullet of mass m and speed v is shot at a height of $4a/3$ as shown in the figure.



The bullet becomes embedded in the cube. Find the minimum value of v required to topple the cube. Assume $m \ll M$.

Sol. Let ω be the angular velocity of cube just after collision about an axis passing through D . From conservation of the angular momentum about an axis passing through D

$$mv\left(\frac{4a}{3}\right) = I_D \cdot \omega$$

Here $I_D = I_{cm} + mr^2$

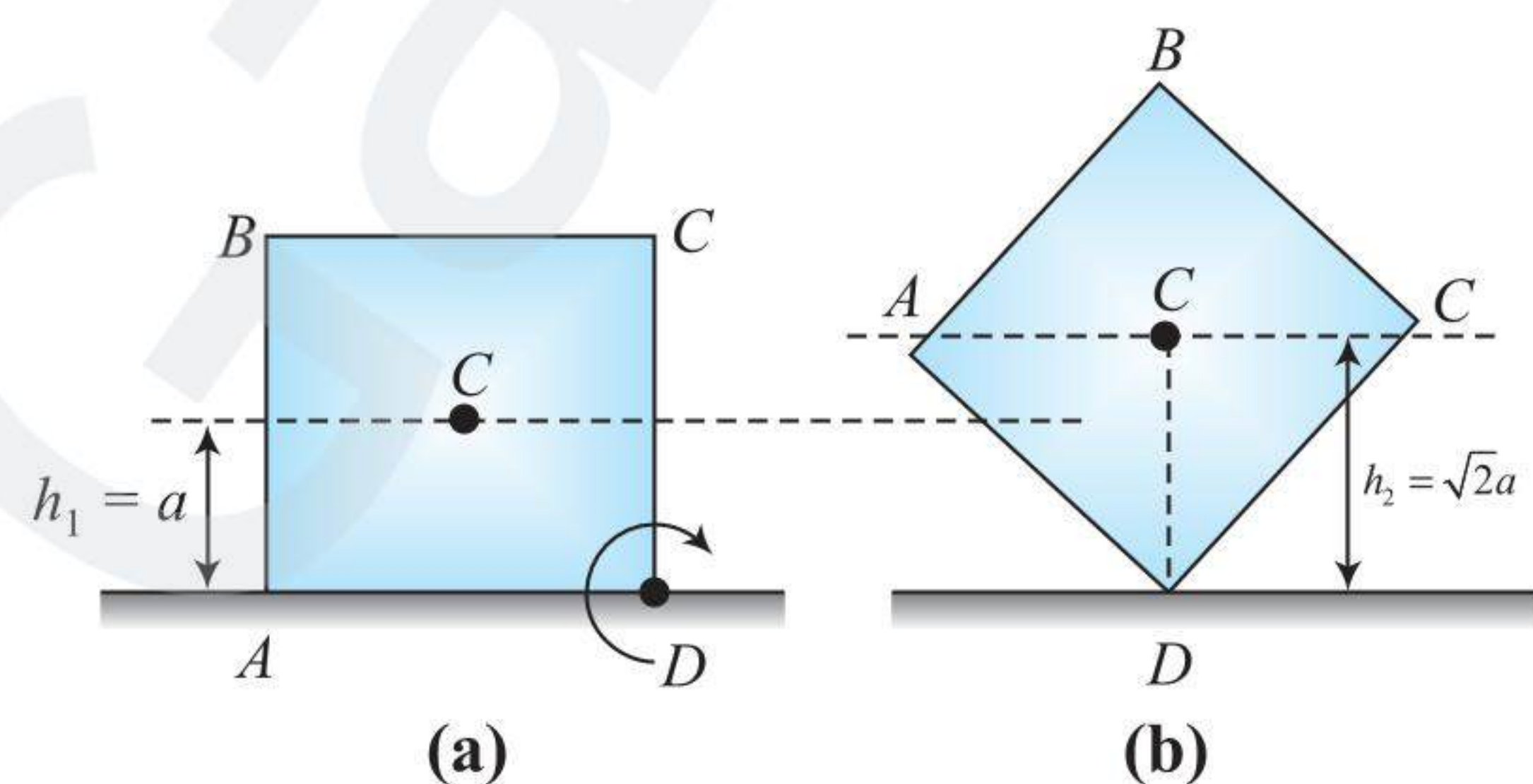
where r = perpendicular distance of axis of rotation passing through D from centre of mass (CM) of the cube $= \sqrt{2}a$

$$\text{or } \frac{4mva}{3} = \left[M\left(\frac{4a^2 + 4a^2}{12}\right) + M(2a^2) \right] \omega$$

$$\text{or } \frac{4mva}{3} = \frac{8}{3} Ma^2 \cdot \omega$$

$$\omega = \frac{1}{2} \frac{mv}{Ma} \quad \dots(i)$$

The cube will topple if its CM is just able to reach in a vertical height h_2 as shown in Fig. (b).



Hence, applying conservation of mechanical energy

$$\frac{1}{2} I_D \omega^2 = Mg(h_2 - h_1) \quad \left(\because I_D = \frac{8}{3} Ma^2 \right)$$

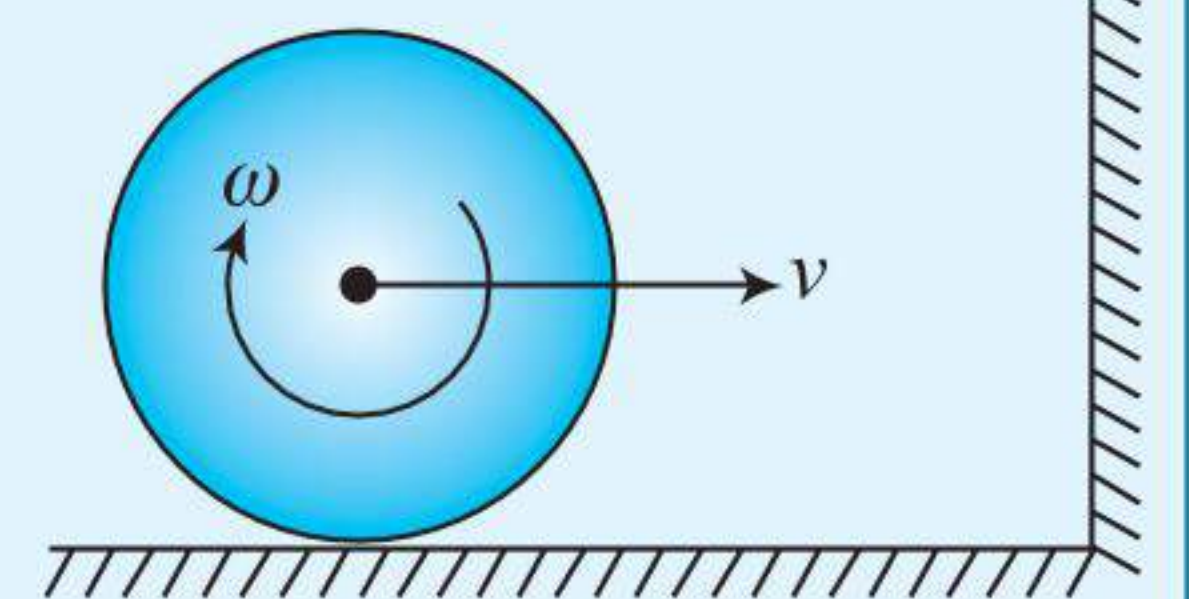
$$\text{or } \frac{1}{2} \left(\frac{8}{3} Ma^2 \right) \cdot \left\{ \frac{1}{2} \frac{mv}{Ma} \right\}^2 = Mg(\sqrt{2} - 1)a$$

$$\text{or } \frac{1}{3} \left(\frac{m}{M} \right)^2 v^2 = ag(\sqrt{2} - 1)$$

$$\text{or } v = \frac{M}{m} [3ag(\sqrt{2} - 1)]^{\frac{1}{2}}$$

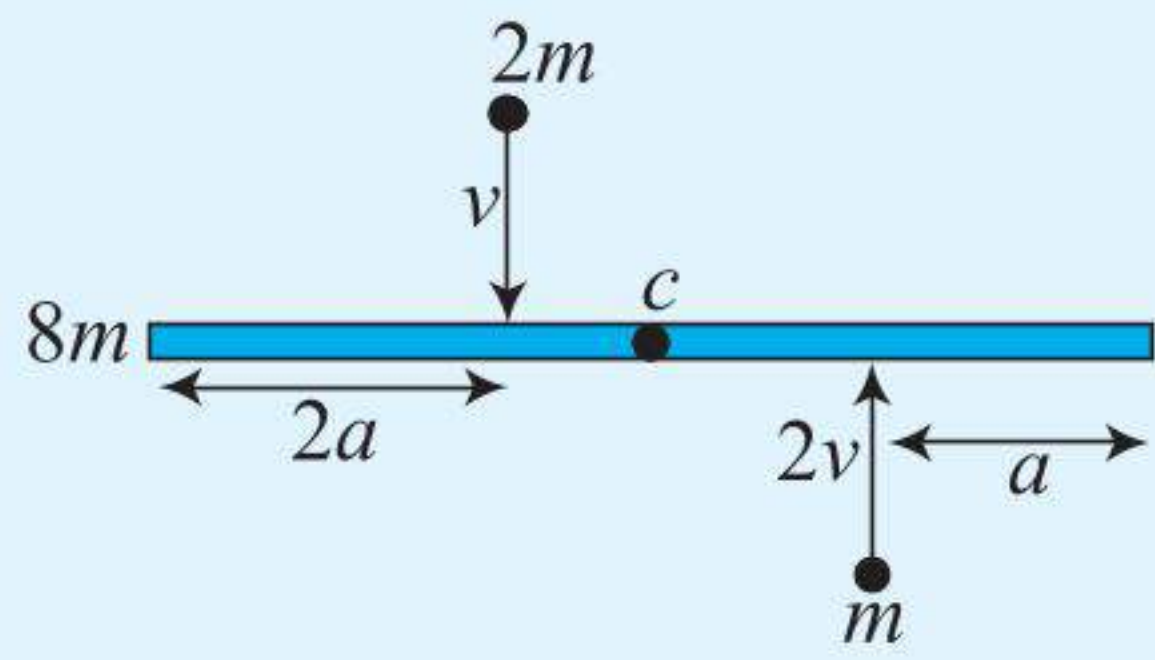
CONCEPT APPLICATION EXERCISE 4.7

1. A sphere rolling on a horizontal rough surface collides elastically with a smooth vertical wall, as shown in figure. State which of the following statements is true or false (T/F).

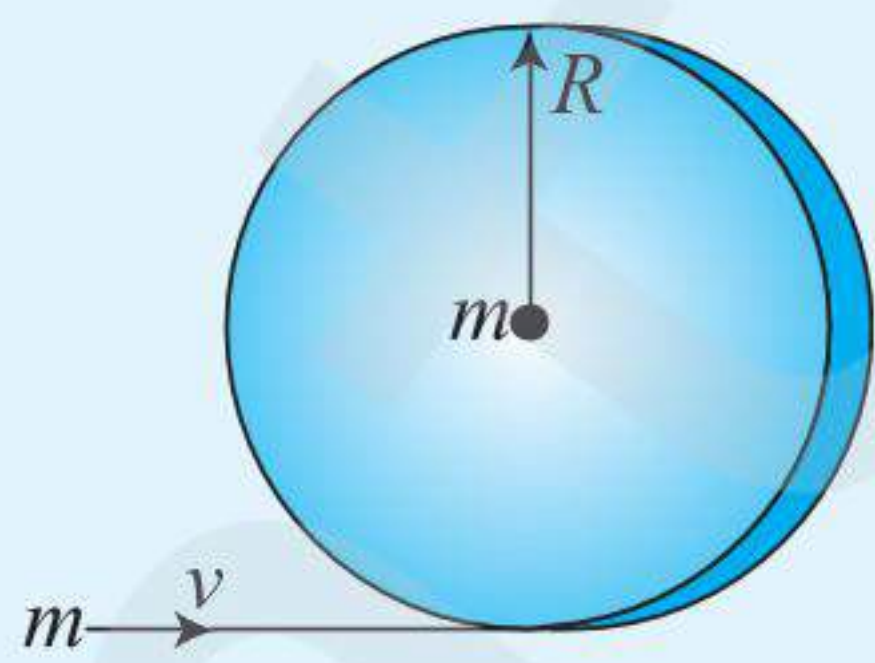
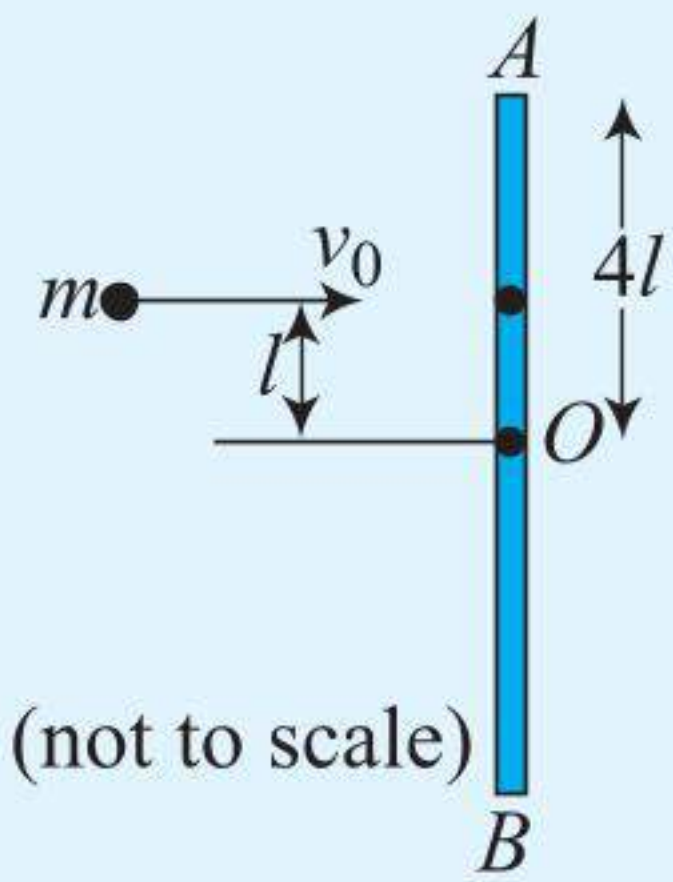


- (a) After collision, the velocity of the centre of mass gets reversed.
- (b) Angular momentum of the sphere about the point of contact with the wall is conserved.
- (c) Angular momentum of the sphere about a stationary point on the horizontal surface is conserved.
- (d) Just after collision the point of contact with the horizontal surface is moving towards the wall.
- (e) After collision the friction force acts on the sphere such that it decreases the linear speed and increases the angular speed.
- (f) Finally, when the sphere starts rolling, it is moving away from the wall.

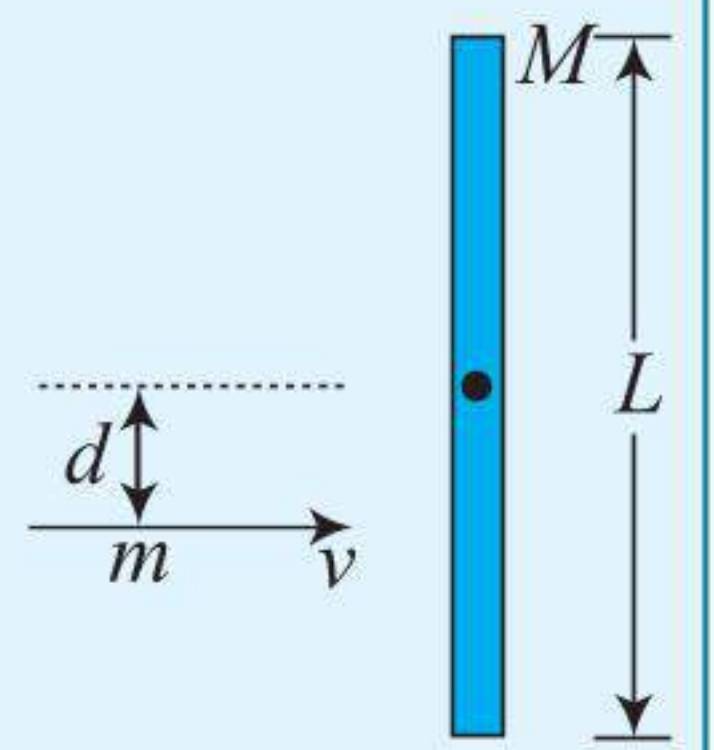
2. A uniform bar of length $6a$ and mass $8m$ lies on a smooth horizontal table. Two point masses m and $2m$ moving in opposite direction in the same horizontal plane with speed $2v$ and v , respectively, strike the bar and stick to the bar after collision. Calculate



- (a) the velocity of the centre of mass after collision
 (b) the angular velocity of the centre of mass after collision
 (c) total energy
3. A rod AB of mass M and length $8l$ lies on a smooth horizontal surface. A particle, of mass ' m ' and velocity v_0 , strikes the rod perpendicular to its length, as shown in figure. As a result of the collision, the centre of mass of the rod attains a speed of $v_0/8$ and the particle rebounds back with a speed of $v_0/4$. Find the following:
- (a) The ratio M/m .
 (b) The angular velocity of the rod about O .
 (c) The coefficient of restitution ' e ' for the collision.
 (d) The velocities of the ends ' A ' and ' B ' of the rod, namely, v_A and v_B , respectively.
4. A circular wooden hoop of mass m and radius R rests flat on a frictionless surface. A bullet, also of mass m and moving with a velocity v , strikes the hoop and gets embedded in it. The thickness of the hoop is much smaller than R . Find the angular velocity with which the system rotates after the bullet strikes the hoop.



5. A stick of length l and mass m lies on a frictionless horizontal surface on which it is free to move in anyway. A ball of mass m moving with speed v collides elastically with the stick. What must be the mass of the ball so that it remains at rest immediately after collision?



6. A thin uniform rod of length l is initially at rest with respect to an inertial frame of reference. The rod is tapped at one end perpendicular to its length. How far the centre of mass translates while the rod completes one revolution about its centre of mass. Neglect gravitational effect.
7. A thin spherical shell of radius R lying on a rough horizontal surface is hit sharply and horizontally by a cue. Where should it be hit so that the shell does not slip on the surface?
8. A wheel rolling along a rough horizontal surface with an angular velocity ω strikes a rough vertical wall, normally. Find the initial angular velocity, as it tends to roll up the wall (neglect any slippage) due to impulse.

ANSWERS

1. (a) T (b) T (c) F (d) F (e) T
2. (a) zero (b) $\frac{v}{5a}$ (b) $\frac{3mv^2}{5}$
3. (a) 10 (b) $\frac{3v_0}{128l}$ (c) $\frac{51}{128}$ (d) $\frac{7}{32}v_0, \frac{v_0}{32}$ 4. $\frac{v}{R}$
5. $\frac{ML^2}{L^2 + 12d^2}$ 6. $\frac{\pi l}{3}$ 7. $\frac{2R}{3}$ 10. $\frac{\omega_0}{3}$

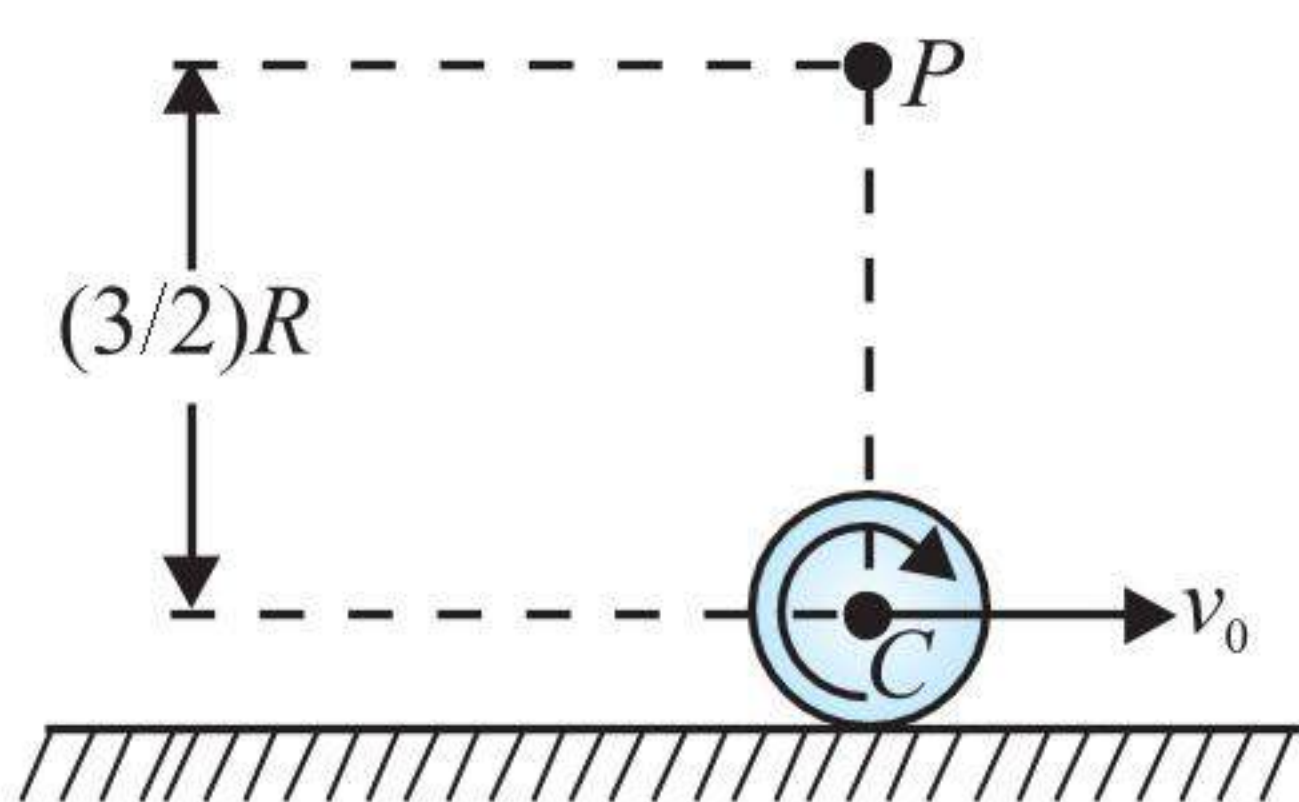
Exercises

Single Correct Answer Type

1. Two bodies with moments of inertia I_1 and I_2 ($I_1 > I_2$) have equal angular momenta. If their kinetic energies of rotation are E_1 and E_2 , respectively, then

- (1) $E_1 = E_2$ (2) $E_1 < E_2$
(3) $E_1 > E_2$ (4) $E_1 \geq E_2$

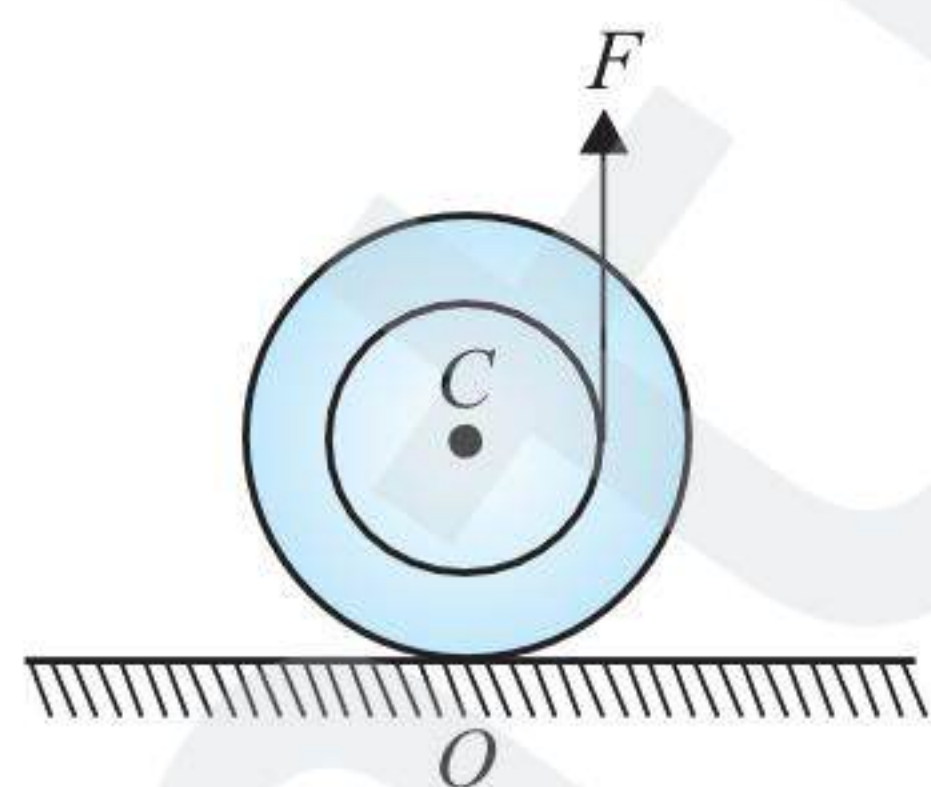
2. A disc of mass M and radius R rolls without slipping on a horizontal surface. If the velocity of its centre is v_0 , then the total angular momentum of the disc about a fixed point P at a height $3/2 R$ above the centre C



- (1) increases continuously as the disc moves away
(2) decreases continuously as the disc moves away
(3) is equal to $2MRv_0$
(4) is equal to MRv_0

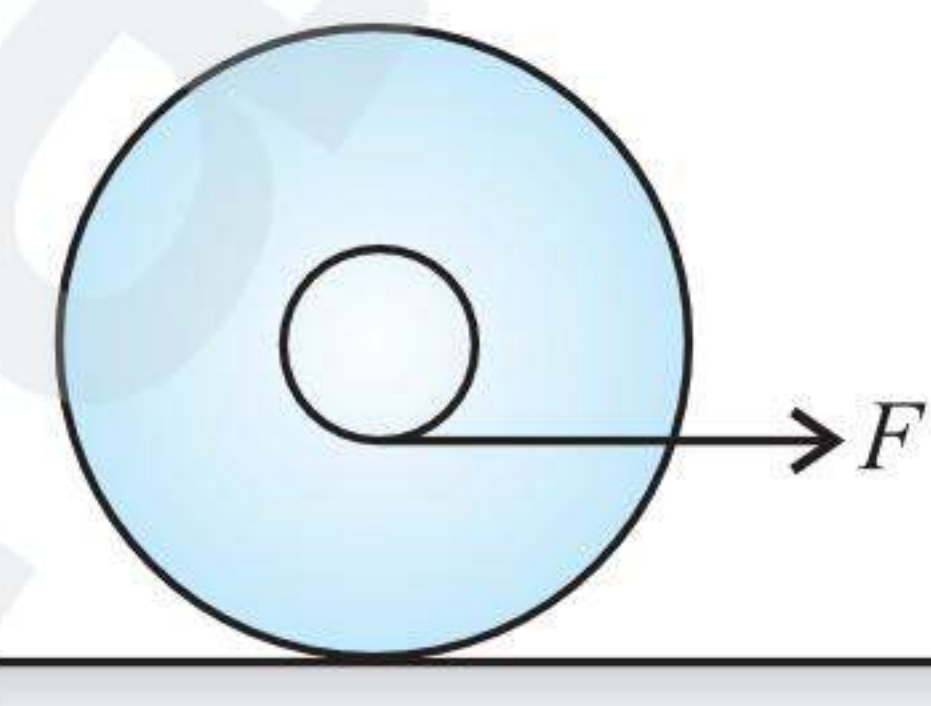
3. A yo-yo is placed on a rough horizontal surface and a constant force F , which is less than its weight, pulls it vertically. Due to this

- (1) friction force acts towards left, so it will move towards left
(2) friction force acts towards right, so it will move towards right
(3) it will move towards left, so friction force acts towards left
(4) it will move towards right so friction force acts towards right



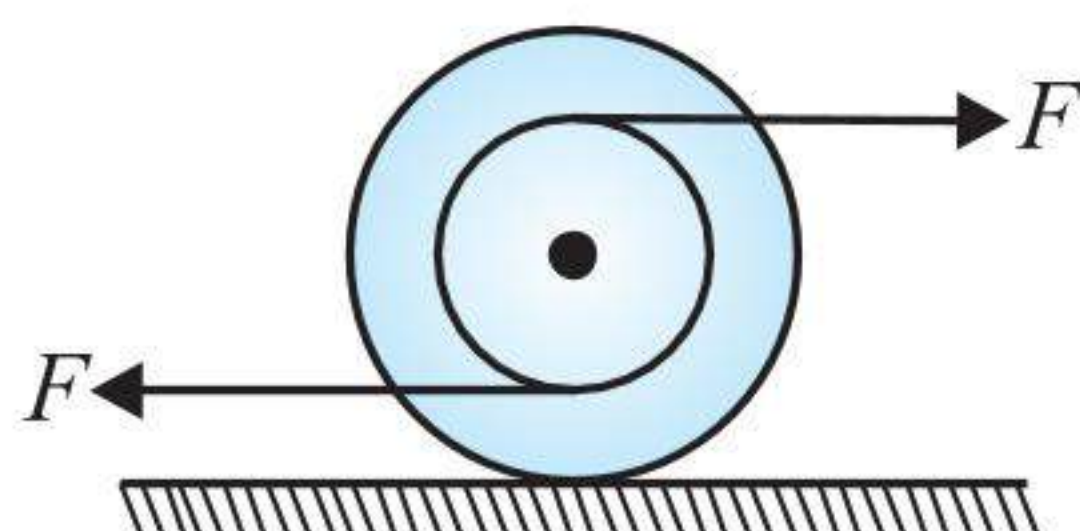
4. A yo-yo, arranged as shown, rests on a frictionless surface. When a force F is applied to the string, the yo-yo

- (1) moves to the left and rotates counterclockwise
(2) moves to the right and rotates counterclockwise
(3) moves to the left and rotates clockwise
(4) moves to the right and rotates clockwise



5. A spool is pulled horizontally by two equal and opposite forces as shown in figure. Which of the following statements are correct?

- (1) The centre of mass moves towards left.
(2) The centre of mass moves towards right.
(3) The centre of mass remains stationary.
(4) The net torque about the centre of mass of the spool is zero.



6. A ring, cylinder and solid sphere are placed on the top of a rough incline on which the sphere can just roll without slipping. When all of them are released at the same instant from the same position, then

- (1) all of them reach the ground at the same instant
(2) the sphere reaches first and the ring at the last
(3) the sphere reaches first and the cylinder and ring reach together
(4) none of the above

7. Suppose you are standing on the edge of a spinning platform and step off at right angles to the edge (radially outward). Now consider it the other way. You are standing on the ground next to a spinning carousel and you step onto the platform at right angles to the edge (radially inward).

- (1) There is no change in rotational speed of the carousel in either situation.
(2) There is a change in rotational speed in the first situation but not the second.
(3) There is a change in rotational speed in the second situation but not the first.
(4) There is a change in rotational speed in both instances.

8. A ball is given a velocity v and angular velocity such that the ball rolls purely on a plank whose upper surface is rough enough to prevent slipping but lower surface in contact with the ground is smooth. No other force is acting on system.

- (1) The plank will recoil back.
(2) The plank will also move forward but with a lesser velocity than that of the ball.
(3) The plank will also move forward but with a greater velocity than that of the ball.
(4) The plank will remain at rest.

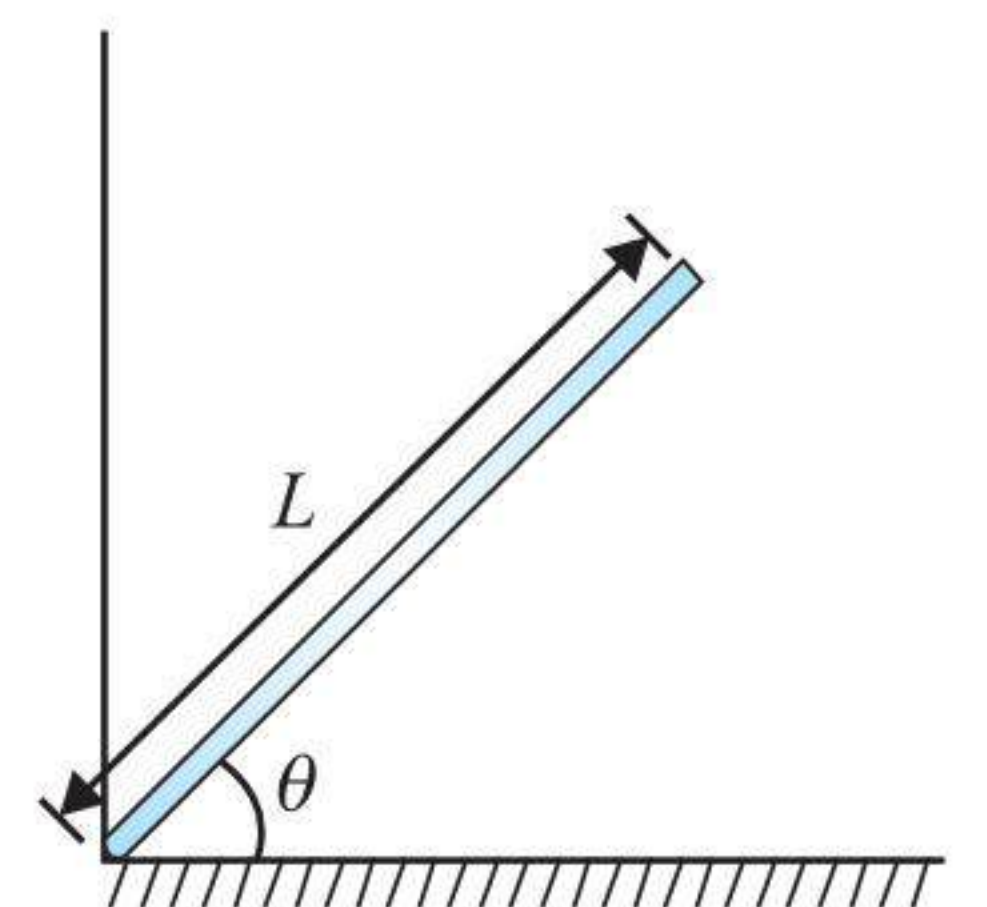
9. A solid homogenous sphere is moving on a rough horizontal surface, partially rolling and partially sliding. During this kind of motion of the sphere

- (1) Total kinetic energy is conserved
(2) Angular momentum of the sphere about the point of contact is conserved
(3) Only the rotational kinetic energy about the centre of mass is conserved
(4) Angular momentum about the centre of mass is conserved

10. A body is rolling without slipping on a horizontal plane. The rotational energy of the body is 40% of the total kinetic energy. Identify the body.

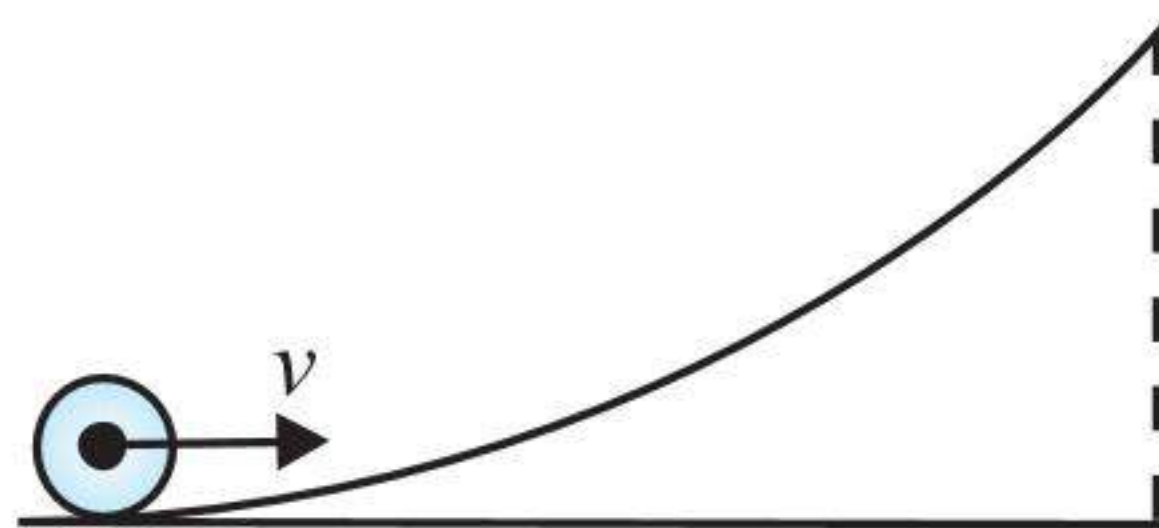
- (1) Ring (2) Hollow cylinder
(3) Solid cylinder (4) Hollow sphere

11. A uniform pole of length L and mass M is pivoted on the ground with a frictionless hinge. The pole makes an angle θ with the horizontal. The moment of inertia of the pole about one end is $(1/3)ML^2$. If it starts falling from the position shown in the accompanying figure, the linear acceleration of the free end of the pole immediately after release would be



- (1) $\left(\frac{2}{3}\right) g \cos \theta$ (2) $\left(\frac{2}{3}\right) g$
 (3) g (4) $\left(\frac{3}{2}\right) g \cos \theta$

12. A small object of uniform density rolls up a curved surface with an initial velocity v . It reaches up to a maximum height of $3v^2/4g$ w.r.t. the initial position. The object is



- (1) ring (2) solid sphere
 (3) hollow sphere (4) disc
13. A solid iron sphere A rolls down an inclined plane, while an identical hollow sphere B of same mass slides down the plane in a frictionless manner. At the bottom of the inclined plane, the total kinetic energy of sphere A is
- (1) less than that of B
 (2) equal to that of B
 (3) more than that of B
 (4) sometimes more and sometimes less

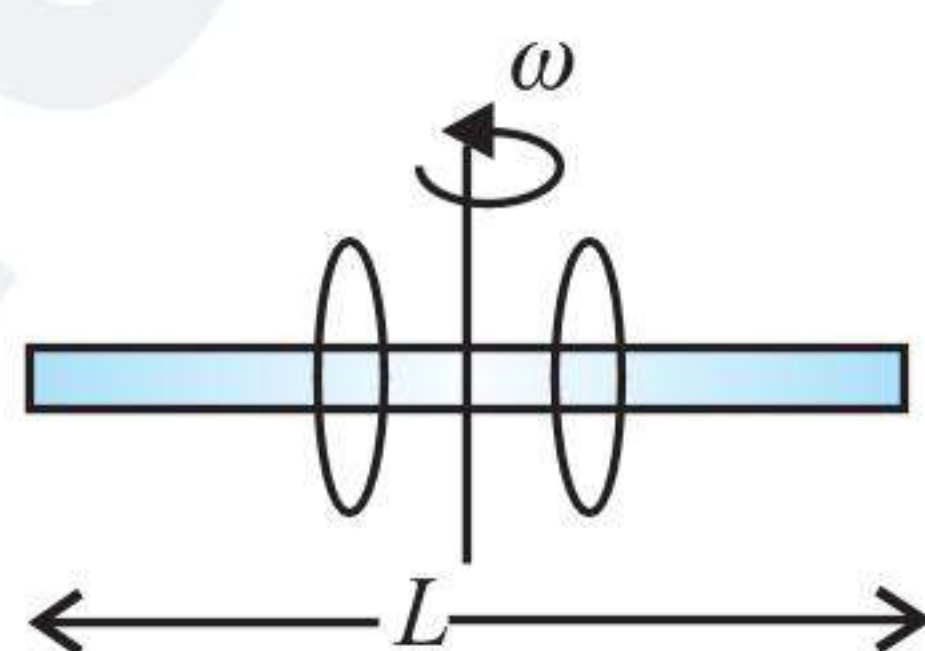
14. A string is wrapped around a cylinder of mass M and radius R . The string is pulled vertically upwards to prevent the centre of mass from falling as the cylinder unwinds the string. The tension in the string is

- (1) $\frac{Mg}{6}$ (2) $\frac{Mg}{3}$
 (3) $\frac{Mg}{2}$ (4) $\frac{2Mg}{3}$

15. In problem 14, work done on the cylinder for reaching an angular speed ω is

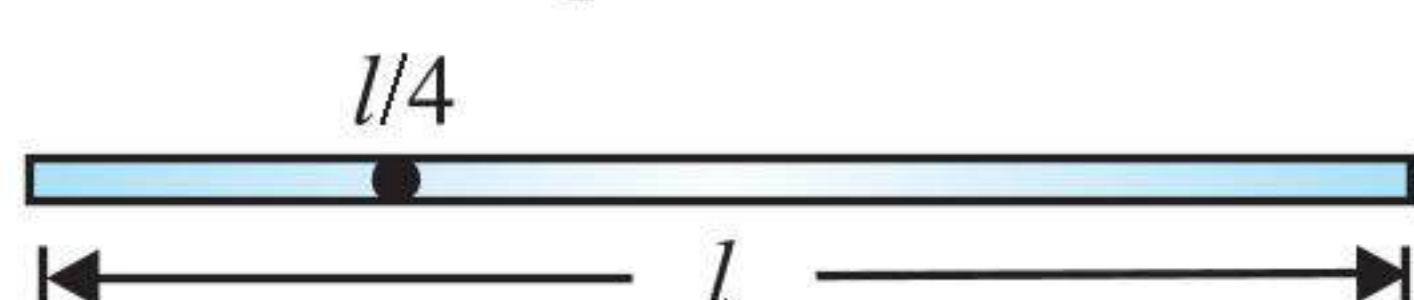
- (1) $\frac{MR^2\omega^2}{4}$ (2) $\frac{MR^2\omega^2}{2}$
 (3) $\frac{MR^2\omega^2}{3}$ (4) $\frac{2MR^2\omega^2}{3}$

16. A smooth uniform rod of length L and mass M has two identical beads (1 and 2) of negligible size, each of mass m , which can slide freely along the rod. Initially the two beads are at the centre of the rod and the system is rotating with angular velocity ω_0 about an axis perpendicular to the rod and is passing through its midpoint. There are no external forces when the beads reach the ends of the rod, the angular velocity of the system is



- (1) $\frac{M\omega_0}{M+6m}$ (2) $\frac{M\omega_0}{m}$
 (3) $\frac{M\omega_0}{M+12m}$ (4) ω_0

17. A uniform thin rod of length l and mass m is hinged at a distance $l/4$ from one of the end and released from horizontal position as shown in figure. The angular velocity of the rod as it passes the vertical position is

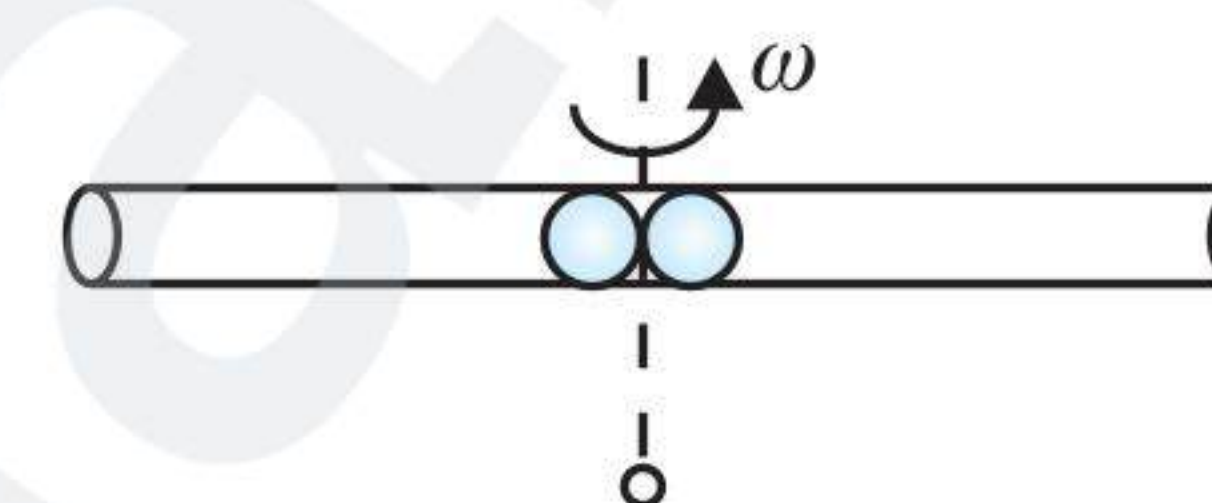


- (1) $2\sqrt{\frac{5g}{7l}}$ (2) $2\sqrt{\frac{6g}{7l}}$
 (3) $\sqrt{\frac{3g}{7l}}$ (4) $2\sqrt{\frac{g}{l}}$

18. A sphere is released on a smooth inclined plane from the top. When it moves down, its angular momentum is

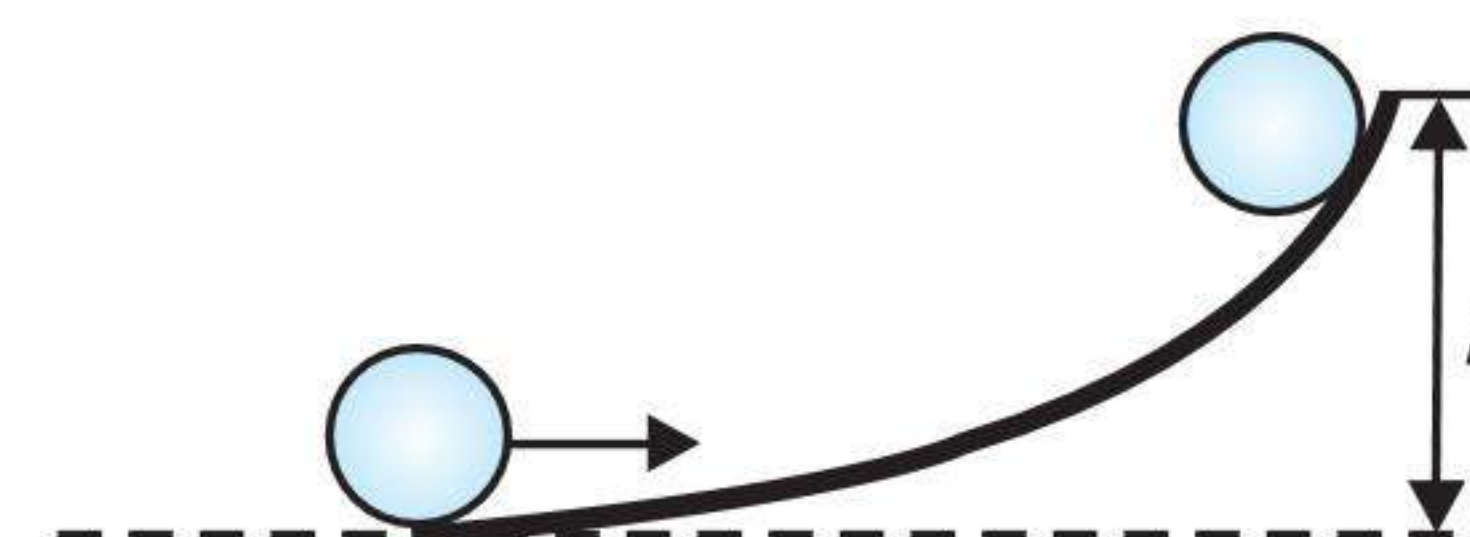
- (1) conserved about every point
 (2) conserved about the point of contact only
 (3) conserved about the centre of the sphere only
 (4) conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball

19. A smooth tube of certain mass is rotated in a gravity-free space and released. The two balls shown in figure move towards the ends of the tube. For the whole system, which of the following quantities is not conserved.



- (1) Angular momentum (2) Linear momentum
 (3) Kinetic energy (4) Angular speed

20. In the figure shown, a ball without sliding on a horizontal surface. It ascends a curved track up to height h and returns. The value of h is h_1 for sufficiently rough curved track to avoid sliding and is h_2 for smooth curved track, then



- (1) $h_1 = h_2$ (2) $h_1 < h_2$
 (3) $h_1 > h_2$ (4) $h_2 = 2h_1$

21. A solid sphere, a hollow sphere and a disc, all having the same mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are same and not sufficient to allow pure rolling. The least time will be taken in reaching the bottom by

- (1) the solid sphere (2) the hollow sphere
 (3) the disc (4) all will take the same time

22. In the previous problem, the smallest kinetic energy at the bottom of the incline will be achieved by

- (1) the solid sphere (2) the hollow sphere
 (3) the disc (4) all will achieve the same kinetic energy

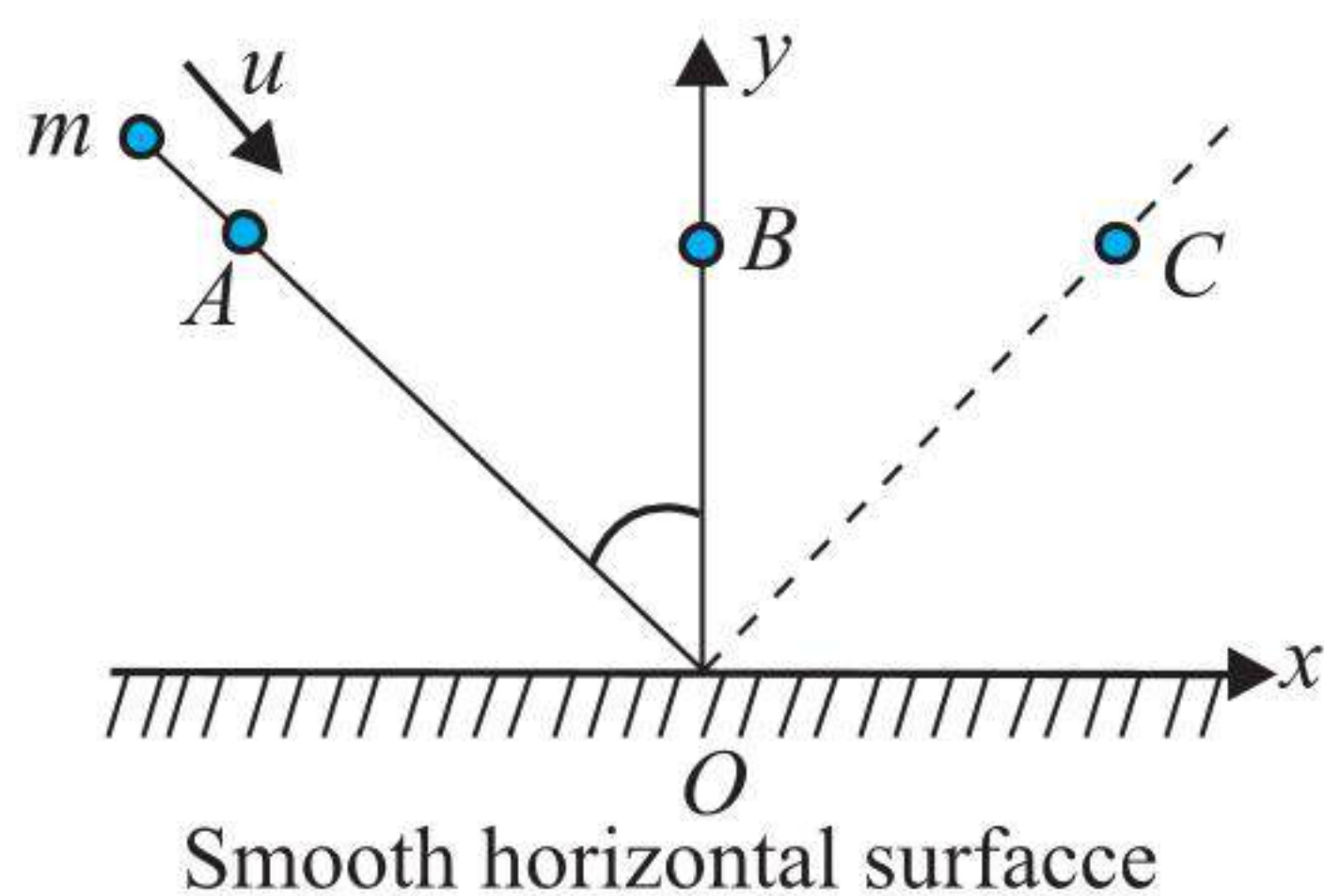
23. An impulse $J = mv$ at one end of a stationary uniform frictionless rod of mass m and length l which is free to rotate in a gravity-free space. The impact is elastic. Instantaneous axis of rotation of the rod will pass through

- (1) its centre of mass
 (2) the centre of mass of the rod plus ball
 (3) the point of impact of the ball on the rod
 (4) the point which is at a distance $2l/3$ from the striking end

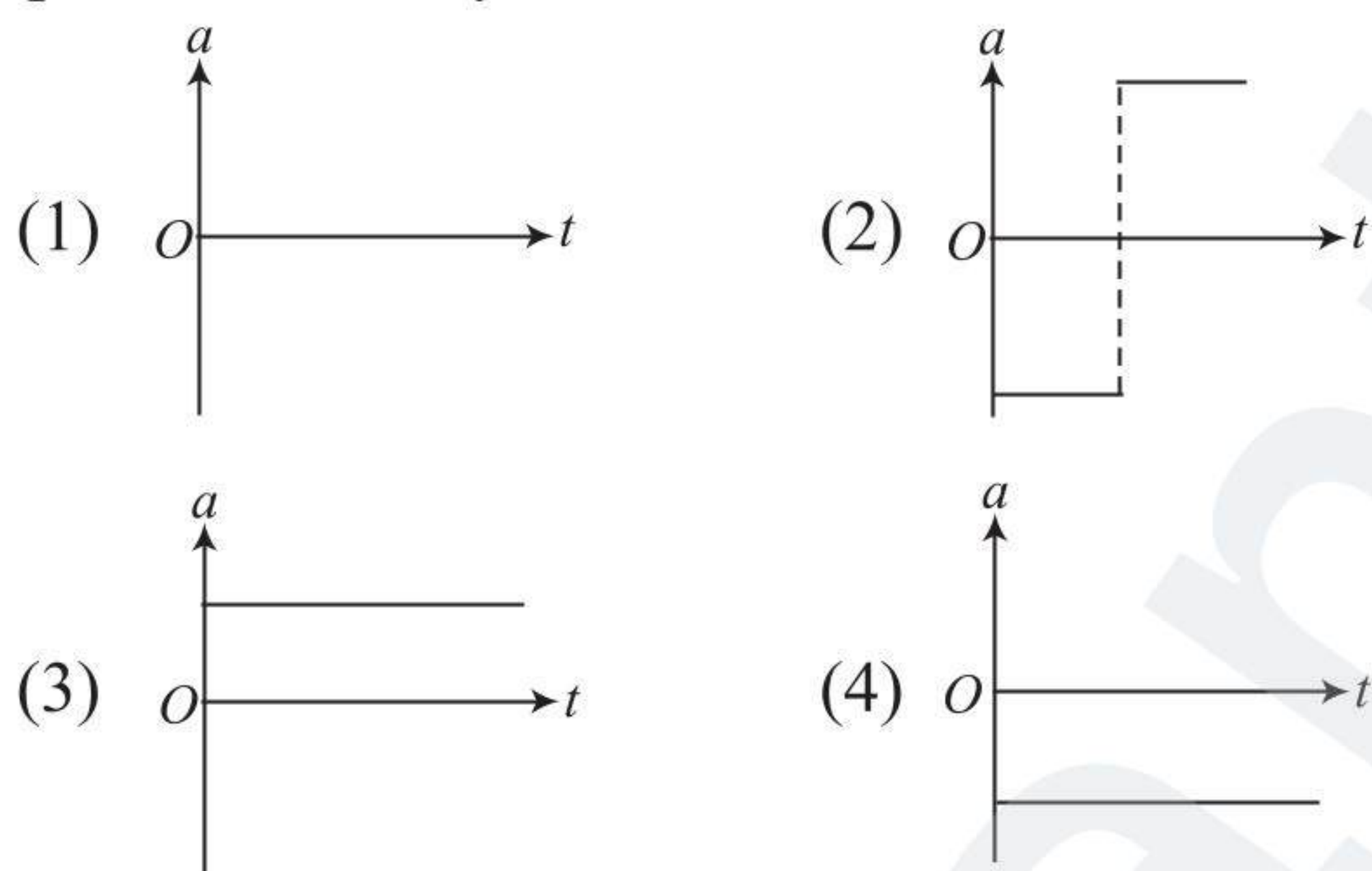
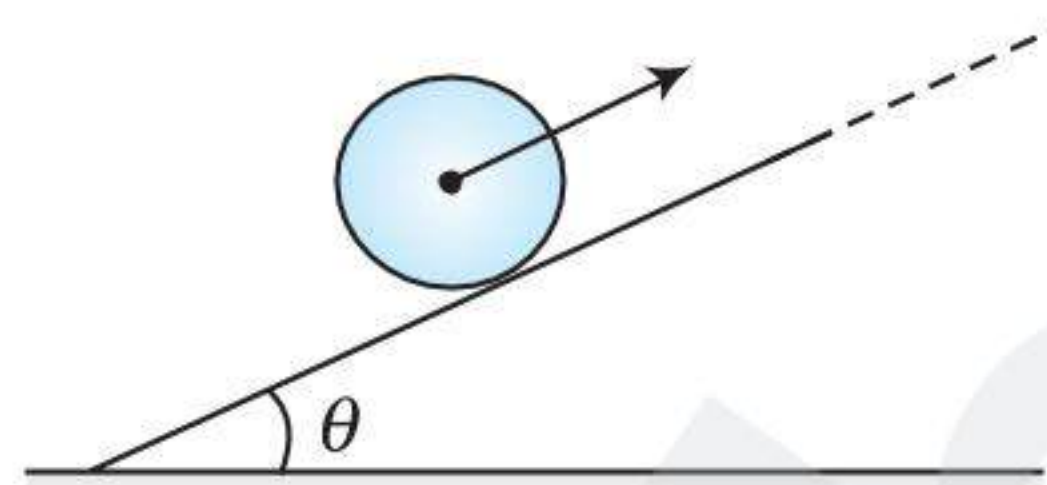
24. A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground. Then pick up the incorrect alternative.

- (1) Angular momentum of the particle about O is increasing.

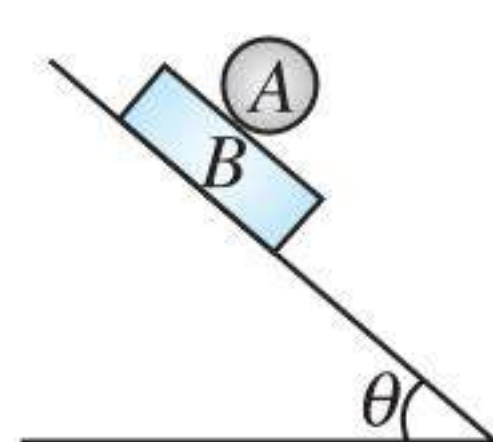
- (2) The moment of inertia of the particle about O is decreasing.
 (3) The moment of inertia of the particle about O is increasing.
 (4) The angular velocity of the particle about O is increasing.
25. A ball of mass m moving with constant velocity u collides with a smooth horizontal surface at O as shown in figure. Neglect gravity and friction. The y -axis is drawn normal to the horizontal surface at the point of impact O and x -axis is horizontal as shown. About which point will the angular momentum of ball be conserved?



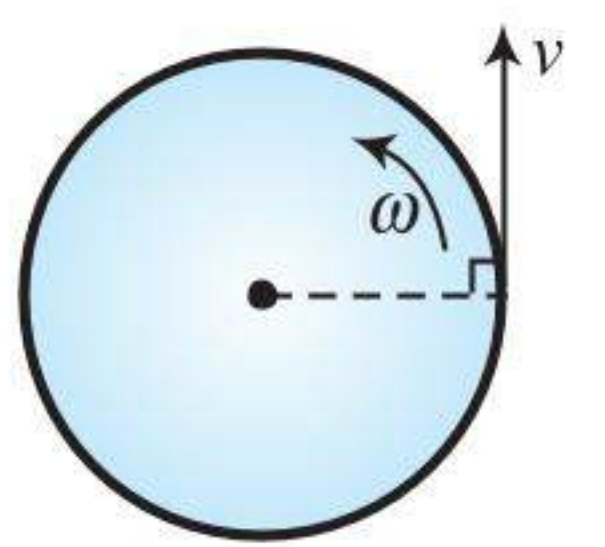
- (1) Point A (2) Point B
 (3) Point C (4) None of these
26. A uniform solid sphere rolls up (without slipping) the rough fixed inclined plane, and then back down. Which is the correct graph of acceleration a of centre of mass of solid sphere as function of time t (for the duration sphere is on the incline)? Assume that the sphere rolling up has a positive velocity.



27. A rolling object rolls without slipping down an inclined plane (angle of inclination θ), then the minimum acceleration it can have is
 (1) $g \sin \theta$ (2) $\frac{2g \sin \theta}{3}$
 (3) $\frac{g \sin \theta}{2}$ (4) zero
28. Three identical solid spheres move down three incline A , B and C are all of the same dimensions. A is without friction, the friction between B and a sphere is sufficient to cause rolling without slipping, the friction between C and a sphere causes rolling with slipping. The kinetic energies of A , B , C at the bottom of the inclines are E_A , E_B , E_C :
 (1) $E_A = E_B = E_C$ (2) $E_A = E_B > E_C$
 (3) $E_A > E_B > E_C$ (4) $E_A > E_B = E_C$
29. A rolling body is kept on a plank B . There is sufficient friction between A and B and no friction between B and the inclined plane. Then body:



- (1) A rolls
 (2) A does not experience any friction
 (3) A and B has equal acceleration and unequal velocities
 (4) A rolls depending upon the angle of inclination θ
30. A solid sphere rolls down two different inclined planes of the same height but of different inclinations
 (1) the speeds will be same but time of descent will be different
 (2) in both cases, the speeds and time of descent will be same
 (3) speeds and time of descent both will be different
 (4) the speeds will be different but time of descent will be same
31. A child with mass m is standing at the edge of a playground merry-go-round (A large uniform disc which rotates in horizontal plane about a fixed vertical axis in parks) with moment of inertia I , radius R , and initial angular velocity ω as shown in the figure. The child jumps off the edge of the merry-go-round with a velocity v with respect to the ground in direction tangent to periphery of the disc as shown. The new angular velocity of the merry-go-round is



- (1) $\sqrt{\frac{I\omega^2 - mv^2}{I}}$ (2) $\sqrt{\frac{(I + mR^2)\omega^2 - mv^2}{I}}$
 (3) $\frac{I\omega - mvR}{I}$ (4) $\frac{(I + mR^2)\omega - mvR}{I}$

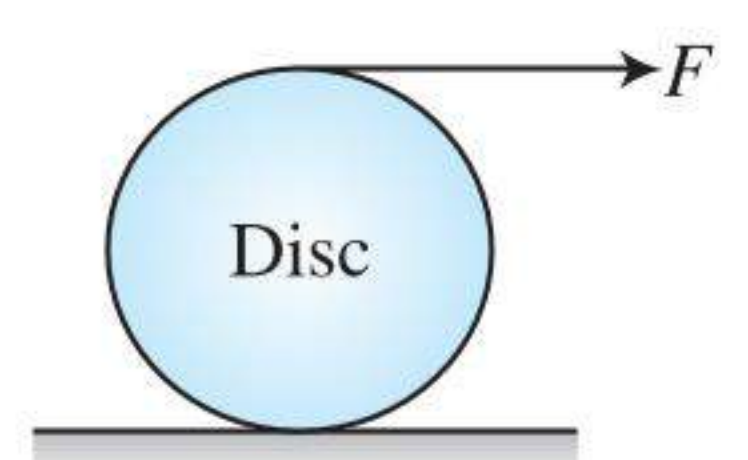
32. A thick walled hollow sphere has outer radius R . It rolls down an inclined plane without slipping and its speed at the bottom is v . If the inclined plane is frictionless and the sphere slides down without rolling, its speed at the bottom is $5v/4$. What is the radius of gyration of the sphere?

- (1) $\frac{R}{\sqrt{2}}$ (2) $\frac{R}{2}$
 (3) $\frac{3R}{4}$ (4) $\frac{\sqrt{3}R}{4}$

33. If a rigid body rolls on a surface without slipping, then
 (1) linear speed is maximum at the highest point but minimum at the point of contact
 (2) linear speed is minimum at highest point but maximum at the point of contact
 (3) linear speed is same at all points of the rigid body
 (4) angular speed is different at different points of a rigid body

34. Two discs, each having moment of inertia 5 kg m^2 about its central axis, rotating with speeds 10 rad s^{-1} and 20 rad s^{-1} , are brought in contact face to face with their axes of rotation coincided. The loss of kinetic energy in the process is
 (1) 2 J (2) 5 J
 (3) 125 J (4) 0 J

35. A force F acts tangentially at the highest point of a disc of mass m kept on a rough horizontal plane. If the disc rolls without slipping, the acceleration of centre of the disc is



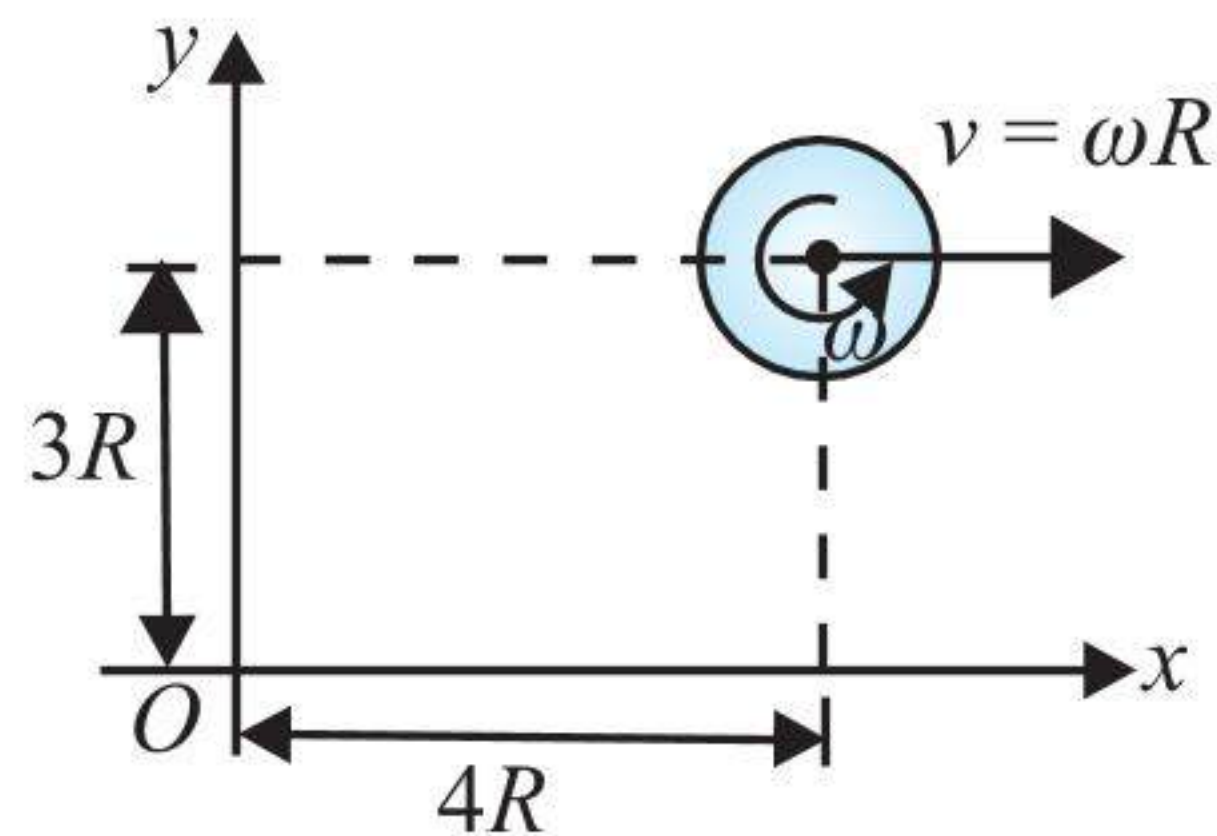
(1) $\frac{2F}{3m}$

(2) $\frac{10F}{7m}$

(3) zero

(4) $\frac{4F}{3m}$

36. A disc of mass m and radius R moves in the x - y plane as shown in figure. The angular momentum of the disc about the origin O at the instant shown is



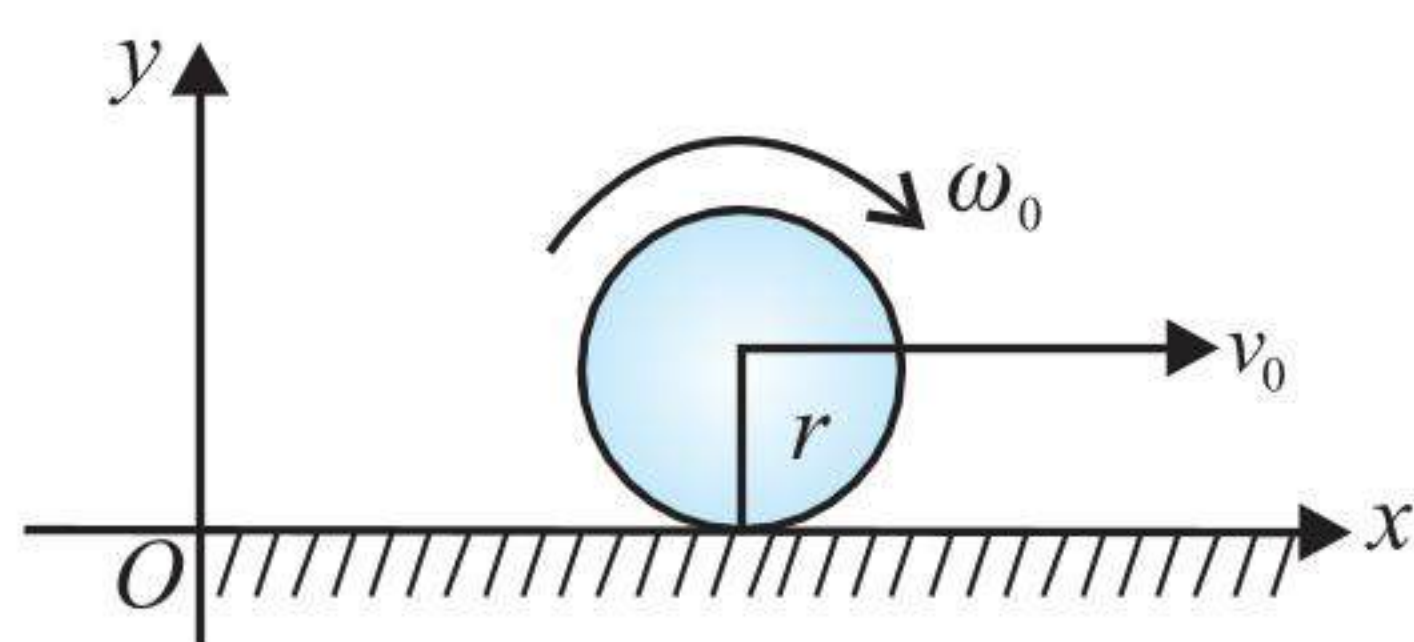
(1) $-\frac{5}{2}mR^2\omega\hat{k}$

(2) $\frac{7}{3}mR^2\omega\hat{k}$

(3) $-\frac{9}{2}mR^2\omega\hat{k}$

(4) $\frac{5}{2}mR^2\omega\hat{k}$

37. A uniform sphere of mass m , radius r and moment of inertia I about its centre moves along the x -axis as shown in figure. Its centre of mass moves with velocity $= v_0$, and it rotates about its centre of mass with angular velocity $= \omega_0$. Let $\vec{L} = (I\omega_0 + mv_0r)(-\hat{k})$. The angular momentum of the body about the origin O is



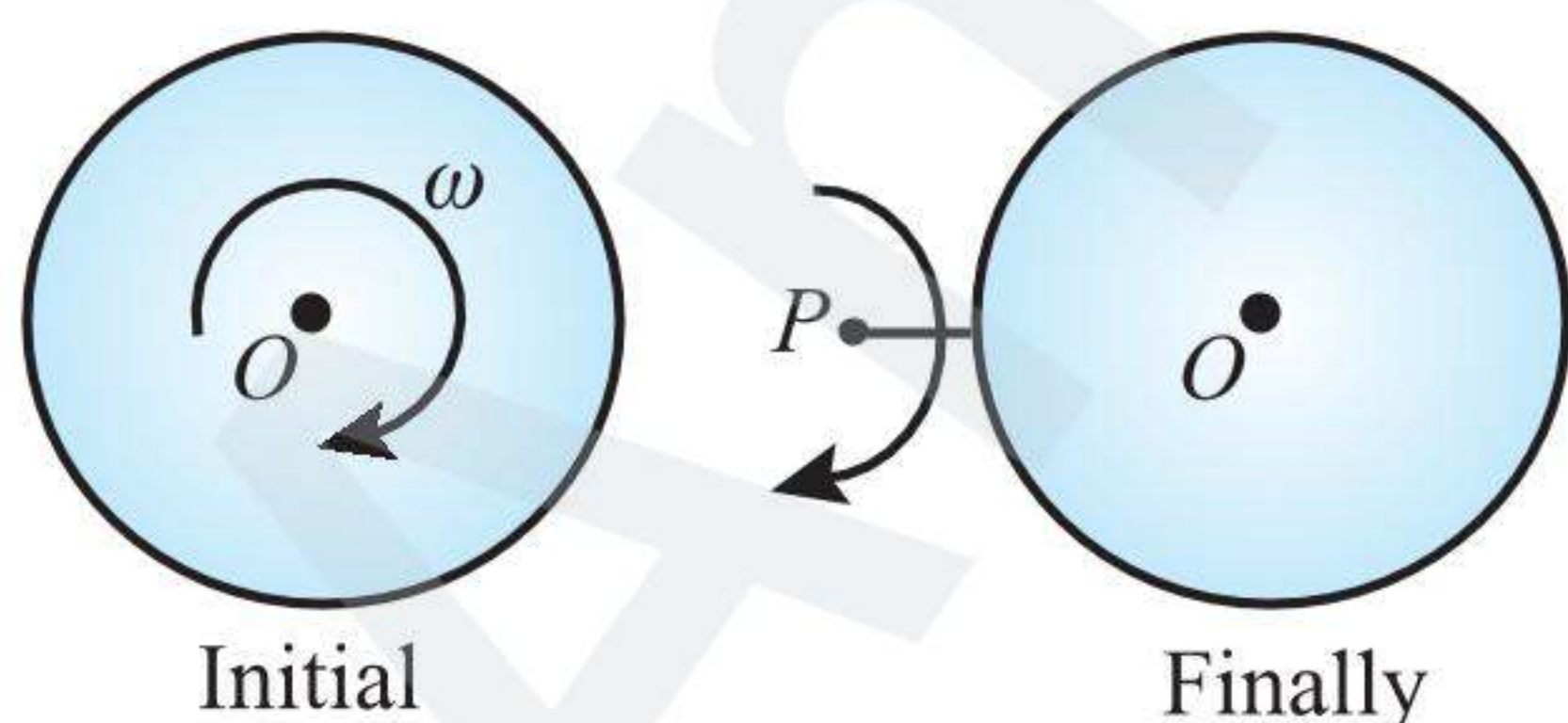
(1) $\vec{L}$, only if $v_0 = \omega_0 r$

(2) greater than $\vec{L}$, if $v_0 > \omega_0 r$

(3) less than $\vec{L}$, if $v_0 > \omega_0 r$

(4) $\vec{L}$, for all values of ω_0 and v_0

38. A disc is freely rotating with an angular speed ω on a smooth horizontal plane. If it is hooked at a rigid page P and rotates without bouncing about a point on its circumference. Its angular speed after the impact will be equal to



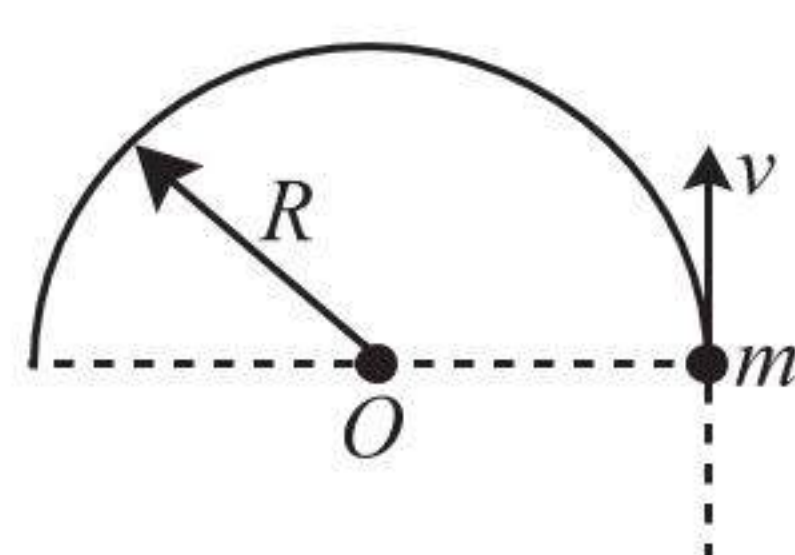
(1) $\frac{2\omega}{3}$

(2) $\frac{\omega}{3}$

(3) $\frac{\omega}{2}$

(4) none of these

39. A small bead of mass m moving with velocity v gets threaded on a stationary semicircular ring of mass m and radius R kept on a horizontal table. The ring can freely rotate about its centre. The bead comes to rest relative to the ring. What will be the final angular velocity of the system?



(1) $\frac{v}{R}$

(2) $\frac{2v}{R}$

(3) $\frac{v}{2R}$

(4) $\frac{3v}{R}$

40. A block of mass m is attached to a pulley disc of equal mass m and radius r by means of a slack string as shown. The pulley is hinged about its centre on a horizontal table and the block is projected with an initial velocity of 5 m/s. Its velocity when the string becomes taut will be



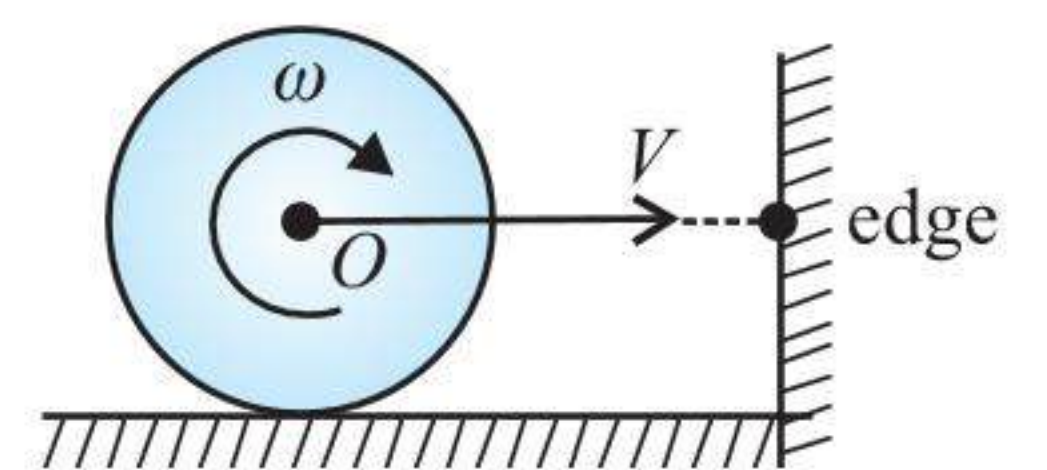
(1) 3 m/s

(2) 2.5 m/s

(3) $\frac{5}{3}$ m/s

(4) $\frac{10}{3}$ m/s

41. A uniform solid sphere of radius r is rolling on a smooth horizontal surface with velocity V and angular velocity $\omega = (V = \omega r)$. The sphere collides with a sharp edge on the wall as shown in figure. The coefficient of friction between the sphere and the edge $\mu = 1/5$. Just after the collision the angular velocity of the sphere becomes equal to zero. The linear velocity of the sphere just after the collision is equal to



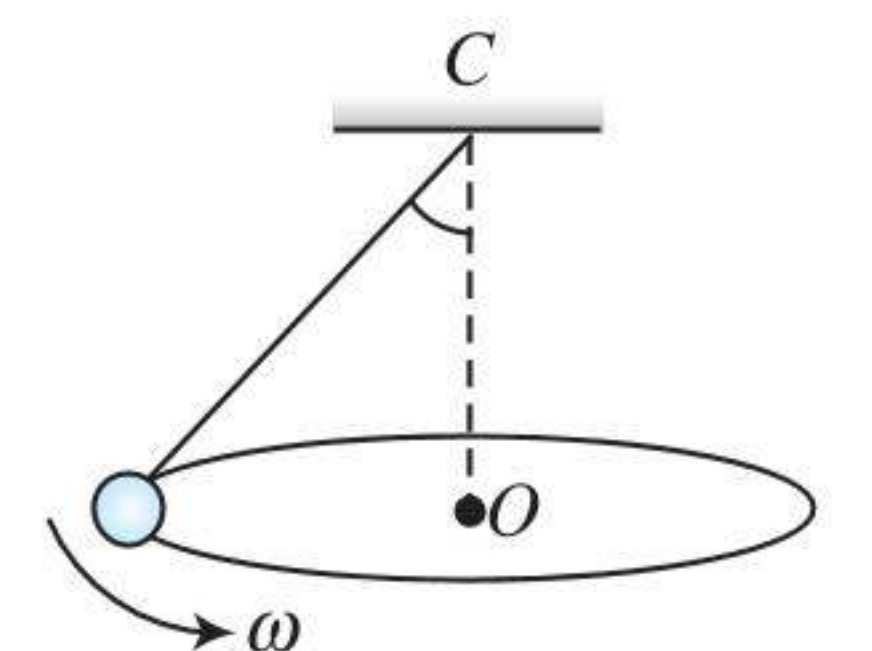
(1) V

(2) $\frac{V}{5}$

(3) $\frac{3V}{5}$

(4) $\frac{V}{6}$

42. A conical pendulum consists of a simple pendulum moving in a horizontal circle as shown in the figure. C is the pivot, O the centre of the circle in which the pendulum bob moves and ω the constant angular velocity of the bob. If $\vec{L}$ is the angular momentum about point C , then



(1) $\vec{L}$ is constant

(2) only direction of $\vec{L}$ is constant

(3) only magnitude of $\vec{L}$ is constant

(4) none of the above

43. A ring of radius R is first rotated with an angular velocity ω_0 and then carefully placed on a rough horizontal surface. The coefficient of friction between the surface and the ring is μ . Time after which its angular speed is reduced to half is

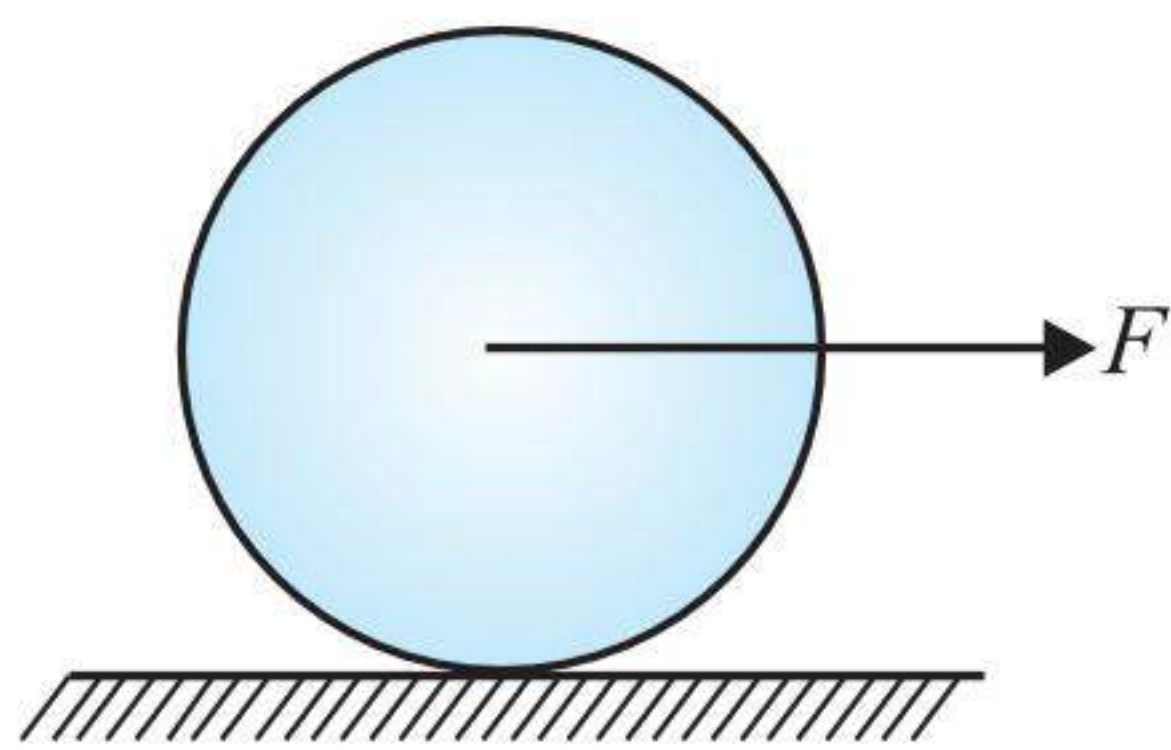
(1) $\frac{\omega_0 \mu R}{2g}$

(2) $\frac{\omega_0 g}{2\mu R}$

(3) $\frac{2\omega_0 R}{\mu g}$

(4) $\frac{\omega_0 R}{2\mu g}$

44. A solid sphere of mass m is lying at rest on a rough horizontal surface. The coefficient of friction between the ground and sphere is μ . The maximum value of F , so that the sphere will not slip, is equal to



- (1) $\frac{7}{5} \mu mg$ (2) $\frac{4}{7} \mu mg$
 (3) $\frac{5}{7} \mu mg$ (4) $\frac{7}{2} \mu mg$

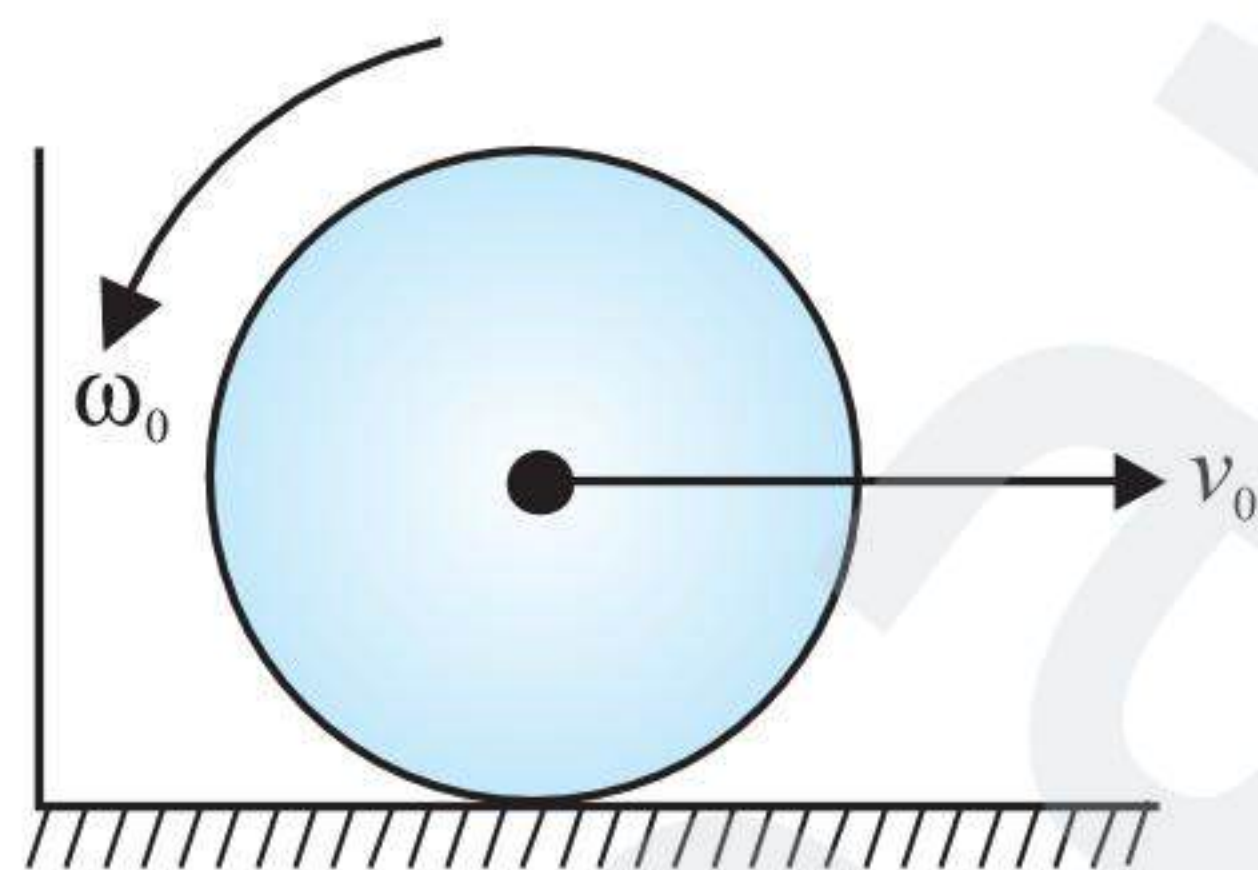
45. A slender rod of mass M and length L rests on a horizontal frictionless surface. The rod is pivoted about one of its ends. The impulse of the force exerted on the rod by the pivot when the rod is struck by a blow of impulse J perpendicular to the rod at other end is

- (1) J (2) $\frac{J}{2}$
 (3) $\frac{J}{3}$ (4) information is insufficient

46. A solid cylinder of mass M and radius R is resting on a horizontal platform (which is parallel to the x - y plane) with its axis fixed along Y -axis and free to rotate about its axis. The platform is given a motion in the X -direction given by $x = A \cos(\omega t)$. There is no slipping between the cylinder and the platform. The maximum torque acting on the cylinder during its motion is

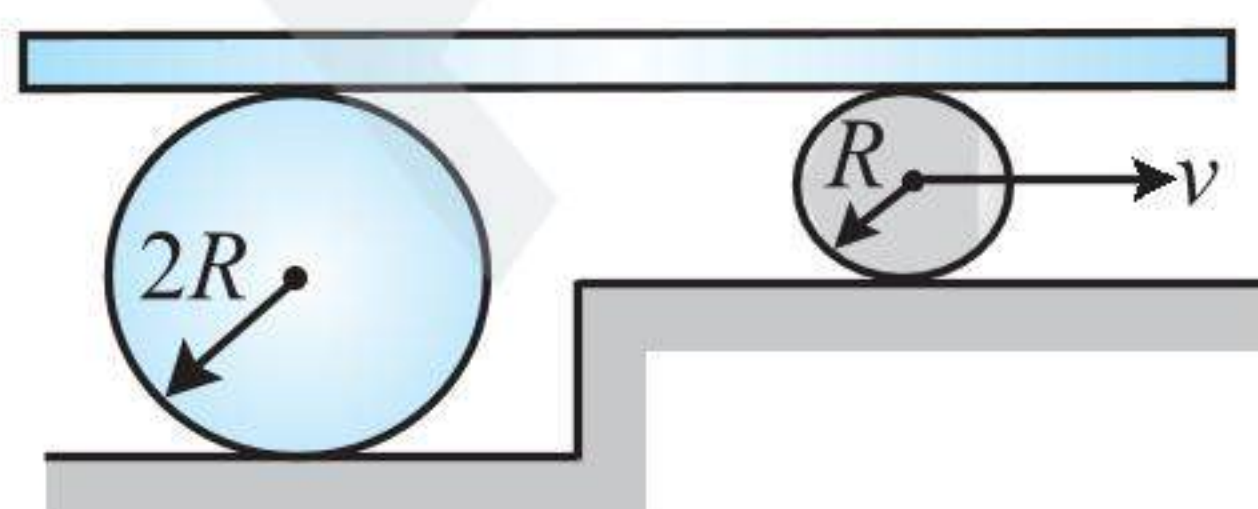
- (1) $\frac{M\omega^2 AR}{3}$ (2) $\frac{M\omega^2 AR}{2}$
 (3) $\frac{2}{3} \times M\omega^2 AR$ (4) the situation is not possible

47. A uniform circular disc of radius r is placed on a rough horizontal surface and given a linear velocity v_0 and angular velocity ω_0 as shown. The disc comes to rest after moving some distance to the right. It follows that



- (1) $3v_0 = 2\omega_0 r$ (2) $2v_0 = \omega_0 r$
 (3) $v_0 = \omega_0 r$ (4) $2v_0 = 3\omega_0 r$

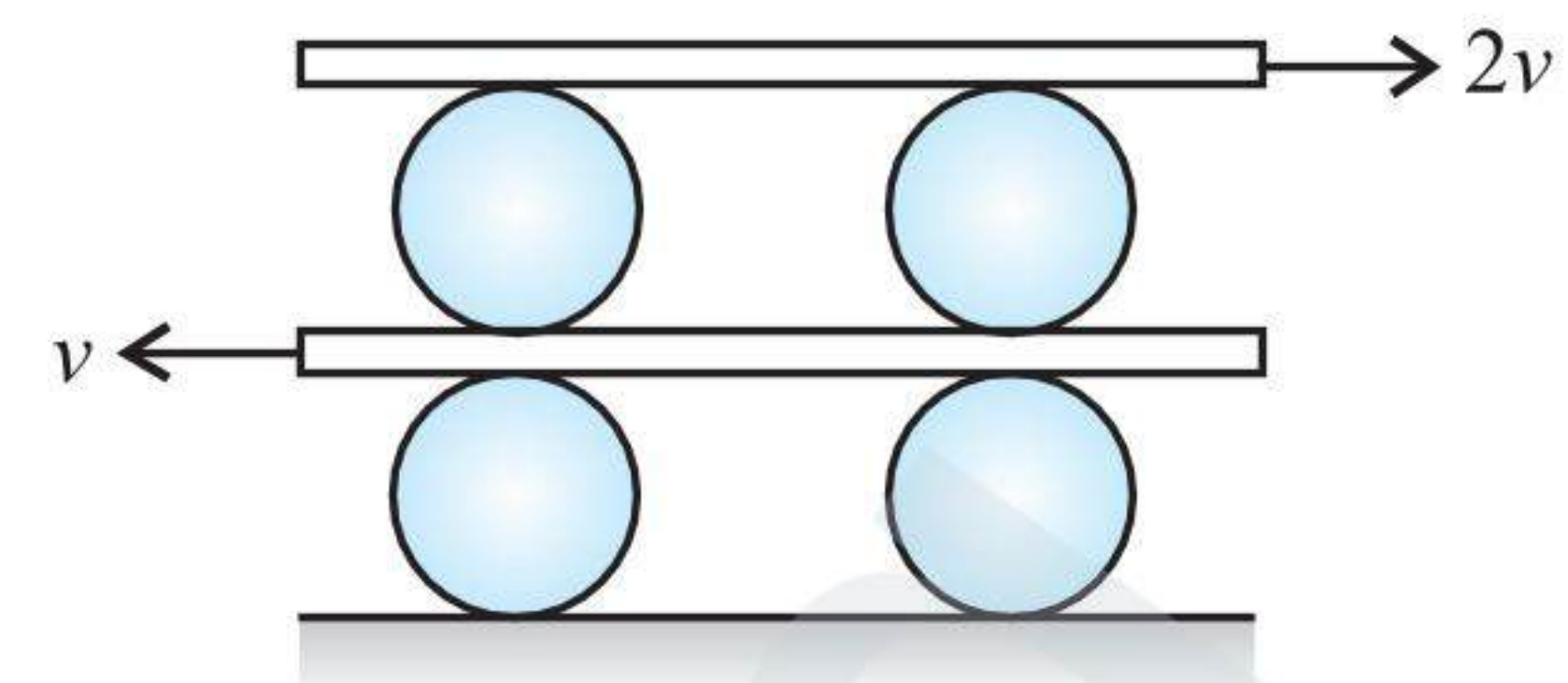
48. Velocity of the centre of a small cylinder is v . There is no slipping anywhere. The velocity of the centre of the larger cylinder is



- (1) $2v$ (2) v
 (3) $\frac{3v}{2}$ (4) none of these

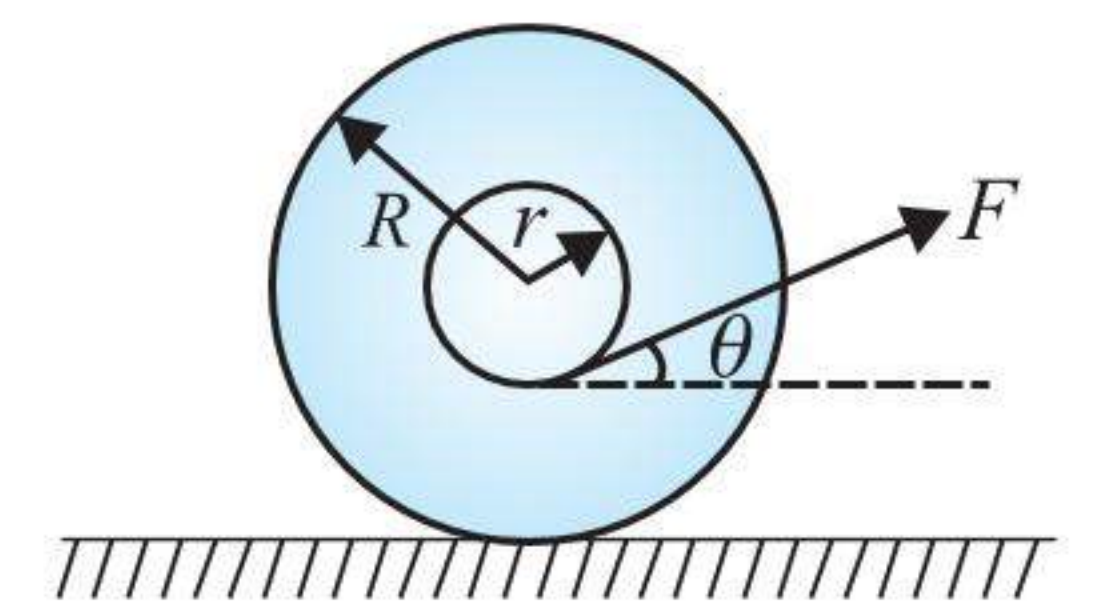
49. A system of identical cylinders and plates is shown in figure. All the cylinders are identical and there is no slipping at any

contact. The velocity of lower and upper plates are v and $2v$, respectively, as shown in figure. Then the ratio of angular speeds of the upper cylinders to lower cylinders is



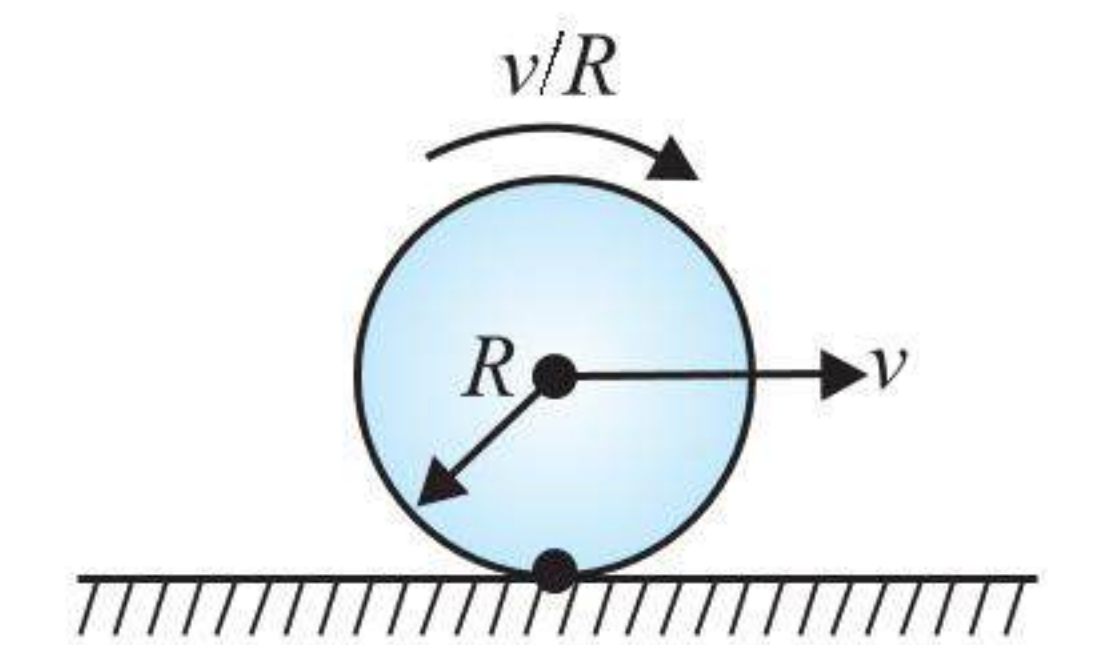
- (1) $1/3$ (2) 3
 (3) 1 (4) none of these

50. The spool shown in the figure is placed on a rough horizontal surface and has inner radius r and outer radius R . The angle θ between the applied force and the horizontal can be varied. The critical angle (θ) for which the spool does not roll and remains stationary is given by



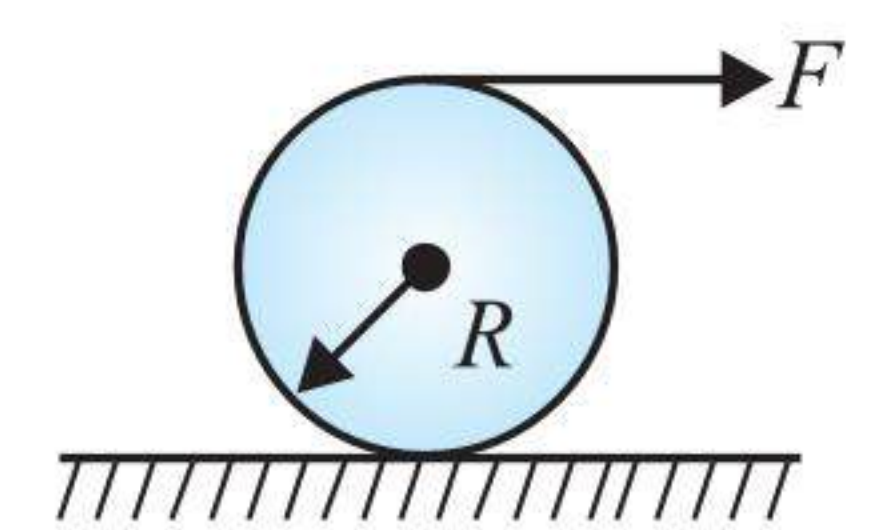
- (1) $\theta = \cos^{-1}\left(\frac{r}{R}\right)$ (2) $\theta = \cos^{-1}\left(\frac{2r}{R}\right)$
 (3) $\theta = \cos^{-1}\sqrt{\frac{r}{R}}$ (4) $\theta = \sin^{-1}\left(\frac{r}{R}\right)$

51. A disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in figure. At any instant, for the lowermost point of the disc,



- (1) velocity is v , acceleration is zero
 (2) velocity is zero, acceleration is zero
 (3) velocity is v , acceleration is v^2/R
 (4) velocity is zero, acceleration is v^2/R

52. An object of mass M and radius R is performing pure rolling motion on a smooth horizontal surface under the action of a constant force F as shown in figure. The object may be



- (1) disk (2) ring
 (3) solid cylinder (4) hollow sphere

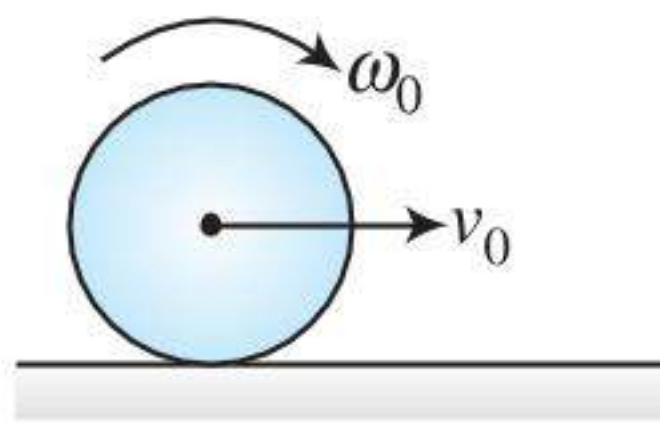
53. A solid cylinder is placed on the end of an inclined plane. It is found that the plane can be tipped at an angle θ before the cylinder starts to slide. When the cylinder turns on its sides and is allowed to roll, it is found that the steepest angle at which the cylinder performs pure rolling is ϕ . The ratio $\tan \phi / \tan \theta$ is

- (1) 3 (2) $1/3$
 (3) 1 (4) $1/2$

54. A uniform box of height 2 m and having a square base of side 1 m, weight 150 kg, is kept on one end on the floor of a truck. The maximum speed with which the truck can round a curve of radius 20 m without causing the block to tip over is (assume that friction is sufficient so that there is no sliding).

- (1) 15 m/s
 (2) 10 m/s
 (3) 8 m/s
 (4) depends on the value of coefficient of friction

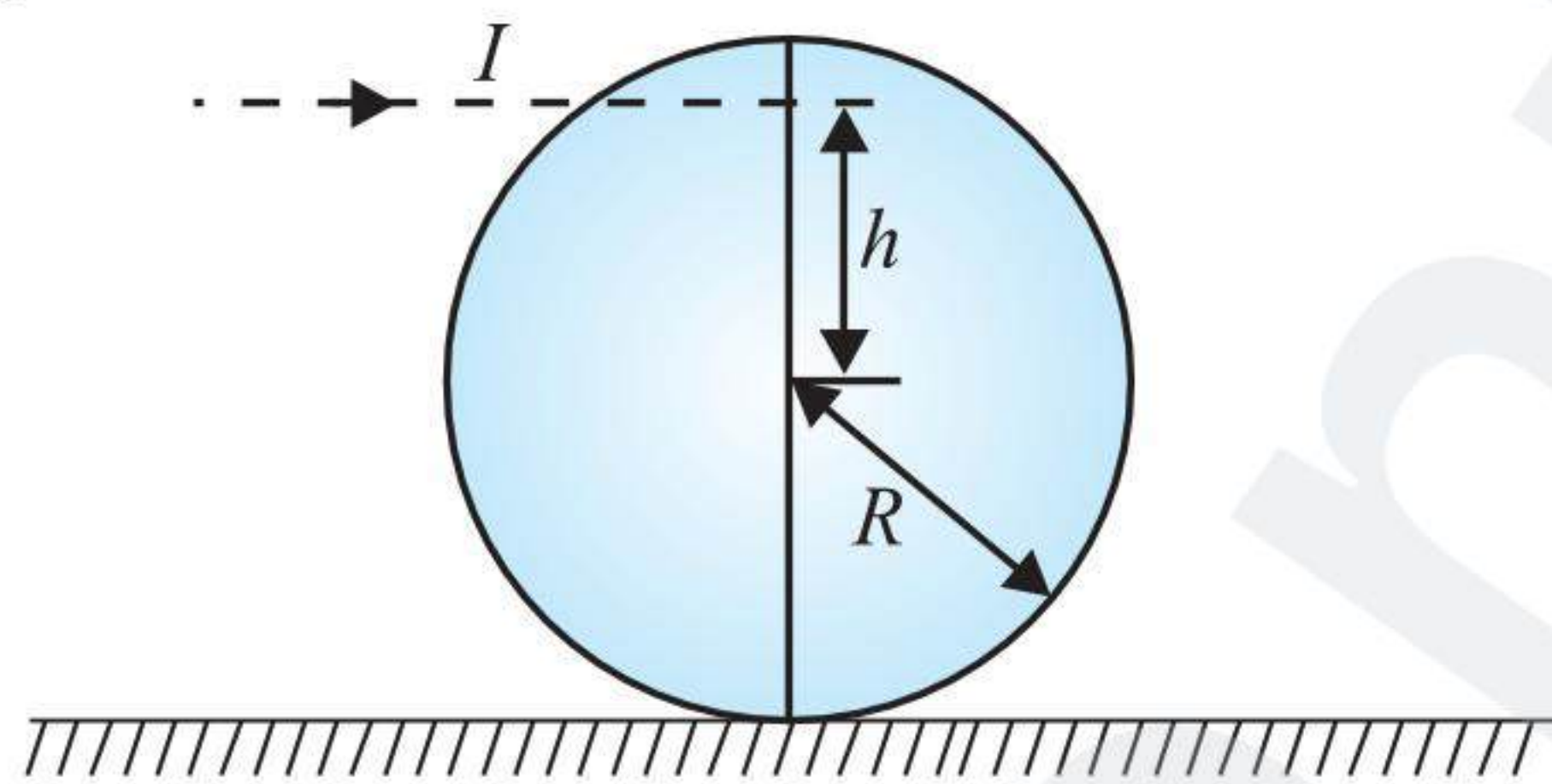
55. A sphere of mass M and radius R is moving on a rough fixed surface, having coefficient of friction μ as shown in figure. It will attain a minimum linear velocity after time ($v_0 > \omega_0 R$)



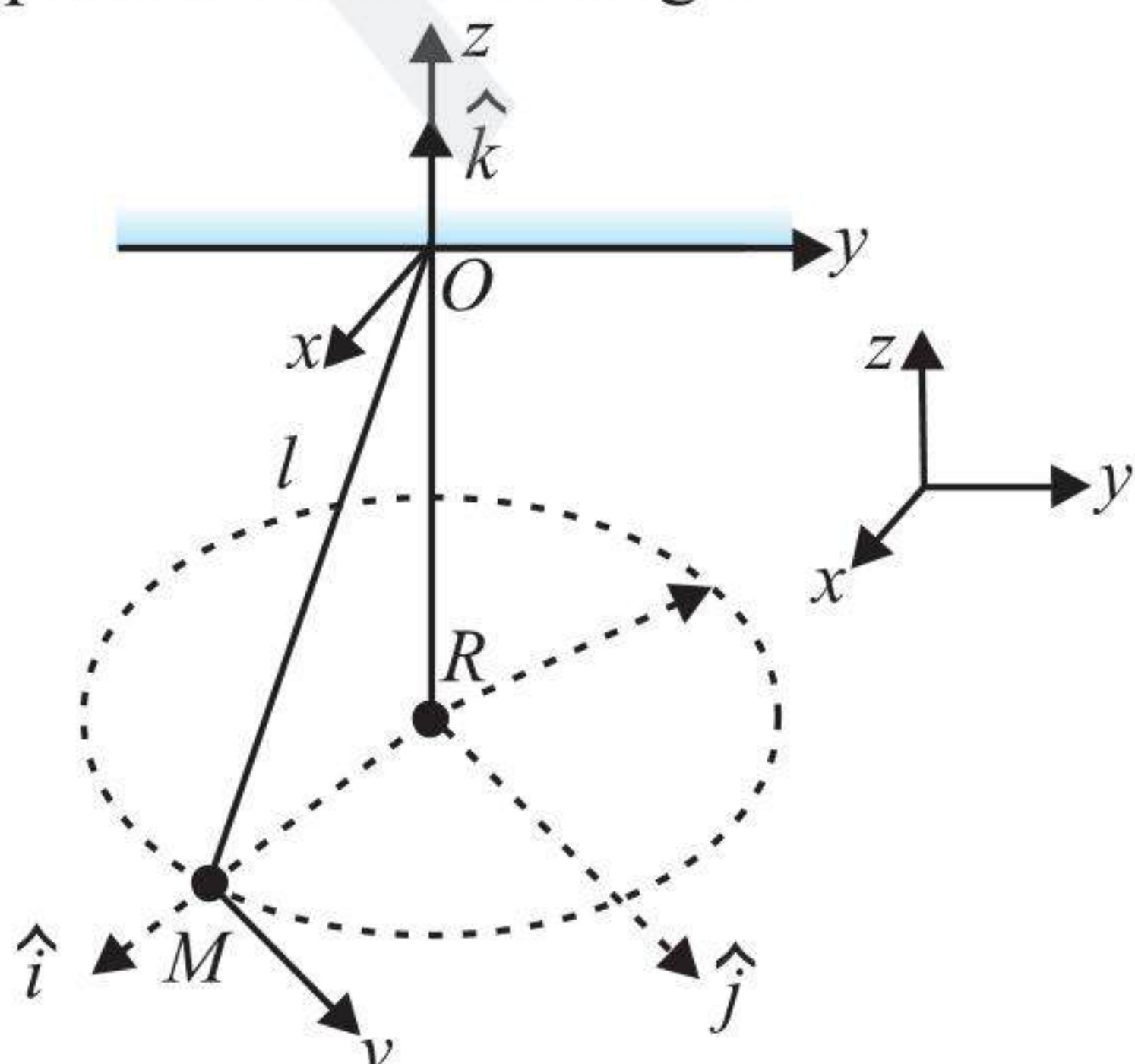
- (1) $v_0/\mu g$ (2) $\omega_0 R/\mu g$
 (3) $2(v_0 - \omega_0 R)/5\mu g$ (4) $2(v_0 - \omega_0 R)/7\mu g$
56. A solid sphere and a hollow sphere of equal mass and radius are placed over a rough horizontal surface after rotating it about its mass centre with same angular velocity ω_0 . Once the pure rolling starts let v_1 and v_2 be the linear speeds of their centres of mass. Then
- (1) $v_1 = v_2$ (2) $v_1 > v_2$
 (3) $v_1 < v_2$ (4) data is insufficient
57. In the above problem, let t_1 and t_2 be the times when pure rolling of solid sphere and of hollow sphere is started. Then
- (1) $t_1 = t_2$ (2) $t_1 < t_2$
 (3) $t_1 > t_2$ (4) none of these
58. A uniform ball of radius r rolls without slipping down from the top of a sphere of radius R . The angular velocity of the ball when it breaks from the sphere is

- (1) $\sqrt{\frac{5g(R+r)}{17r^2}}$ (2) $\sqrt{\frac{10g(R+r)}{17r^2}}$
 (3) $\sqrt{\frac{5g(R-r)}{10r^2}}$ (4) $\sqrt{\frac{10g(R+r)}{7r^2}}$

59. A solid sphere rests on a horizontal surface. A horizontal impulse is applied at height h from centre. The sphere starts rolling just after the application of impulse. The ratio h/r will be

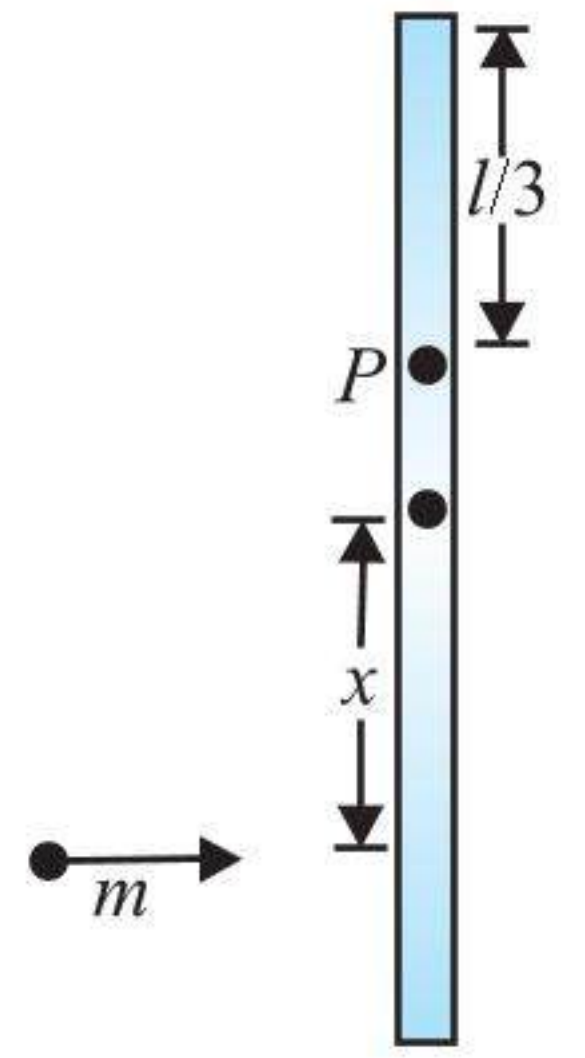


- (1) $\frac{1}{2}$ (2) $\frac{2}{5}$
 (3) $\frac{1}{5}$ (4) $\frac{2}{3}$
60. A conical pendulum consists of a mass M suspended from a strong sling of length l . The mass executes a circle of radius R in a horizontal plane with speed v . At time t , the mass is at position $R\hat{i}$ and has $v\hat{j}$ velocity. At time t , the angular momentum vector of mass M about the point from which the string passes on the ceiling is



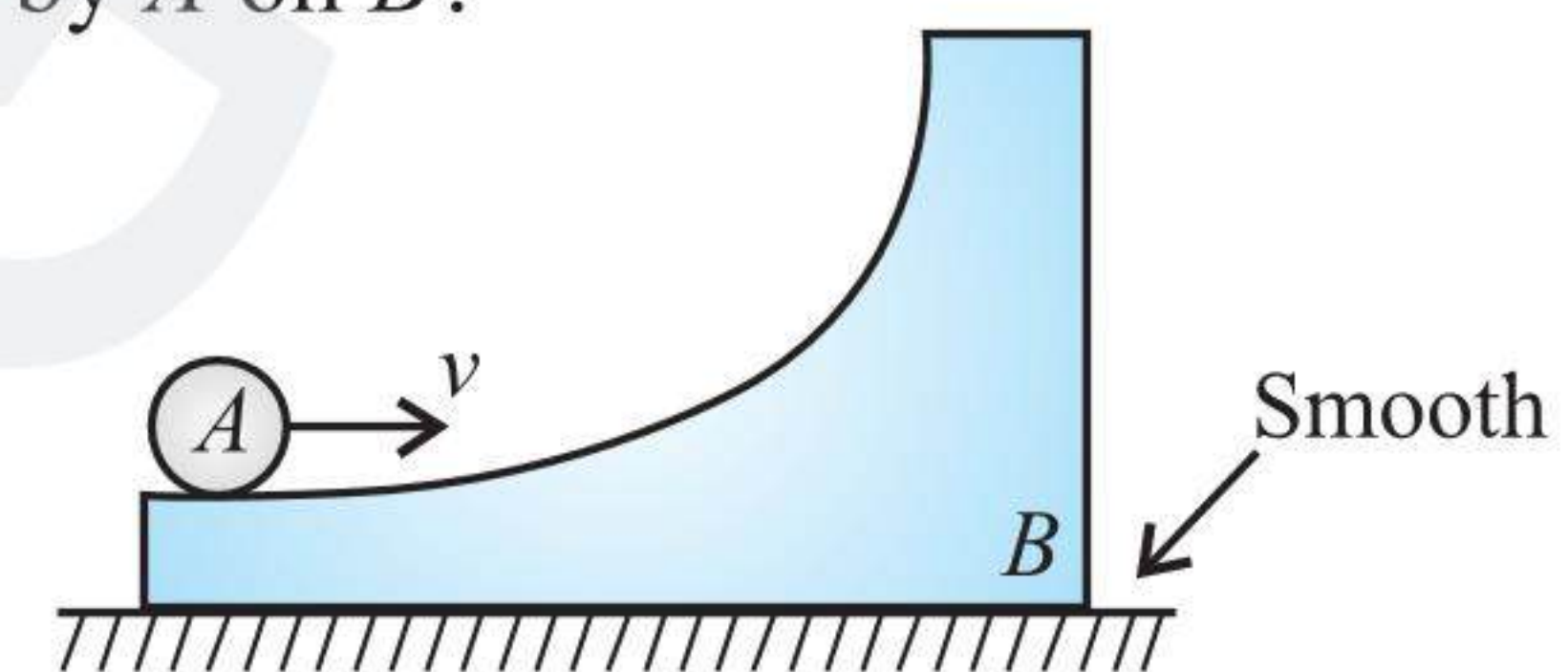
- (1) $MvR\hat{k}$ (2) $Mvl\hat{k}$
 (3) $Mvl\left[\frac{\sqrt{l^2 - R^2}}{l}\hat{i} + \frac{R}{l}\hat{k}\right]$ (4) $-Mvl\left[\frac{\sqrt{l^2 - R^2}}{l}\hat{i} + \frac{R}{l}\hat{k}\right]$

61. A thin uniform rod of mass m and length l is kept on a smooth horizontal surface such that it can move freely. At what distance from centre of rod should a particle of mass m strike on the rod such that the point P at a distance $l/3$ from the end of the rod is instantaneously at rest just after the elastic collision?



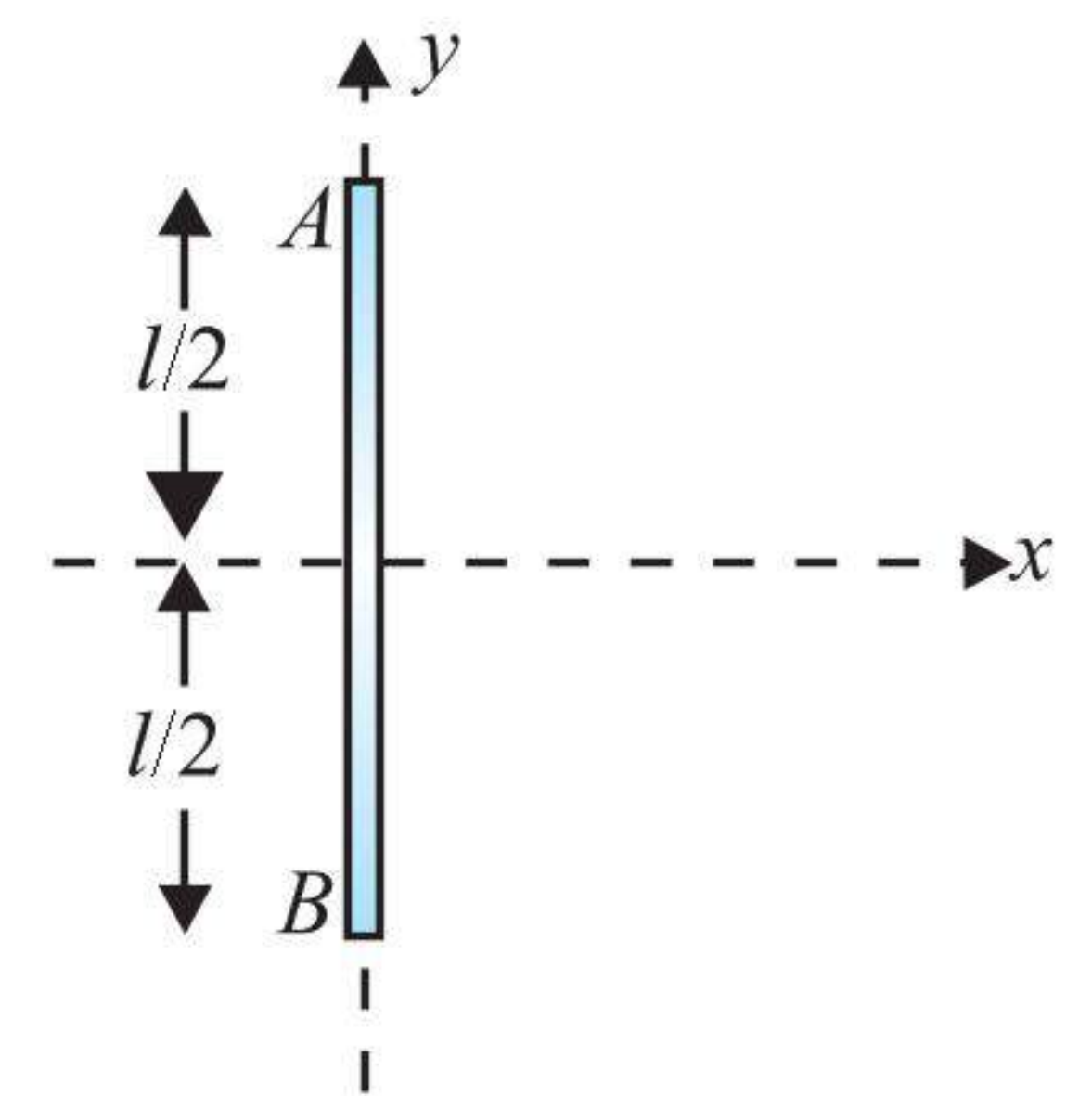
- (1) $l/2$ (2) $l/3$
 (3) $l/6$ (4) $l/4$

62. In the figure shown, a ring A is initially rolling without sliding with a velocity v on the horizontal surface of the body B (of same mass as A). All surfaces are smooth. B has no initial velocity. What will be the maximum height reached by A on B ?



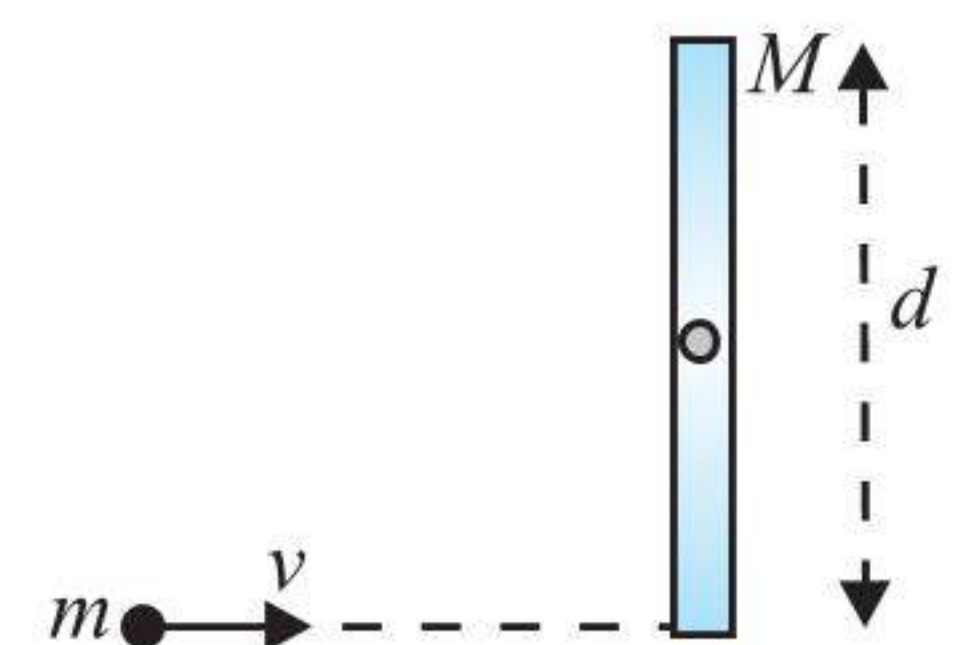
- (1) $\frac{3v^2}{4g}$ (2) $\frac{v^2}{4g}$
 (3) $\frac{v^2}{2g}$ (4) $\frac{v^2}{3g}$

63. A uniform rod of mass m and length l is placed over a smooth horizontal surface along the y -axis and is at rest as shown in figure. An impulsive force F is applied for a small time Δt along x -direction at point A . The x -coordinate of end A of the rod when the rod becomes parallel to x -axis for the first time is [initially, the coordinate of centre of mass of the rod is $(0, 0)$]



- (1) $\frac{\pi l}{12}$ (2) $\frac{l}{2}\left(1 + \frac{\pi}{12}\right)$
 (3) $\frac{l}{2}\left(1 - \frac{\pi}{6}\right)$ (4) $\frac{l}{2}\left(1 + \frac{\pi}{6}\right)$

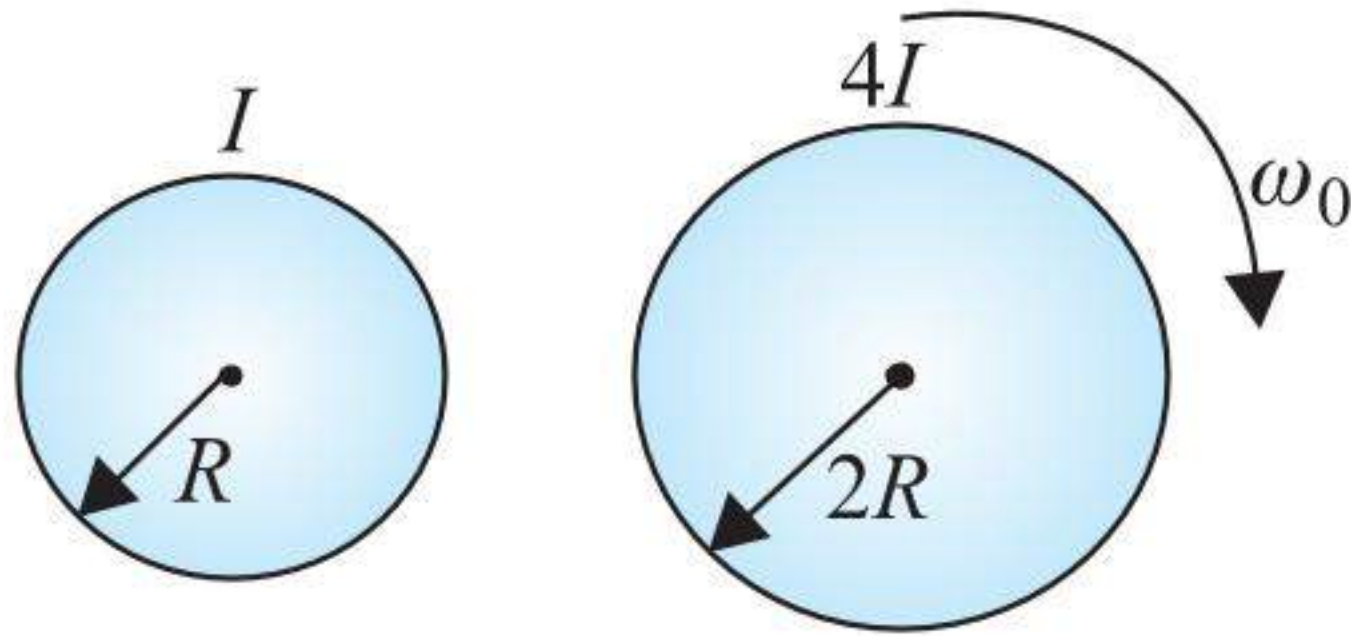
64. A mass m is moving at speed v perpendicular to a rod of length d and mass $M = 6m$ which pivots around a frictionless axle running through its centre. It strikes and sticks to the end of the rod. The moment of inertia of the rod about its centre is $Md^2/12$. Then the angular speed of the system just after the collision is



- (1) $\frac{2v}{3d}$ (2) $\frac{2v}{d}$
 (3) $\frac{v}{d}$ (4) $\frac{3v}{2d}$

65. Two cylinders having radii $2R$ and R and moment of inertia $4I$ and I about their central axes are supported by

axes perpendicular to their planes. The large cylinder is initially rotating clockwise with angular velocity ω_0 . The small cylinder is moved to the right until it touches the large cylinder and is caused to rotate by the frictional force between the two. Eventually slipping ceases and the two cylinders rotate at constant rates in opposite directions. During this



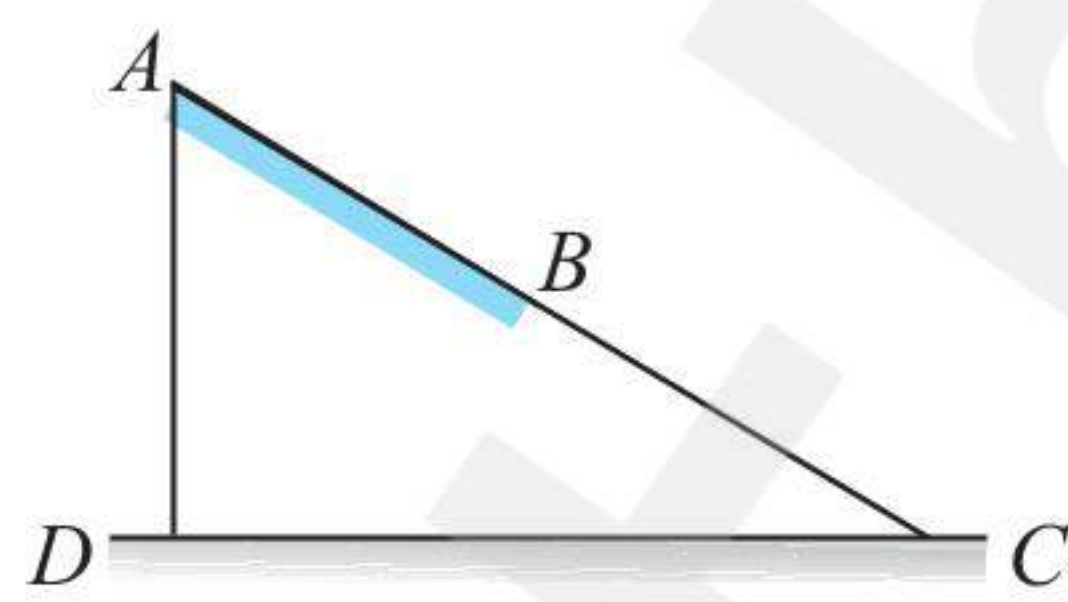
- (1) angular momentum of system is conserved
- (2) kinetic energy is conserved
- (3) neither the angular momentum nor the kinetic energy is conserved
- (4) both the angular momentum and kinetic energy are conserved

66. In the above problem, the final angular velocity of the small cylinder is

- (1) $\frac{\omega_0}{4}$
- (2) ω_0
- (3) $\frac{\omega_0}{2}$
- (4) $\frac{\omega_0}{8}$

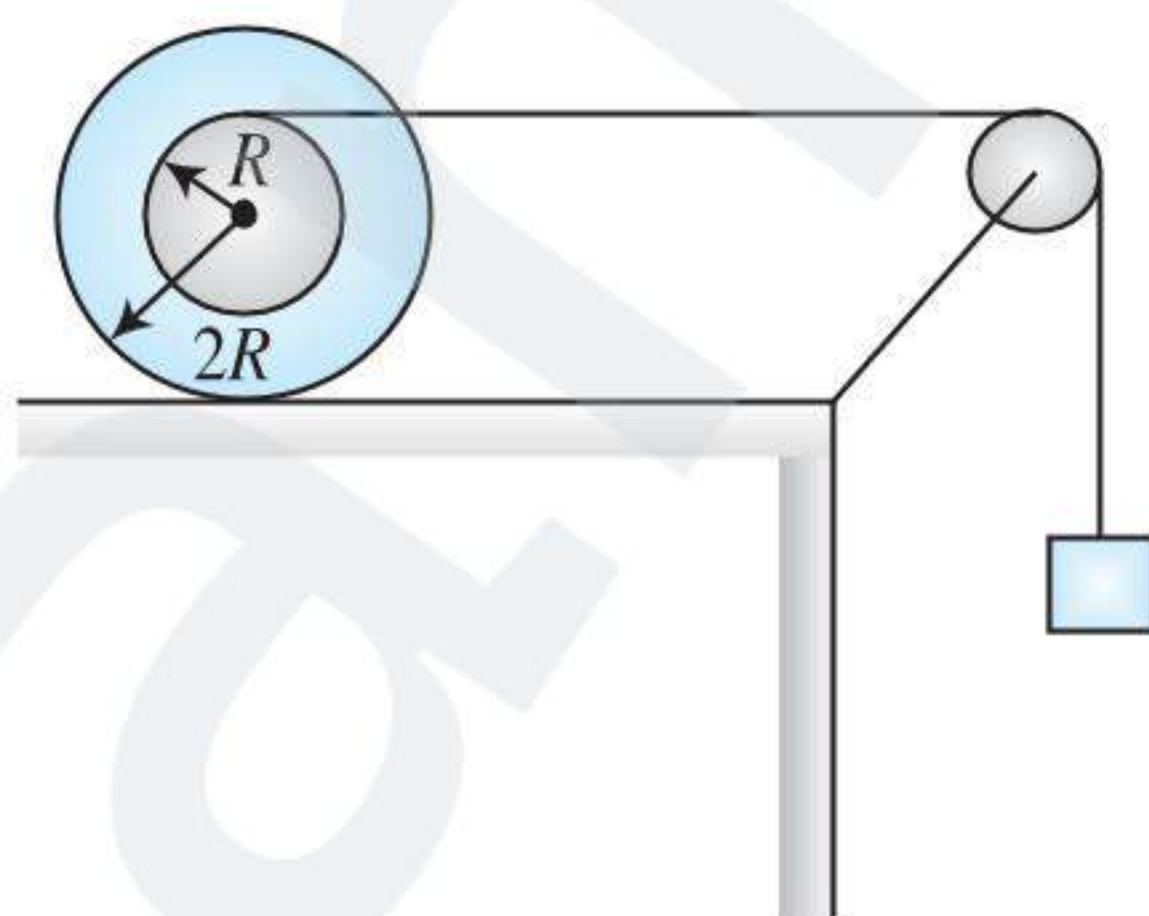
67. Portion AB of the wedge shown in the figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B . If $AB = BC$, then ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches point C is

- (1) $3/5$
- (2) 5
- (3) $7/5$
- (4) $8/3$



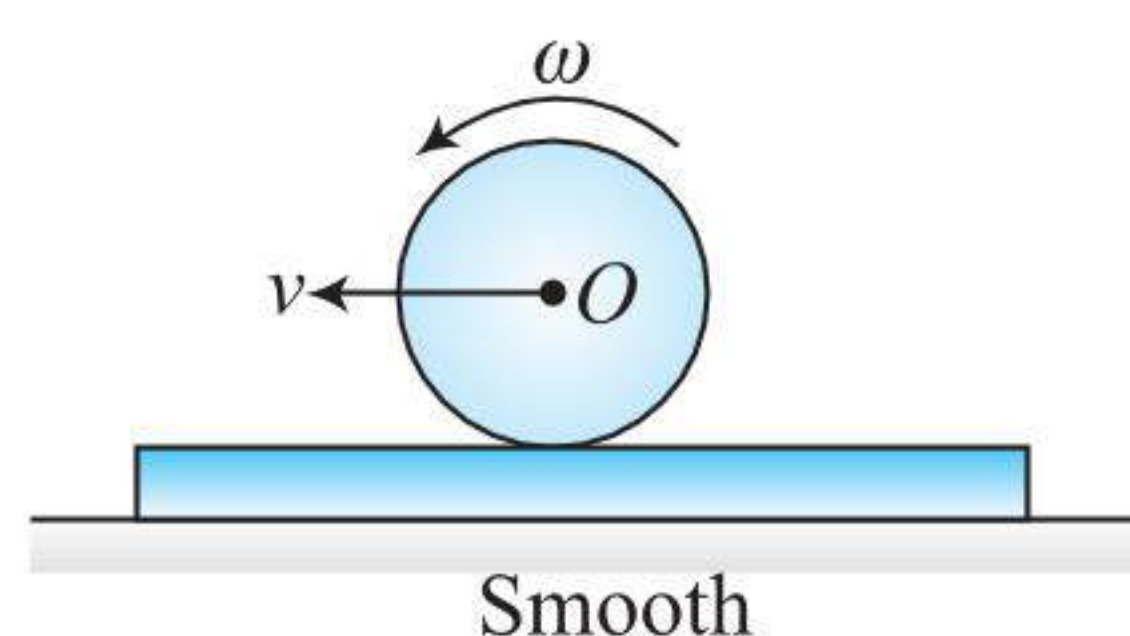
68. In the figure shown mass of both, the spherical body and block is m . Moment of inertia of the spherical body about centre of mass is $2mR^2$. The spherical body rolls on the horizontal surface. There is no slipping at any surfaces in contact. The ratio of kinetic energy of the spherical body to that of block is

- (1) $3/4$
- (2) $1/3$
- (3) $2/3$
- (4) $1/2$

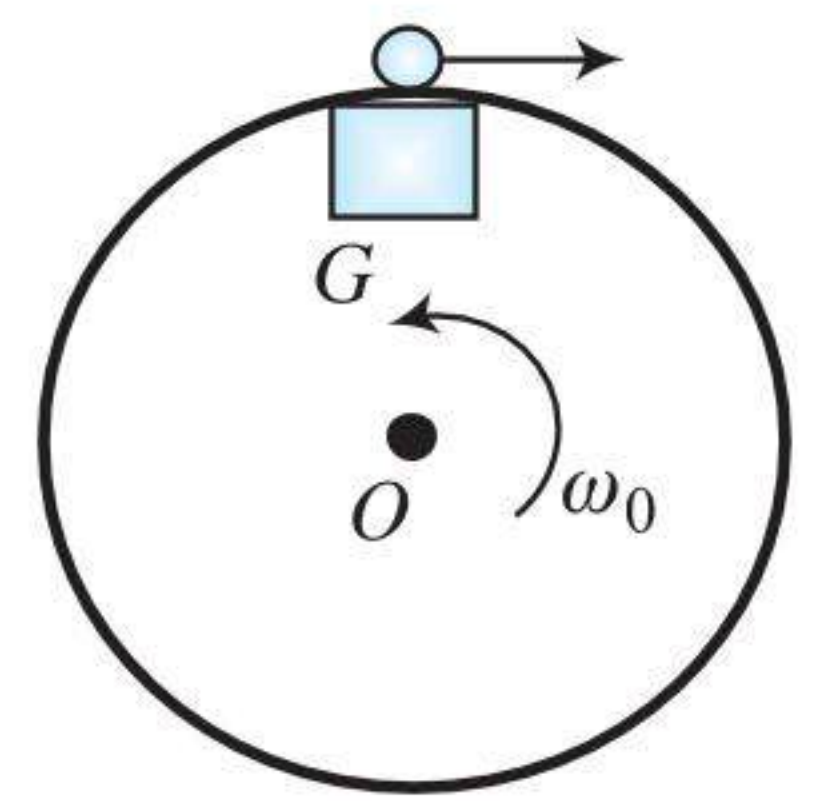


69. A cylinder executes pure rolling without slipping with a constant velocity on a plank, whose upper surface is rough enough, but lower surface is smooth. The plank is kept at rest on a smooth horizontal surface by the application of an external force F . Choose the correct alternative.

- (1) The direction of F is towards right.
- (2) The direction of F is towards left.
- (3) The value of F is zero.
- (4) The direction of F depends on the ratio of the relative masses of disc and plank.



70. A horizontal turn table in the form of a disc of radius r carries a gun at G and rotates with angular velocity ω_0 about a vertical axis passing through the centre O . The increase in angular velocity of the system if the gun fires a bullet of mass m with a tangential velocity v with respect to the gun is (moment of inertia of gun + table about O is I_0)



- (1) $\frac{mvr}{I_0 + mr^2}$
- (2) $\frac{2mvr}{I_0}$
- (3) $\frac{v}{2r}$
- (4) $\frac{mvr}{2I_0}$

71. A circular platform is mounted on a vertical frictionless axle. Its radius is $r = 2$ m and its moment of inertia is $I = 200$ kg m². It is initially at rest. A 70 kg man stands on the edge of the platform and begins to walk along the edge at speed $v_0 = 10$ ms⁻¹ relative to the ground. The angular velocity of the platform is

- (1) 1.2 rad s⁻¹
- (2) 0.4 rad s⁻¹
- (3) 2.0 rad s⁻¹
- (4) 0.7 rad s⁻¹

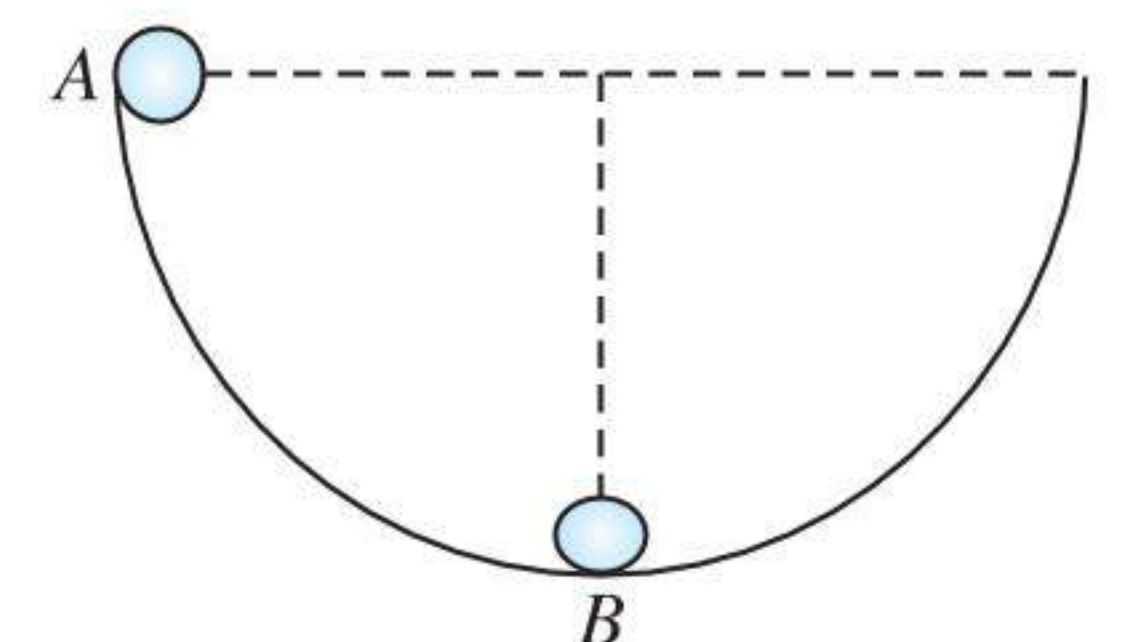
72. In the above problem, when the man has walked once around the platform so that he is at his original position on it, what is his angular displacement relative to ground

- (1) $\frac{6}{5}\pi$
- (2) $\frac{5}{6}\pi$
- (3) $\frac{4}{5}\pi$
- (4) $\frac{5}{4}\pi$

73. A uniform rod AB of mass m and length $2a$ is falling freely without rotation under gravity with AB horizontal. Suddenly the end A is fixed when the speed of the rod is v . The angular speed with which the rod begins to rotate is

- (1) $\frac{v}{2a}$
- (2) $\frac{4v}{3a}$
- (3) $\frac{v}{3a}$
- (4) $\frac{3v}{4a}$

74. A ball of mass m and radius r rolls inside a hemispherical shell of radius R . It is released from rest from point A as shown in the figure. The angular velocity of centre of the ball in position B about the centre of the shell is

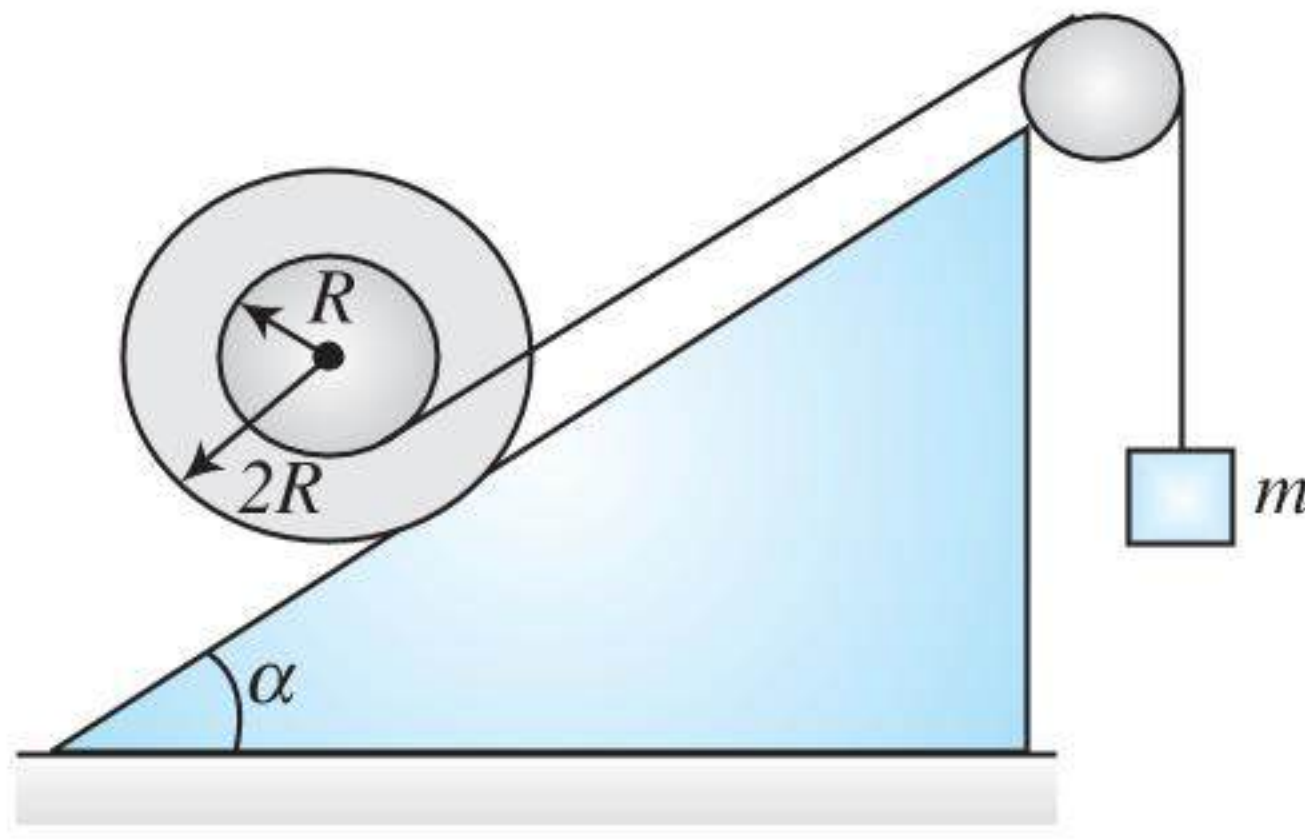


- (1) $2\sqrt{\frac{g}{5(R-r)}}$
- (2) $2\sqrt{\frac{g}{7(R-r)}}$
- (3) $\sqrt{\frac{2g}{5(R-r)}}$
- (4) $\sqrt{\frac{5g}{2(R-r)}}$

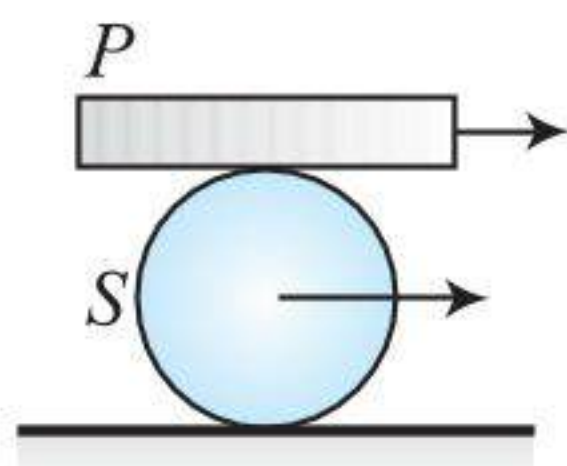
75. In the previous problem the normal force between the ball and the shell in position B is

- (1) $\frac{12}{7}mg$
- (2) $\frac{7}{9}mg$
- (3) $\frac{17}{7}mg$
- (4) $\frac{10}{7}mg$

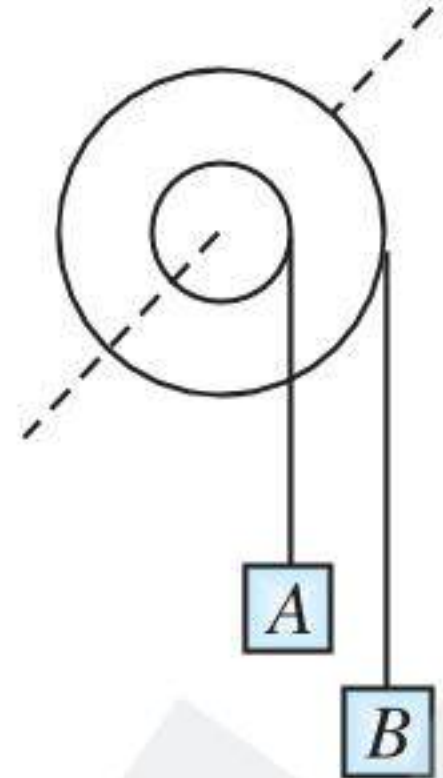
76. A spool of mass M and radius $2R$ lies on an inclined plane as shown in the figure. A light thread is wound around the connecting tube of the spool and its free end carries a weight of mass m . The value of m so that system is in equilibrium is



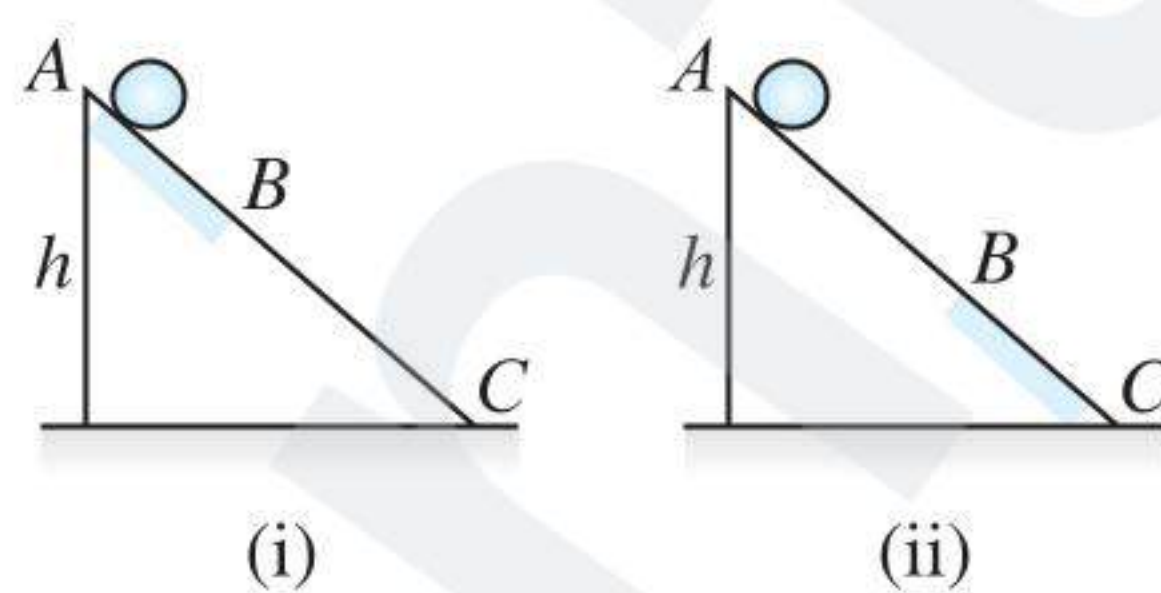
- (1) $2M \sin \alpha$ (2) $M \sin \alpha$
 (3) $2M \tan \alpha$ (4) $M \tan \alpha$
77. A plank P is placed on a solid cylinder S , which rolls on a horizontal surface. The two are of equal mass. There is no slipping at any of the surfaces in contact. The ratio of kinetic energy of P to the kinetic energy of S is



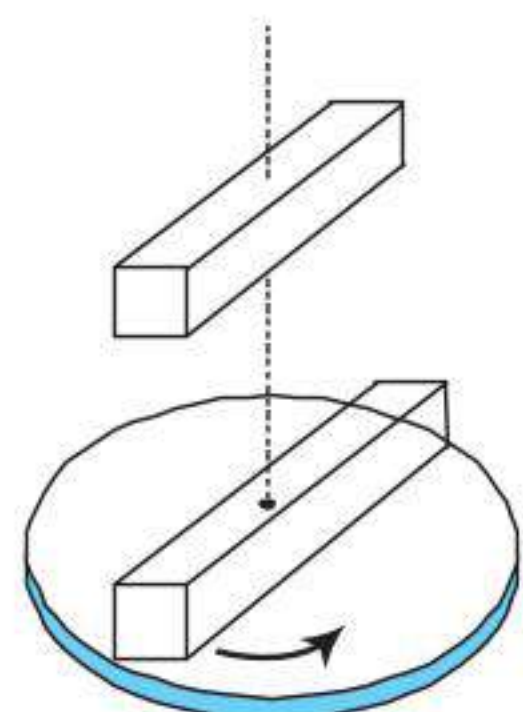
- (1) 1 : 1 (2) 2 : 1
 (3) 8 : 3 (4) 11 : 3
78. Figure shows a small wheel fixed coaxially on a bigger one of double the radius. The system rotates about the common axis. The strings supporting A and B do not slip on the wheels. If x and y be the distance travelled by A and B in the same time interval, then



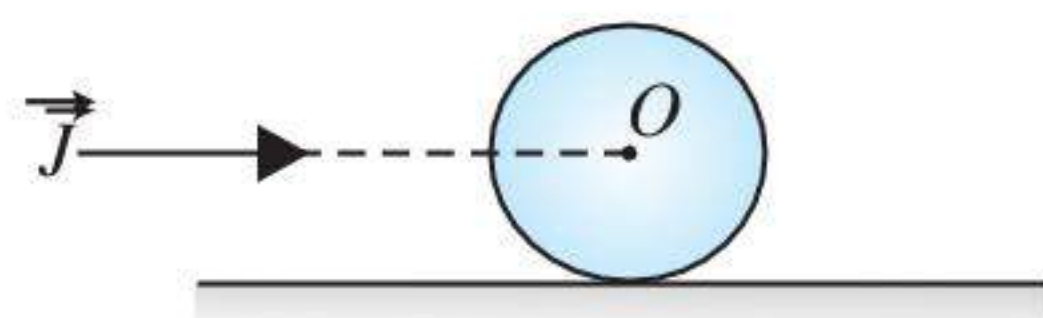
- (1) $x = 2y$ (2) $x = y$
 (3) $y = 2x$ (4) none of these
79. In both the figures all other factors are same, except that in Fig. (i) AB is rough and BC is smooth while in Fig. (ii) AB is smooth and BC is rough. In Fig. (i), if a sphere is released from rest it starts rolling. Now consider Fig. (ii), if same sphere is released from top of the inclined plane, what will be the kinetic energy of the sphere on reaching the bottom



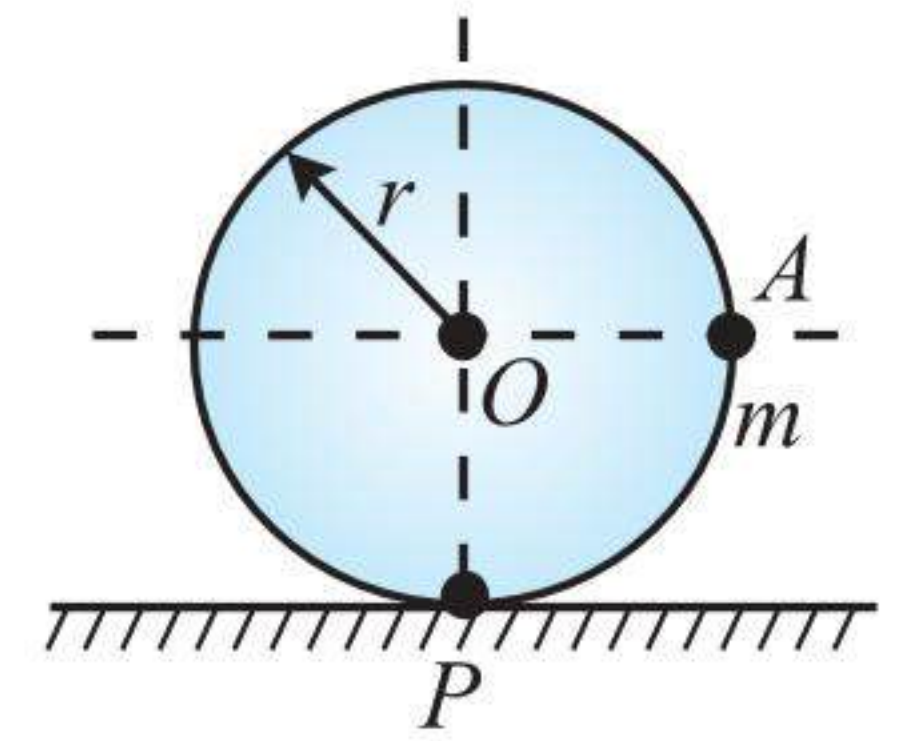
- (1) is same in both the cases
 (2) is greater in case (i)
 (3) is greater in case (ii)
 (4) information insufficient
80. A uniform disk turns at 2.4 rev/s around a frictionless axis. A nonrotating rod, of the same mass as the disk and length equal to the disk's diameter is dropped onto the freely spinning disk (see the figure). They then both turn around the axis with their centres superposed. What will be the angular frequency in rev/s of the combination when they start rotating together?



- (1) 1.2 rev/s (2) 2.0 rev/s
 (3) 1.44 rev/s (4) 0.96 rev/s
81. An impulse J is applied on a ring of mass m along a line passing through its centre O . The ring is placed on a rough horizontal surface. The linear velocity of centre of ring once it starts rolling without slipping is

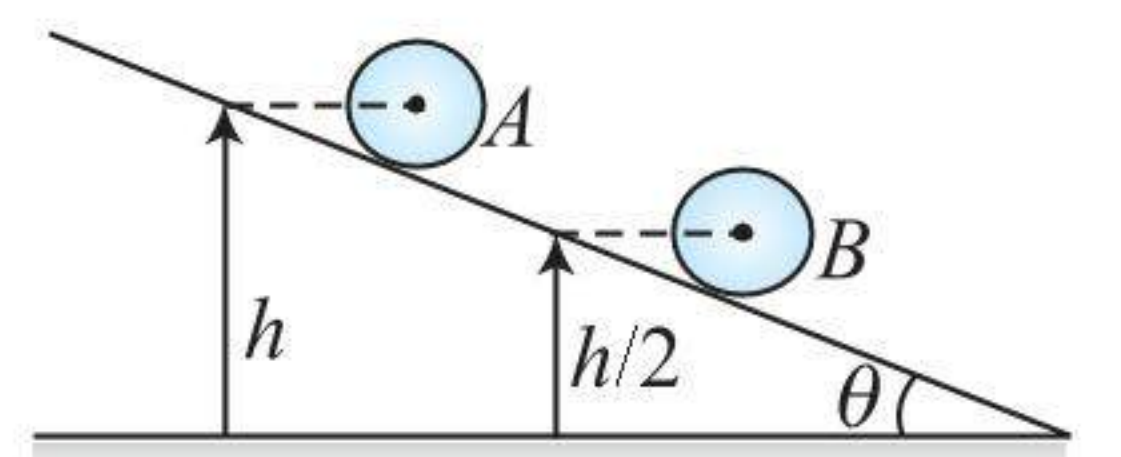


- (1) $\frac{g}{4r}$ (2) $\frac{g}{6r}$
 (3) $\frac{g}{8r}$ (4) $\frac{g}{2r}$
82. A particle of mass ' m ' is rigidly attached at ' A ' to a ring of mass ' $3m$ ' and radius ' r '. The system is released from rest and rolls without sliding. The angular acceleration of ring just after release is



83. A uniform smooth rod (mass m and length l) placed on a smooth horizontal floor is hit by a particle (mass m) moving on the floor, at a distance $l/4$ from one end elastically ($e = 1$). The distance travelled by the centre of the rod after the collision when it has completed three revolutions will be
- (1) $2\pi l$ (2) cannot be determined
 (3) πl (4) none of these

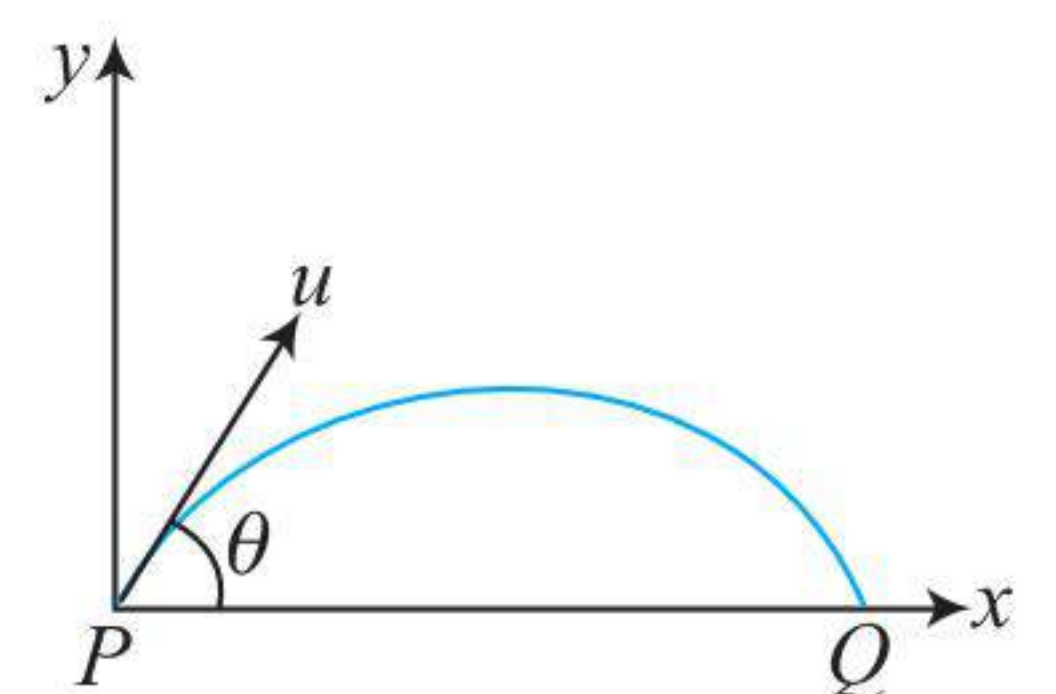
84. Two identical uniform solid spherical balls A and B of mass m each are placed on a the fixed wedge as shown in figure. Ball B is kept at rest and it is released just before two balls collide. Ball A rolls down without slipping on inclined plane and collide elastically with ball B . The kinetic energy of ball A just after the collision with ball B is



- (1) $\frac{mgh}{7}$ (2) $\frac{mgh}{2}$
 (3) $\frac{2mgh}{5}$ (4) $\frac{7mgh}{5}$

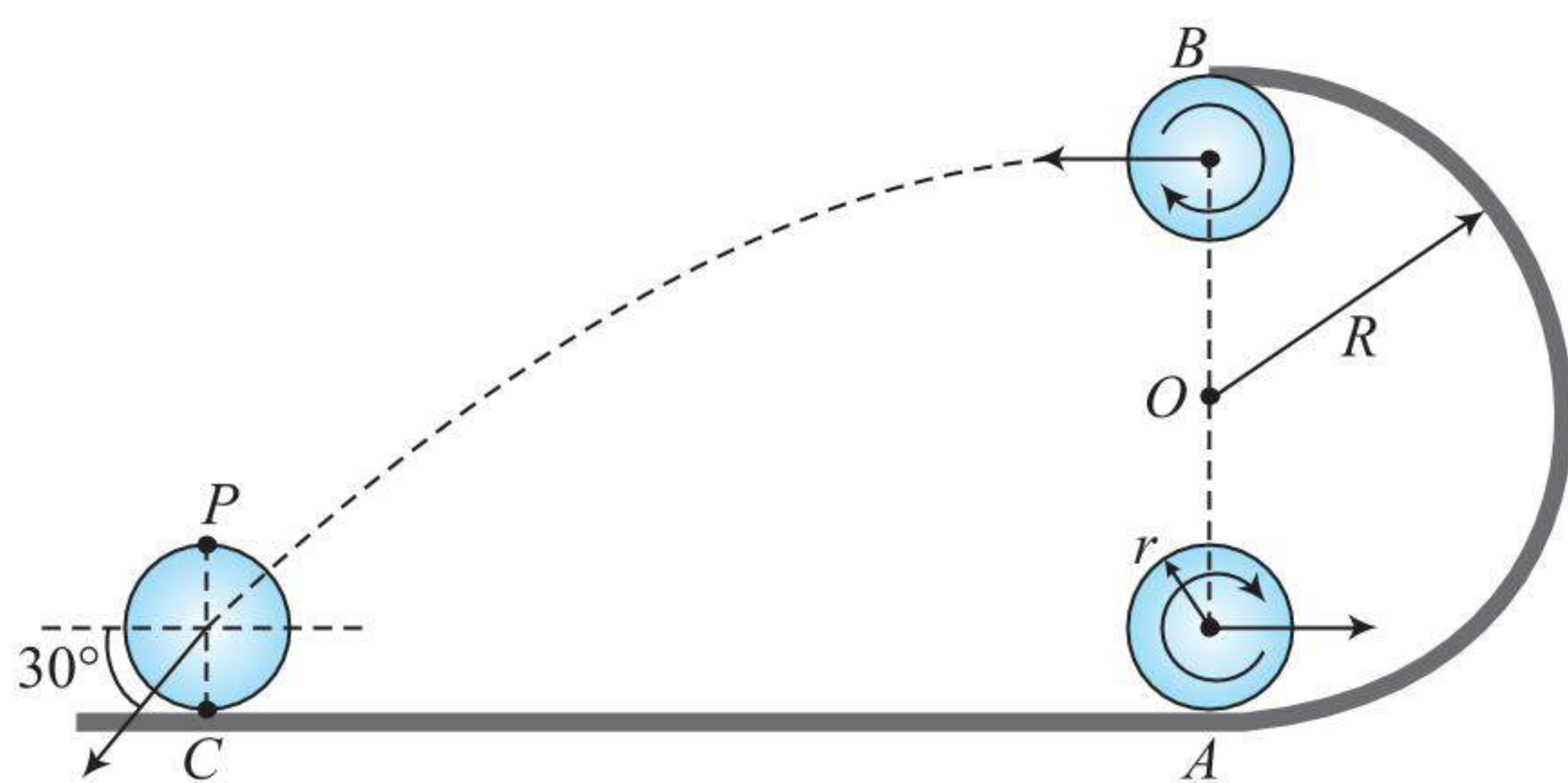
85. A homogenous rod of length $l = \eta x$ and mass M is lying on a smooth horizontal floor. A bullet of mass m hits the rod at a distance x from the middle of the rod at a velocity v_0 perpendicular to the rod and comes to rest after collision. If the velocity of the farther end of the rod just after the impact is in the opposite direction of v_0 , then
- (1) $\eta > 3$ (2) $\eta < 3$
 (3) $\eta > 6$ (4) $\eta < 6$

86. Average torque on a projectile of mass m , initial speed u and angle of projection θ between initial and final positions P and Q as shown in the figure about the point of projection is



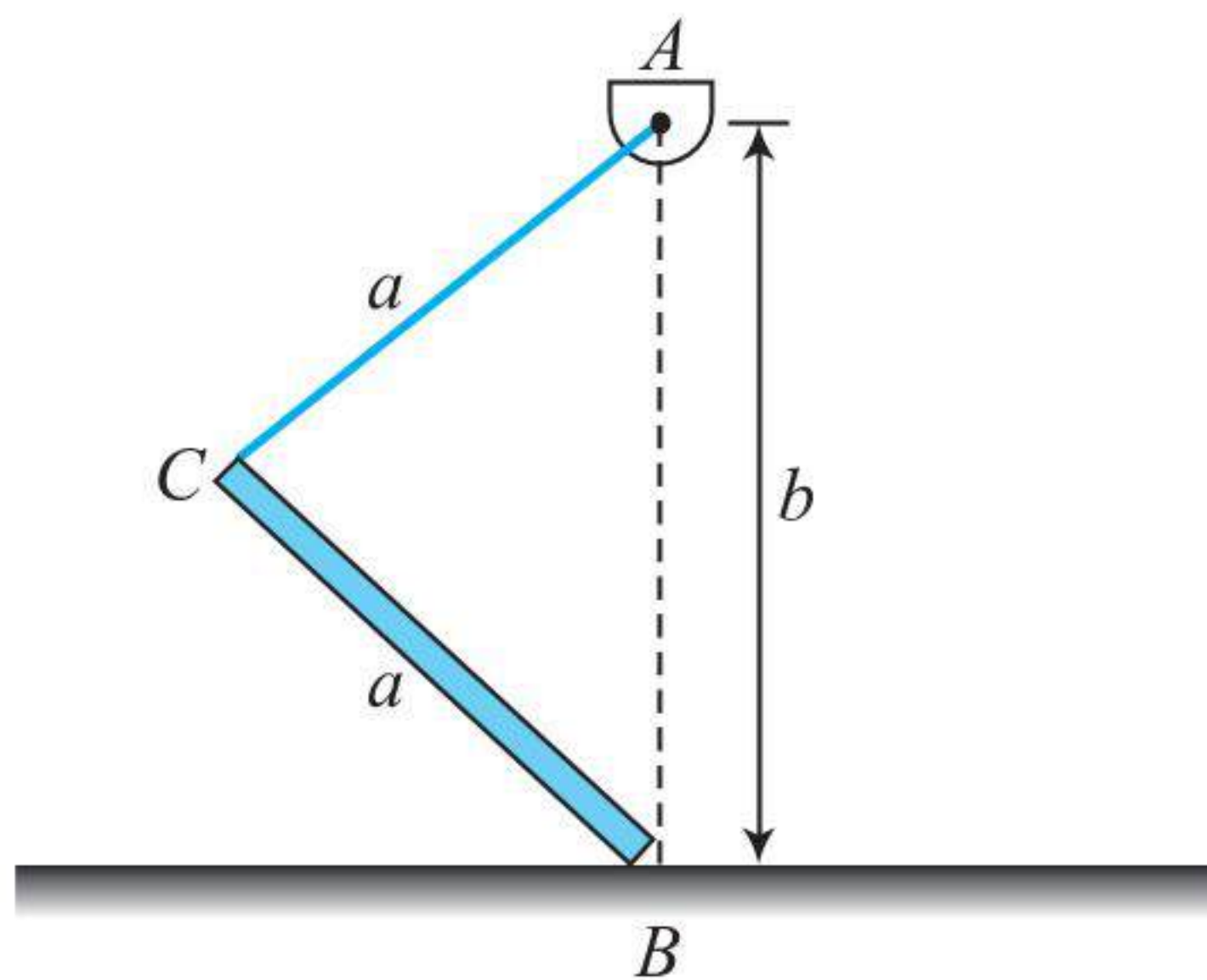
- (1) $\frac{mu^2 \sin 2\theta}{2}$ (2) $mu^2 \cos \theta$
 (3) $mu^2 \sin \theta$ (4) $\frac{mu^2 \cos \theta}{2}$

87. A disc of radius $r = 0.1$ m is rolled from a point A on a track as shown in the figure. The part AB of the track is a semi-circle of radius R in a vertical plane. The disc rolls without sliding and leaves contact with the track at its highest point B . Flying through the air it strikes the ground at point C . The velocity of the center of mass of the disc makes an angle of 30° below the horizontal at the time of striking the ground. At the same instant, velocity of the topmost point P of the disc is found to be 6 m/s. The value of R is (Take $g = 10$ m/s²).



- (1) 1 m (2) 5 m
(3) 2.5 m (4) 2 m

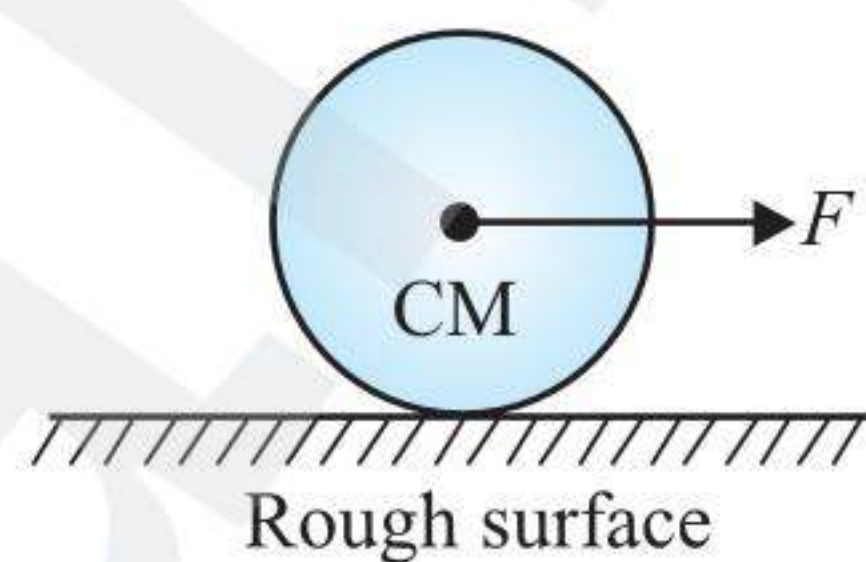
88. A uniform rod BC with length a is attached to a light string AC . End A of the string is fixed to the ceiling and the end B of the rod is on a smooth horizontal surface. B is exactly below point A and length AB is b ($a < b < 2a$). The system is released from rest and the rod begins to slide. Find the speed of the centre of the rod when the string becomes vertical.



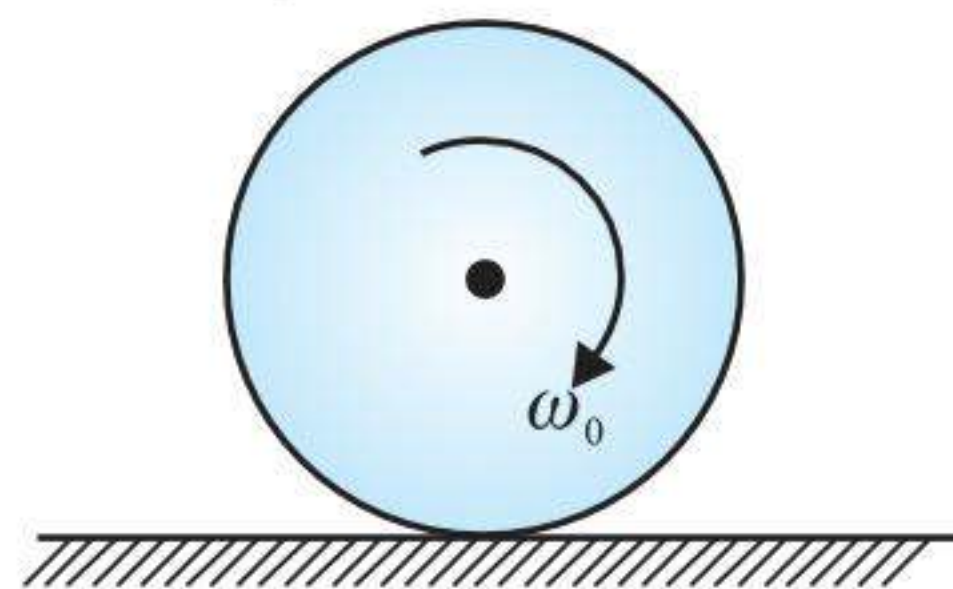
- (1) $\sqrt{g\left(a + \frac{b}{2}\right)}$ (2) $\sqrt{g\left(a - \frac{b}{2}\right)}$
(3) $\sqrt{g\left(a + \frac{b}{3}\right)}$ (4) $\sqrt{g\left(a - \frac{b}{3}\right)}$

Multiple Correct Answers Type

1. A solid sphere of mass M and radius R is pulled horizontally on a rough surface as shown in figure. Choose the incorrect alternatives.



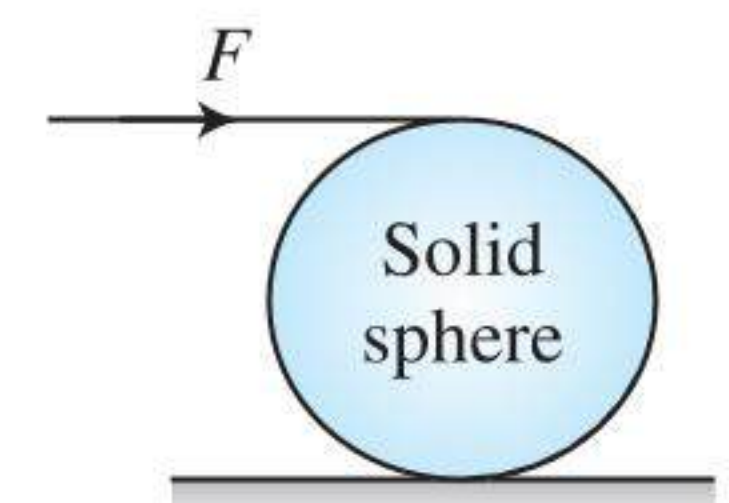
- (1) The magnitude of the frictional force is $F/3$.
(2) The frictional force on the sphere acts forward.
(3) The acceleration of the centre of mass is $2F/3M$.
(4) The acceleration of the centre of mass is F/M .
2. A disc is given an initial angular velocity ω_0 and placed on a rough horizontal surface as shown in figure. The quantities which will not depend on the coefficient of friction is/are
- (1) the time until rolling begins
(2) the displacement of the disc until rolling begins
(3) the velocity when rolling begins
(4) the work done by the force of friction
3. Consider three uniform solid spheres, sphere (i) has radius ' r ' and mass ' m ', sphere (ii) has radius r and mass $3m$, sphere (iii) has radius $3r$ and mass ' m '. All the spheres can be placed at the same point on the same inclined plane where



they will roll without slipping to the bottom. If allowed to roll down the incline, then at the bottom of the incline

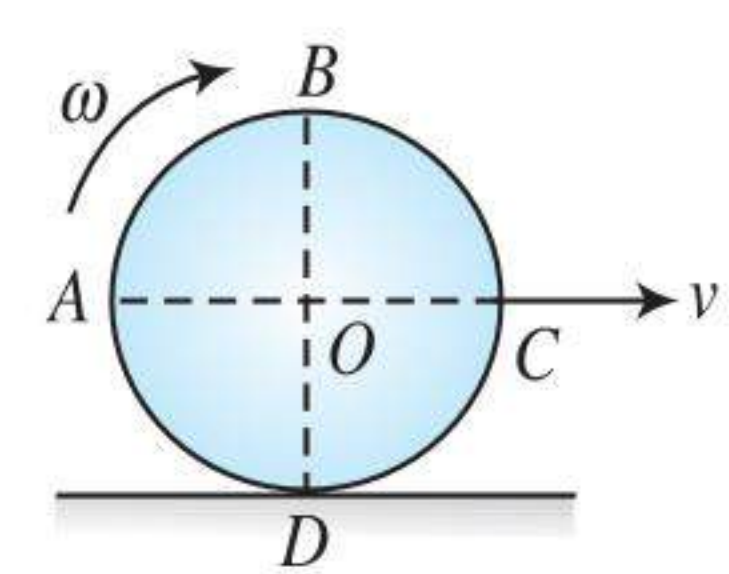
- (1) sphere (i) will have the largest speed
(2) sphere (ii) will have the largest speed
(3) sphere (ii) will have the largest kinetic energy
(4) all the spheres will have equal speed

4. In the situation given, a force F is applied at the top of sphere. Study following statements.



- (1) The sphere will move faster if friction is absent.
(2) The sphere will move faster if friction is present.
(3) The sphere will move with same velocity irrespective of the friction.
(4) Friction supports the motion.

5. A ring rolls without slipping on a horizontal surface. At any instant, its position is as shown in the figure. Then

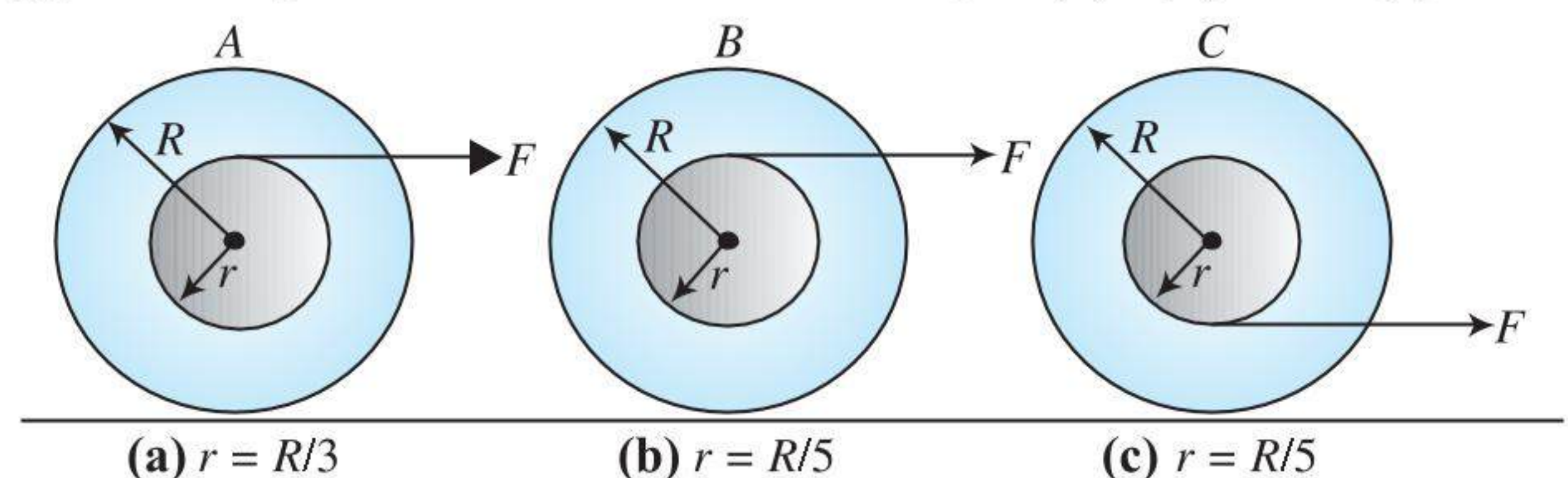


- (1) section ABC has greater kinetic energy than section ADC
(2) section BC has greater kinetic energy than section CD
(3) section BC has the same kinetic energy as section DA
(4) the sections AB , BC , CD and DA have the same kinetic energy

6. A uniform disc is rotating at a constant speed in a vertical plane about a fixed horizontal axis, passing through the centre of the disc. A piece of disc from its rim detaches itself from the disc at the instant when it is on the same horizontal with the centre of the disc and moving upwards. Then about the fixed axis, the angular speed of the

- (1) remaining disc remains unchanged
(2) remaining disc decreases
(3) remaining disc increases
(4) broken away piece decreases initially and later increases

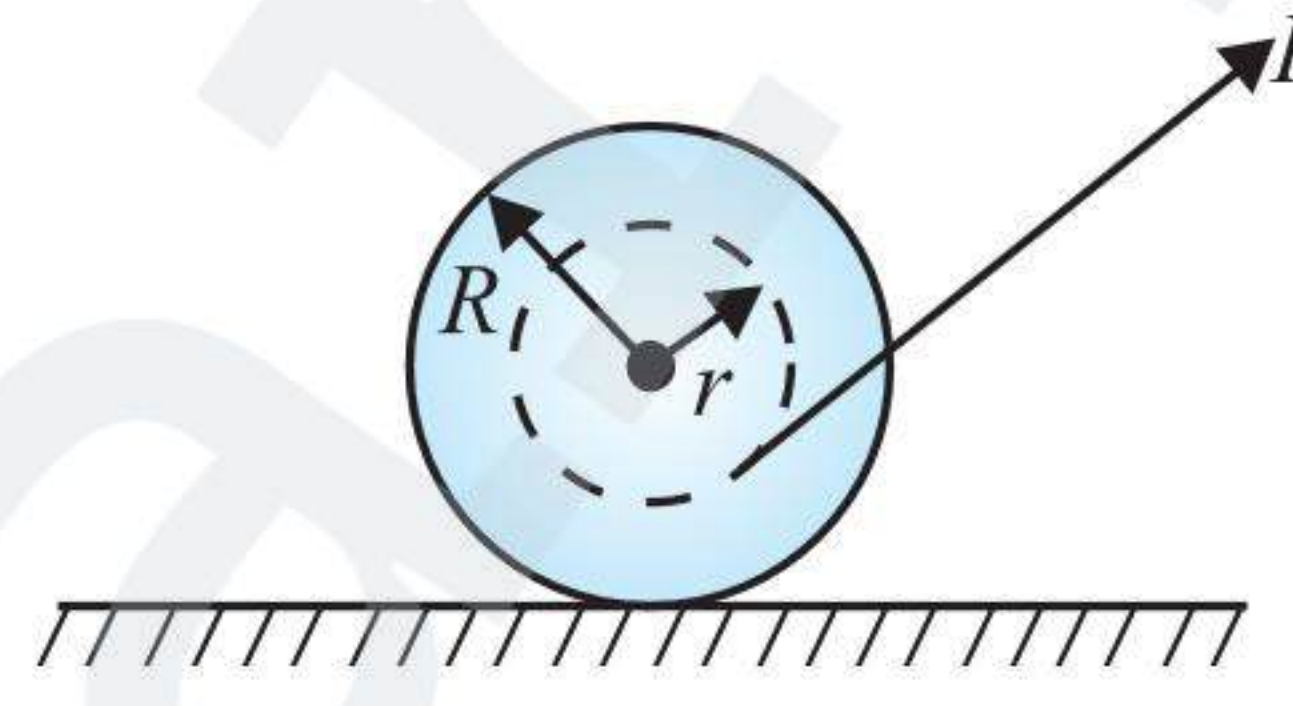
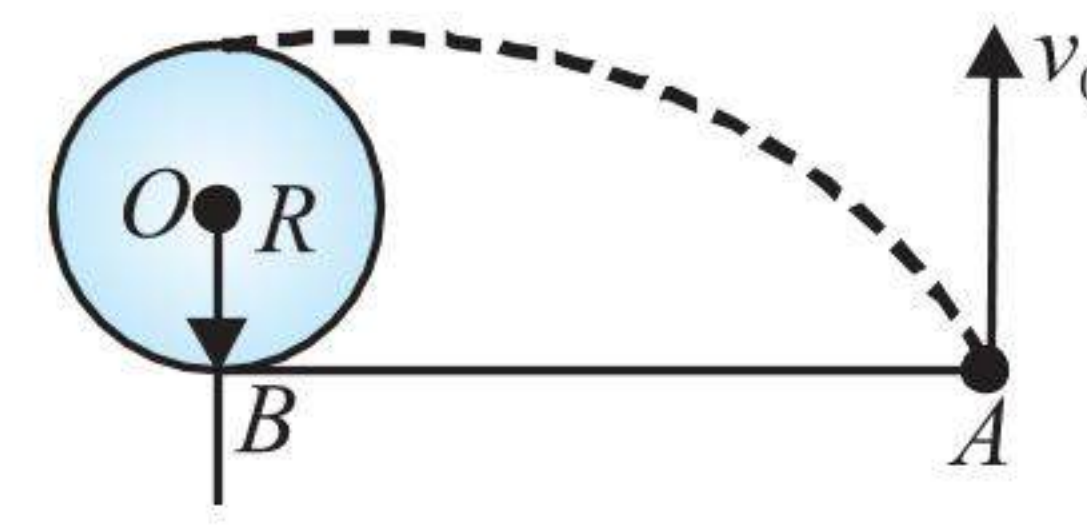
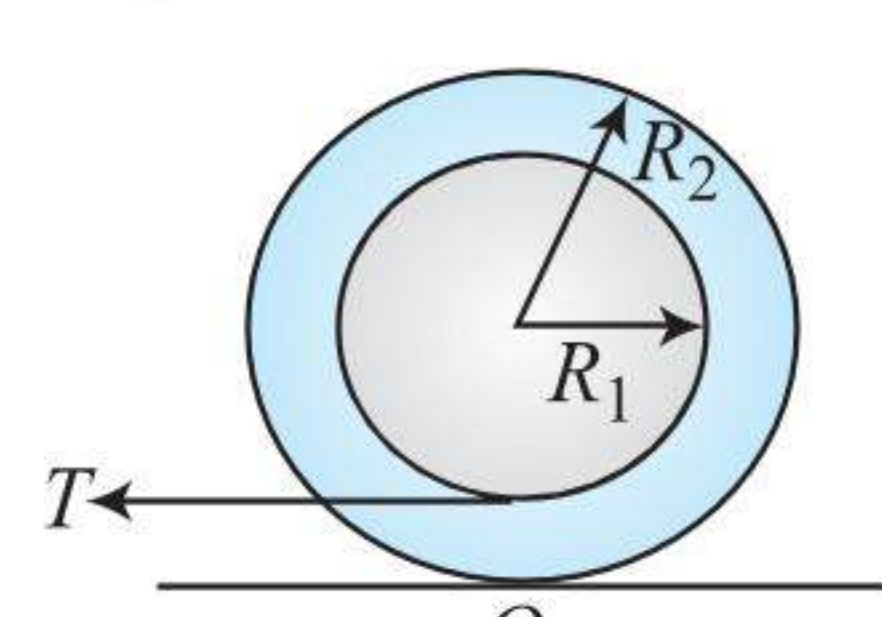
7. Three spools A , B and C each having moment of inertia $I = MR^2/4$ are placed on rough ground and equal force F is applied at positions as shown in Figs. (a), (b) and (c). Then



- (1) frictional force on spool A acts in forward direction
(2) frictional force on spool A acts in backward direction
(3) frictional force on spool B acts in forward direction
(4) frictional force on spool B and C acts in backward direction

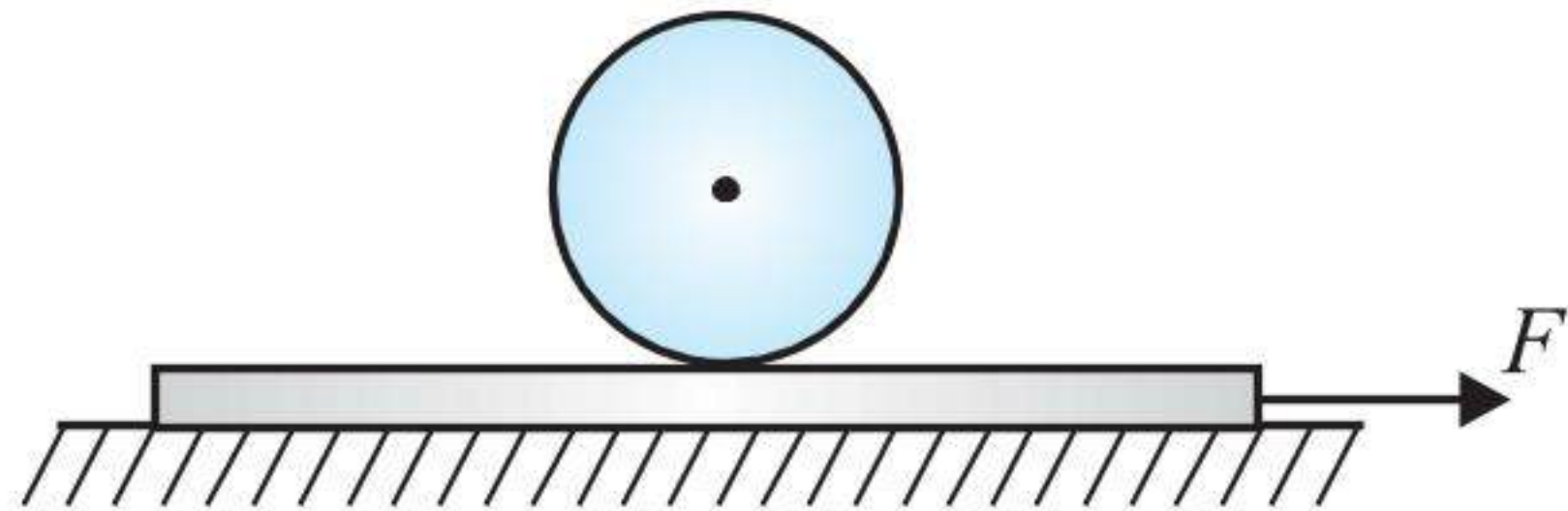
8. A horizontal disc rotates freely about a vertical axis through its centre. A ring, having the same mass and radius as the disc, is now gently placed on the disc. After some time, the two rotate with a common angular velocity, then

- (1) some friction exists between the disc and the ring
(2) the angular momentum of the 'disc plus ring' is conserved
(3) the final common angular velocity is $(2/3)$ rd of the initial angular velocity of the disc
(4) $(2/3)$ rd of the initial kinetic energy changes to heat

9. Two horizontal discs of different radii are free to rotate about their central vertical axes. One is given some angular velocity, the other is stationary. Their rims are now brought in contact. There is friction between the rims. Then
- (1) the force of friction between the rims will disappear when the discs rotate with equal angular speeds
 - (2) the force of friction between the rims will disappear when they have equal linear velocities
 - (3) the angular momentum of the system will be conserved
 - (4) the rotational kinetic energy of the system will not be conserved
10. A constant external torque τ acts for a very brief period Δt on a rotating system having moment of inertia I , then
- (1) the angular momentum of the system will change by $\tau\Delta t$
 - (2) the angular velocity of the system will change by $\tau\Delta t/I$
 - (3) if the system was initially at rest, it will acquire rotational kinetic energy $(\tau\Delta t)^2/2I$
 - (4) the kinetic energy of the system will change by $(\tau\Delta t)^2/2I$
11. Two identical spheres A and B are free to move and to rotate about their centres. They are given the same impulse J . The lines of action of the impulses pass through the centre of A and away from the centre of B , then
- (1) A and B will have the same speed
 - (2) B will have greater kinetic energy than A
 - (3) they will have the same kinetic energy, but the linear kinetic energy of B will be less than that of A
 - (4) the kinetic energy of B will depend on the point of impact of the impulse on B
12. A disc of circumference s is at rest at a point A on a horizontal surface when a constant horizontal force begins to act on its centre. Between A and B there is sufficient friction to prevent slipping and the surface is smooth to the right of B , $AB = s$. The disc moves from A to B in time T . To the right of B ,
- (1) the angular acceleration of the disc will disappear, linear acceleration will remain unchanged
 - (2) linear acceleration of the disc will increase
 - (3) the disc will make one rotation in time $T/2$
 - (4) the disc will cover a distance greater than s in further time T
13. A ring (R), a disc (D), a solid sphere (S) and a hollow sphere with thin walls (H), all having the same mass but different radii, start together from rest at the top of an inclined plane and roll down without slipping. Then
- (1) all of them will reach the bottom of the incline together
 - (2) the body with the maximum radius will reach the bottom first
 - (3) they will reach the bottom in the order S, D, H and R
 - (4) all of them will have the same kinetic energy at the bottom of the incline
14. A solid sphere rolls without slipping on a rough horizontal floor, moving with a speed v . It makes an elastic collision with a smooth vertical wall. After impact
- (1) it will move with a speed v initially
 - (2) its motion will be rolling without slipping
 - (3) its motion will be rolling without slipping initially and its rotational motion will stop momentarily at some instant
 - (4) its motion will be rolling without slipping only after some time
15. A thin uniform rod of mass m and length l is free to rotate about its upper end. When it is at rest, it receives an impulse J at its lowest point, normal to its length. Immediately after impact,
- (1) the angular momentum of the rod is Jl
 - (2) the angular velocity of the rod is $3J/ml$
 - (3) the kinetic energy of the rod is $3J^2/2m$
 - (4) the linear velocity of the midpoint of the rod is $3J/2m$
16. Inner and outer radii of a spool are r and R , respectively. A thread is wound over its inner surface and spool is placed over a rough horizontal surface. Thread is pulled by a force F as shown in figure. In case of pure rolling, which of the following statements are false?
- 
- (1) Thread unwinds, spool rotates anticlockwise and friction acts leftwards.
 - (2) Thread winds, spool rotates clockwise and friction acts leftwards.
 - (3) Thread winds, spool moves to the right and friction acts rightwards.
 - (4) Thread winds, spool moves to the right and friction does not come into existence.
17. A horizontal plane supports a fixed vertical cylinder of radius R and a particle is attached to the cylinder by a horizontal thread AB as shown in figure. The particle initially rest on a horizontal plane. A horizontal velocity v_0 is imparted to the particle, normal to the threading during subsequent motion. Point out the false statements.
- 
- (1) Angular momentum of particle about O remains constant.
 - (2) Angular momentum about B remains constant.
 - (3) Momentum and kinetic energy both remain constant.
 - (4) Kinetic energy remains constant.
18. A uniform rod is resting freely over a smooth horizontal plane. A particle moving horizontally strikes at one end of the rod normally and gets stuck. Then
- (1) the momentum of the particle is shared between the particle and the rod and remains conserved
 - (2) the angular momentum about the mid-point of the rod before and after the collision is equal
 - (3) the angular momentum about the centre of mass of the combination before and after the collision is equal
 - (4) the centre of mass of the rod particle system starts to move translationally with the original momentum of the particle
19. A stepped cylinder having mass 50 kg and a radius of gyration (K) of 0.30 m. The radii R_1 and R_2 are respectively 0.3 m and 0.6 m. A pull T equal to 200 N is exerted on the rope attached to inner cylinder. The coefficient of static and dynamic friction between cylinder and ground are 0.1 and 0.08 respectively.
- 

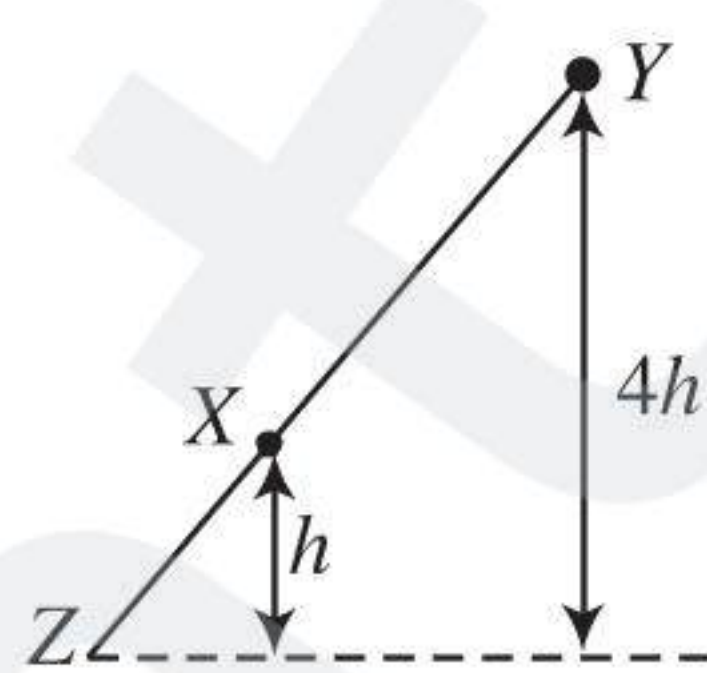
- (1) The angular acceleration is 2.67 rad s^{-2} .
- (2) The force of kinetic friction is 40 N .
- (3) The acceleration is 3.2 m s^{-2} towards left.
- (4) The acceleration is 3.2 m s^{-2} towards right.

20. A plank with a uniform sphere placed on it rests on a smooth horizontal plane. The plank is pulled to the right by a constant force F . If the sphere does not slip over the plank, then



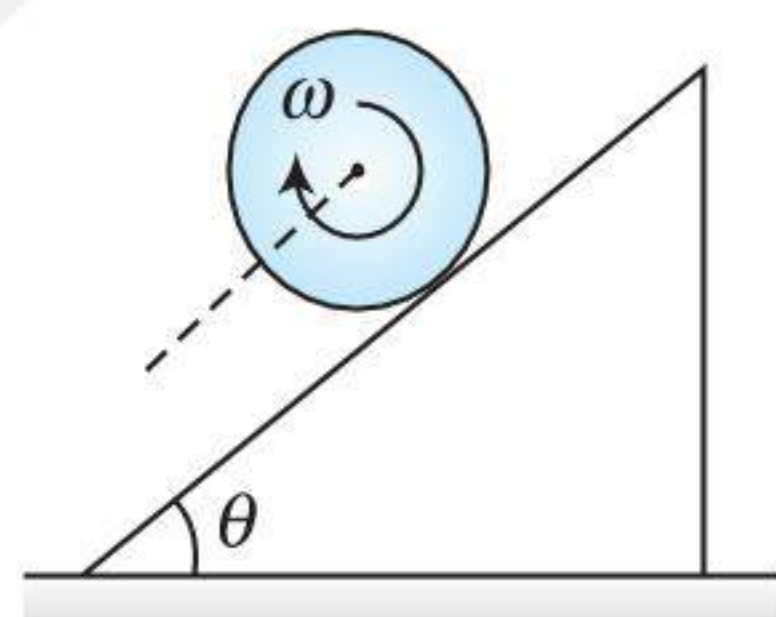
- (1) both have the same acceleration
 - (2) acceleration of the centre of sphere is less than that of the plank
 - (3) work done by friction acting on the sphere is equal to its total kinetic energy
 - (4) total kinetic energy of the system is equal to work done by the force F
21. A sphere A moving with speed u and rotating with an angular velocity ω makes a head-on elastic collision with an identical stationary sphere B . There is no friction between the surfaces of A and B . Choose the correct alternative(s). Disregard gravity.
- (1) A will stop moving but continue to rotate with an angular velocity ω .
 - (2) A will come to rest and stop rotating.
 - (3) B will move with speed u without rotating.
 - (4) B will move with speed u and rotate with an angular velocity ω .

22. Two wheels A and B are released from rest from points X and Y respectively on an inclined plane as shown in the figure. Which of the following statement(s) is/are incorrect?



- (1) Wheel B takes twice as much time to roll from Y to Z than that of wheel A from X to Z .
- (2) At point Z velocity of wheel A is four times that of wheel B .
- (3) Acceleration of the wheel A is twice that of the wheel B .
- (4) Both wheel take same time to arrive at point Z .

23. A solid sphere is given an angular velocity ω and kept on a rough fixed incline plane. Then choose the correct statement.



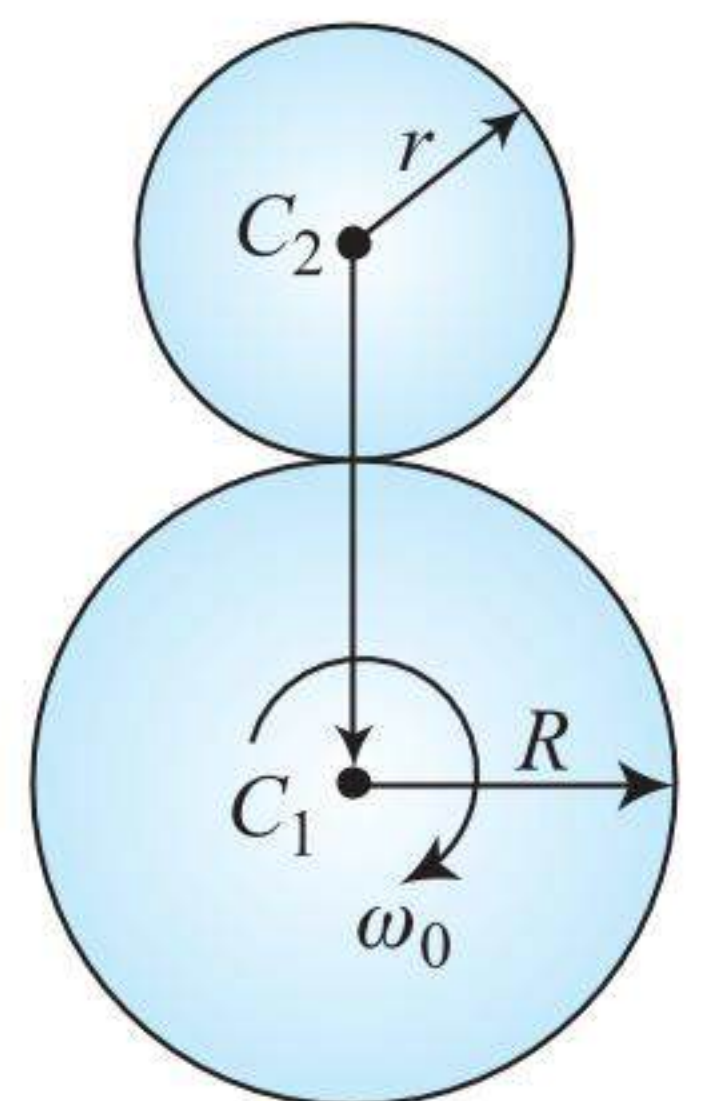
- (1) If $\mu = \tan \theta$ then sphere will be in linear equilibrium for some time and after that pure rolling down the plane will start.
- (2) If $\mu = \tan \theta$ then sphere will move up the plane and frictional force acting all the time will be $2mg \sin \theta$.
- (3) If $\mu = \frac{\tan \theta}{2}$ there will never be pure rolling (consider inclined plane to be long enough).
- (4) If incline plane is not fixed and it is on smooth horizontal surface then linear momentum of the system (wedge and sphere) can be conserved in horizontal direction.

24. A wheel A starts rolling up a rough inclined plane and another identical wheel B starts rolling up a smooth plane

having same inclination with the horizontal. If initial velocity of both the wheels is same then

- (1) A stops ascending earlier than B
- (2) kinetic energy of B never becomes zero
- (3) maximum height ascended by A is less than that by B
- (4) friction acting on A remains constant during the round trip

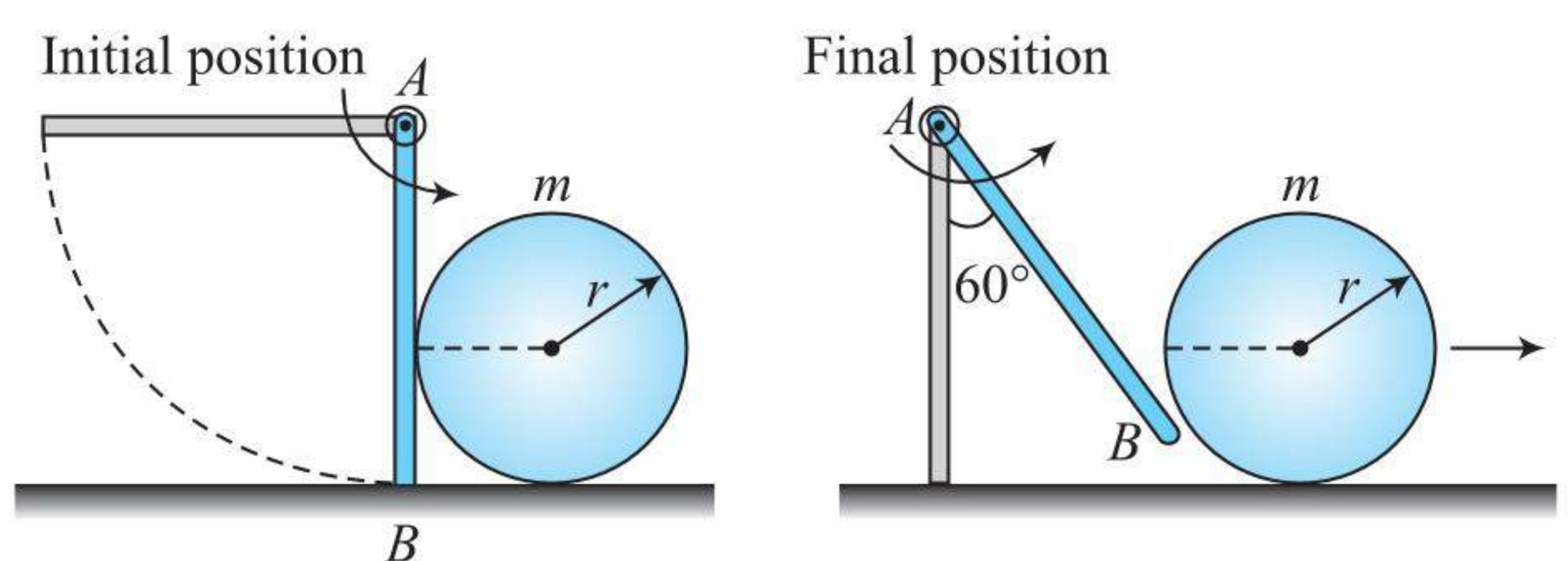
25. Consider a fixed wheel of radius R . A small wheel (in the form, of a uniform solid disc) of radius r is performing pure rolling on periphery of bigger wheel centre of bigger wheel and smaller wheel are joined by rigid rod such that smaller wheel can rotate freely w.r.t. its centre. Rod joining the (centers is rotating with constant angular velocity ω_0 . Whole situation is shown in figure. Choose the correct option(s): (Use $R = 4r$)



- (1) Angular velocity of smaller wheel is ω_0
- (2) Angular velocity of smaller wheel is $5\omega_0$
- (3) Kinetic energy of smaller wheel is $\frac{75}{4}mr^2\omega_0^2$, where m is the mass of the smaller disc
- (4) Radius of curvature of the path of the particle which is lying on the circumference of smaller wheel and at farthest distance from centre of bigger wheel is $\frac{10r}{3}$

26. A uniform rod AB of mass M is attached to a hinge at one end A , and released from rest from the horizontal position. The rod rotates about A , and when it reaches the vertical position the rod strikes a sphere of mass m and radius r initially at rest on the smooth horizontal surface as shown in the adjacent figure. The impact is along the horizontal direction and perfectly elastic. If at the moment of impact the lowest end of the rod is very close to the smooth horizontal surface. After the impact, the sphere moves along the horizontal and the rod, subsequently rises to a maximum of 60° with the vertical. Choose the correct statement(s) from the following, taking into account the information given above.

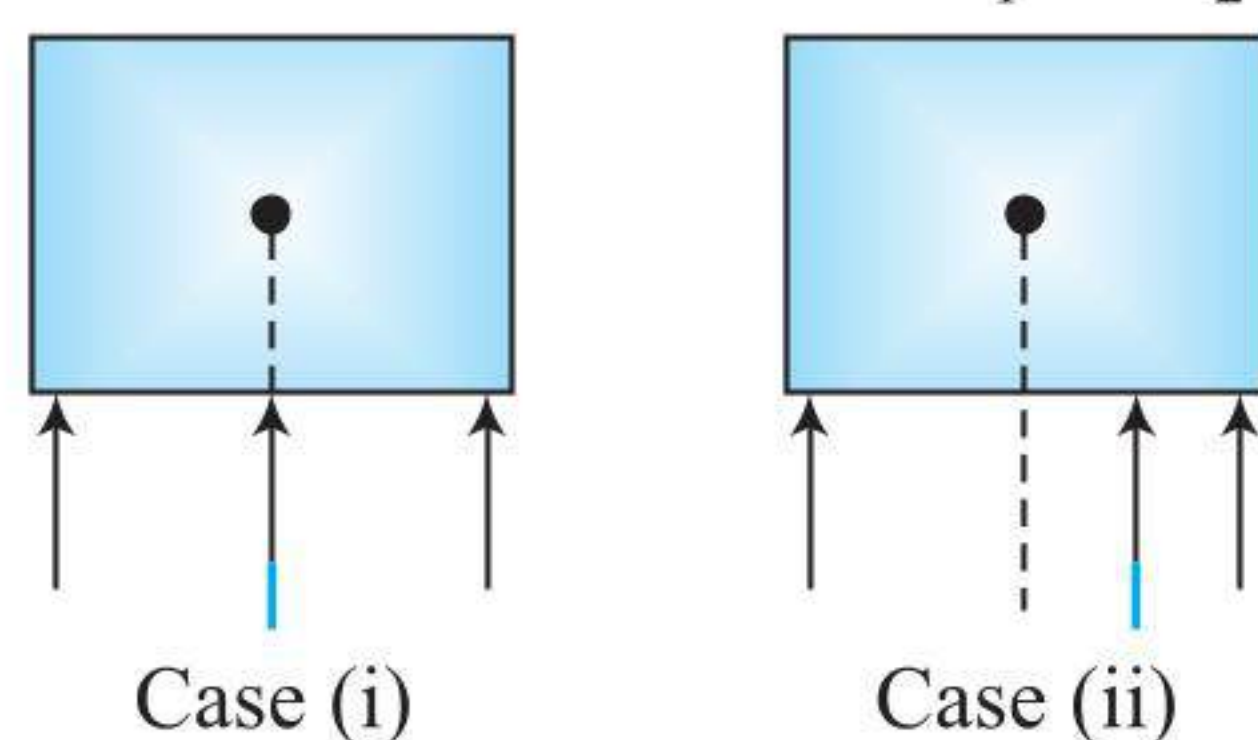
The length of the rod equals $\sqrt{2}r$, $\left(r = \frac{6\sqrt{2}}{10} \text{ m}\right)$



- (1) The ratio $\frac{M}{m}$ is $\frac{3}{2}$
- (2) The ratio $\frac{M}{m}$ is $\frac{2}{3}$
- (3) The speed of the sphere just after collision is 6 m/s
- (4) The speed of the sphere just after collision is 3 m/s

27. Two identical blocks are resting on supports as shown. Two identical bullets travelling with the same speed hit the

blocks and get embedded in them in very short time. The blocks leave the contact immediately after bullet strikes them. In case (i) bullet's velocity is toward centre of mass and in case (ii) bullet's velocity is off centre. The kinetic energies after collision are K_1 & K_2 and maximum height reached by the centre of mass are h_1 & h_2 respectively. Then



- (1) $K_1 < K_2$ (2) $K_1 = K_2$
 (3) $h_1 > h_2$ (4) $h_1 = h_2$

Linked Comprehension Type

For Problems 1–4

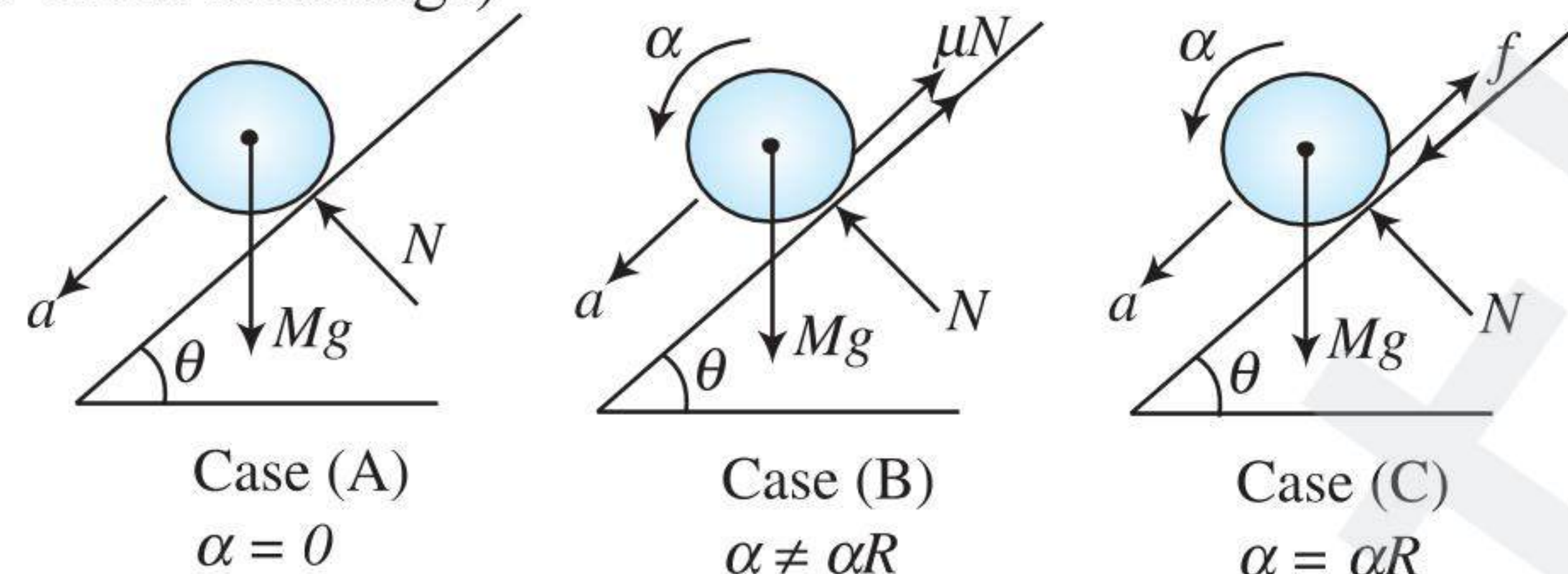
In this passage a brief idea is given of the motion of the rolling bodies on an inclined plane. We will consider three cases: objects are released on an incline plane

Case-A: which is smooth.

Case-B: where friction is insufficient to provide pure rolling.

Case-C: where friction is sufficient to provide pure rolling.

Force diagram for three cases are as follows: (where symbols have their usual meanings)



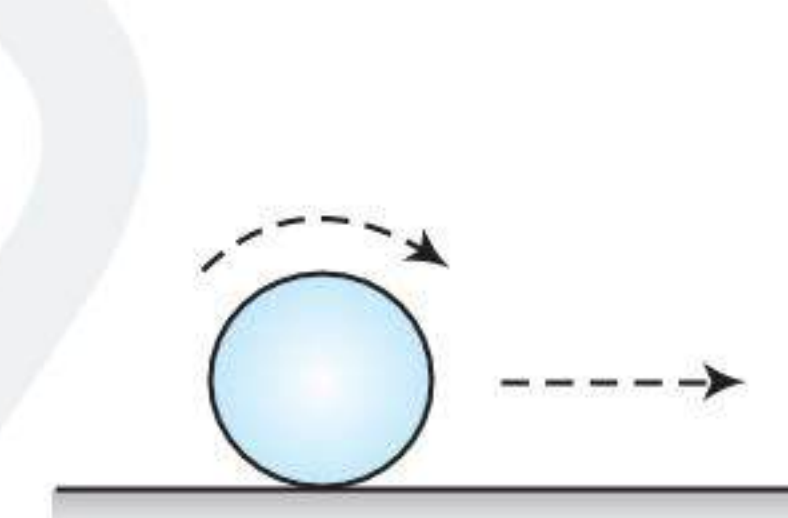
- Three solid uniform spheres are released on an inclined plane as shown in the figure. The distance between the spheres remains constant during motion in
 - all three masses
 - case A and B
 - only case C
 - depends on the mass of the spheres.
- We have four objects: a solid sphere, a hollow sphere, a ring and a disc, all of same radius. When these are released on an inclined plane, it may happen that all of them do not perform pure rolling. But from the information of pure rolling, if one object can be confirmed to be purely rolling then it can be said that rest all will perform pure rolling. This object whose pure rolling confirms pure rolling of all other objects is
 - hollow sphere
 - solid sphere
 - ring
 - disc
- If the four objects given in the above question are of same mass, same radius having the same friction coefficient and are released from the same height, then at the bottom the object which will have least kinetic energy for case B will be the
 - hollow sphere
 - solid sphere
 - ring
 - disc

- Two children A and B use bicycles, having wheels of ring type and disc type respectively. During a race, bicycles are given the same velocity from the bottom of the inclined bridge to ascend the bridge without pedalling, then (assuming pure rolling).

- both bicycles will reach up to same height
- bicycle of child A will reach a greater height
- bicycle of child B will reach a greater height
- depends on the masses of bicycles and the child

For Problems 5–6

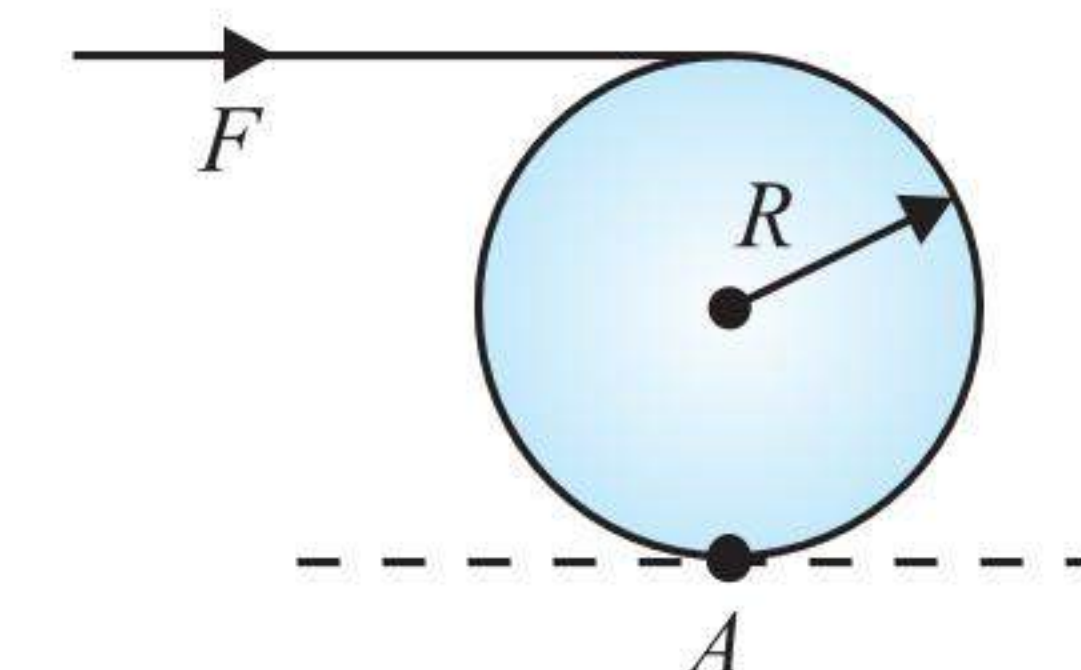
A ball, rolling purely on a horizontal floor with centre's speed v , hits a smooth vertical wall surface elastically. Answer the following questions.



- Just after the collision is over, the velocity of the lowest point of the ball is
 - v , leftward
 - $2v$, leftward
 - v , rightward
 - $2v$, rightward
- The change in angular momentum of the solid ball (mass m , radius R), about the corner point of floor and wall, due to the collision is
 - $2mvR$
 - mvR
 - $mvR/2$
 - None

For Problems 7–9

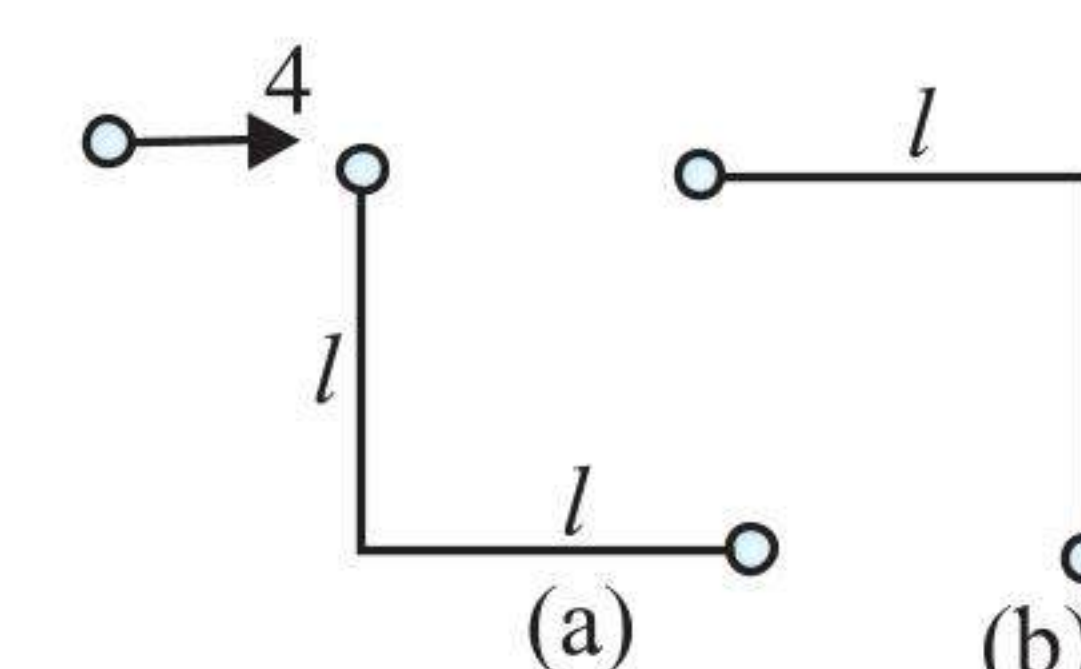
When a body is hinged at a point and a force is acting on the body in such a way that the line of action of force is at some distance from the hinged point, the body will start rotating about the hinged point. The angular acceleration of the body can be calculated by finding the torque of that force about the hinged point. A disc of mass m and radius R is hinged at point A at its bottom and is free to rotate in the vertical plane. A force of magnitude F is acting on the ring at the top most point.



- Tangential acceleration of the centre of mass is
 - $\frac{3F}{4m}$
 - $\frac{F}{m}$
 - $\frac{2F}{3m}$
 - $\frac{4F}{3m}$
- Component of reaction at the hinge in the vertical direction is
 - $\frac{4}{3}mg$
 - mg
 - $\frac{mg}{2}$
 - $\frac{2}{3}mg$
- Component of reaction at the hinge in the horizontal direction is
 - $\frac{F}{4}$
 - F
 - $\frac{F}{3}$
 - $\frac{F}{2}$

For Problems 10–12

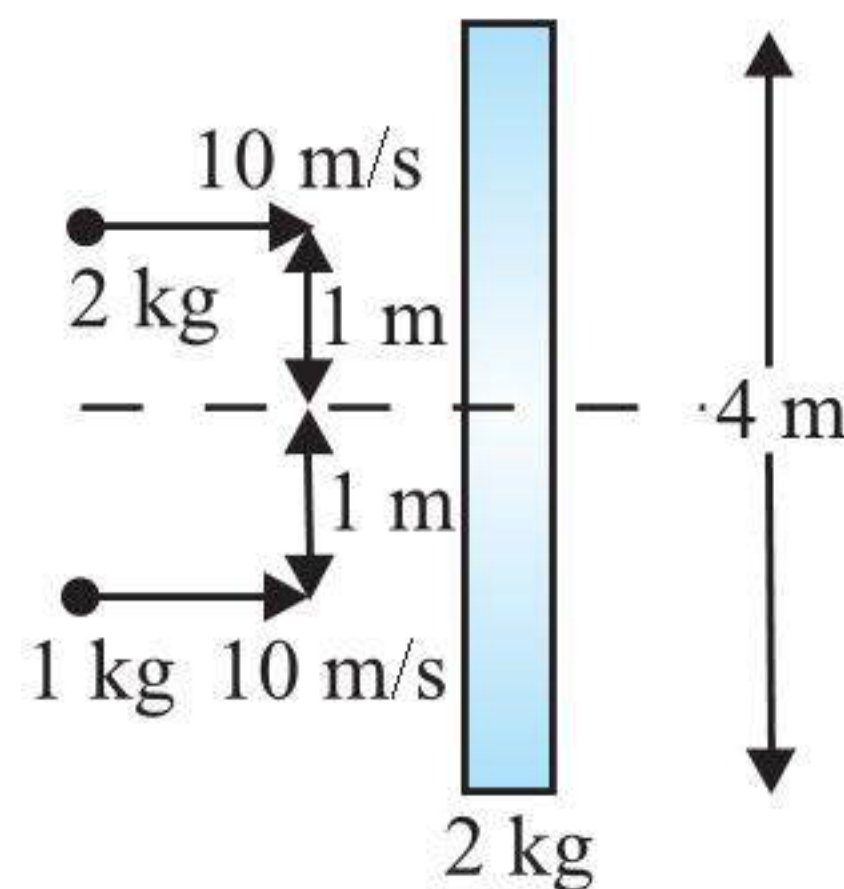
A spherical ball of mass M moving with initial velocity u collides elastically with another ball of mass M , which is fixed at one end of L shaped rigid massless frame as shown in Fig. (a). The L shaped frame contains another mass M connected at the other end.



10. The speed of the striking mass after collision is
 (1) $u/2$ backwards (2) 0
 (3) $u/3$ in same direction (4) $u/7$ backwards
11. The angular speed of L frame immediately after collision is
 (1) $\frac{4u}{7l}$ (2) $\frac{u}{3l}$
 (3) $\frac{4u}{2l}$ (4) $\frac{u}{7l}$
12. How soon will the frame come to the orientation shown in Fig. (b) after collision?
 (1) $\frac{\pi l}{4u}$ (2) $\frac{7\pi l}{8u}$
 (3) $\frac{\pi l}{u}$ (4) $\frac{7\pi l}{4u}$

For Problems 13–15

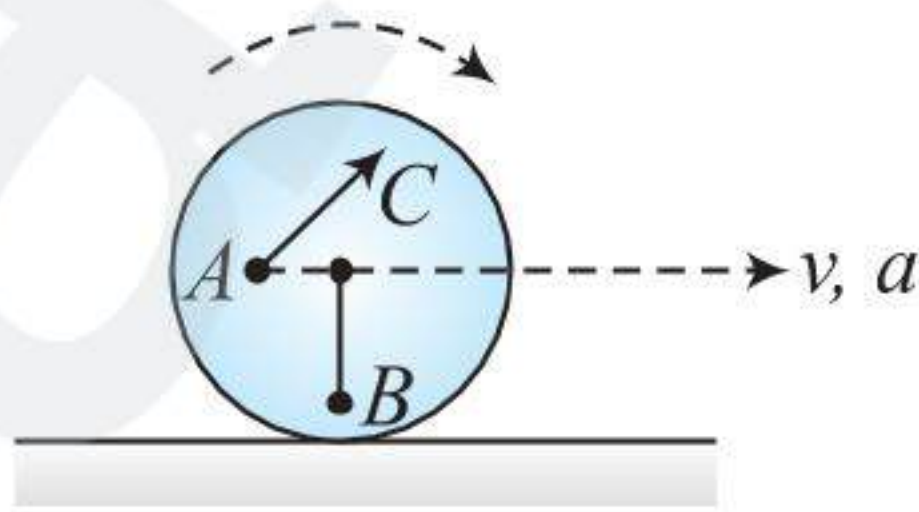
A long slender rod of mass 2 kg and length 4 m is placed on a smooth horizontal table. Two particles of masses 2 kg and 1 kg strike the rod simultaneously and stick to the rod after collision as shown in figure.



13. Velocity of the centre of mass of the rod after collision is
 (1) 12 m/s (2) 9 m/s
 (3) 6 m/s (4) 3 m/s
14. Angular velocity of the rod after collision is
 (1) $\frac{11}{17}$ rad/s (2) $\frac{24}{17}$ rad/s
 (3) $\frac{19}{17}$ rad/s (4) $\frac{30}{17}$ rad/s
15. If the two particles strike the rod in opposite direction, then after collision, as compared to the previous situation the rod will
 (1) rotate faster but translate at the same rate
 (2) show no change in linear or angular velocity
 (3) rotate slower and translate faster
 (4) rotate faster and translate slower

For Problems 16–18

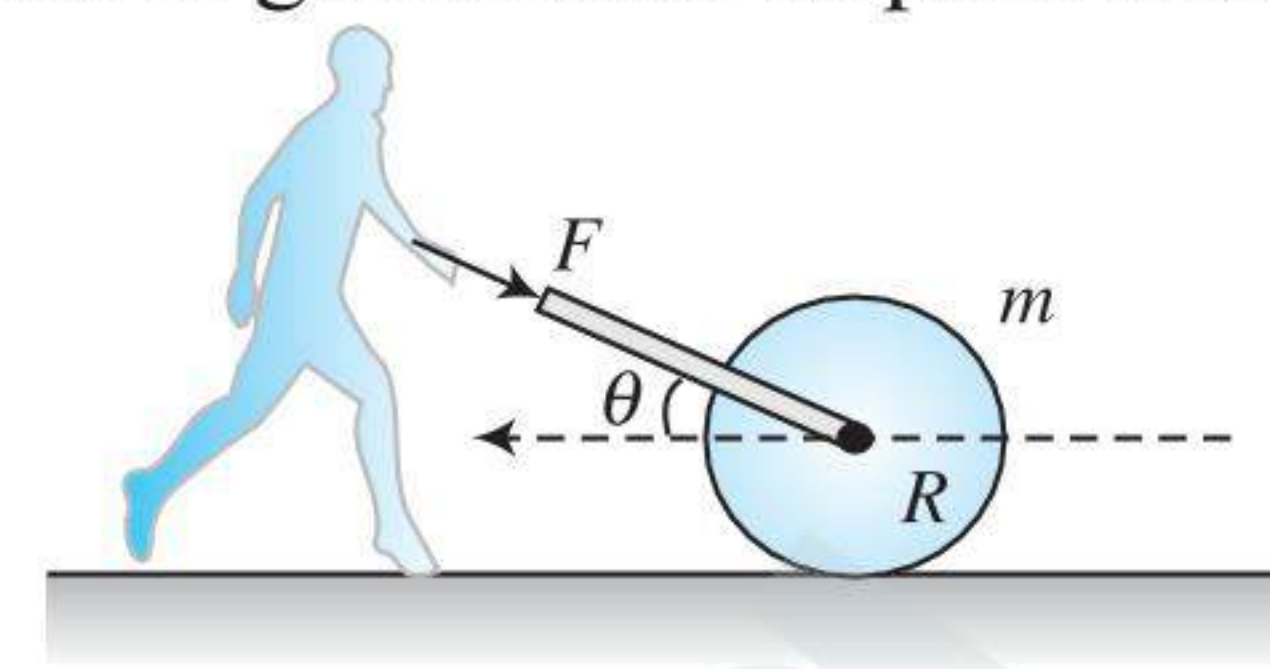
A uniform disc of mass 1 kg and radius 20 cm is rolling purely on a flat horizontal surface. Its centre C is moving with acceleration $a = 20 \text{ ms}^{-2}$ and velocity $v = 4 \text{ m/s}$ at a certain instant. At this instant, points A and B are located on the disc as shown in the diagram, with $AC = BC = 10 \text{ cm}$.



16. What is the kinetic energy of the disc?
 (1) 12 J (2) 8 J
 (3) 20 J (4) 10 J
17. What is the acceleration magnitude of point A ?
 (1) $10\sqrt{17} \text{ ms}^{-2}$ (2) $10\sqrt{37} \text{ ms}^{-2}$
 (3) $10\sqrt{35} \text{ ms}^{-2}$ (4) $60\sqrt{17} \text{ ms}^{-2}$
18. What is the acceleration magnitude of point B ?
 (1) $10\sqrt{17} \text{ ms}^{-2}$ (2) $10\sqrt{37} \text{ ms}^{-2}$
 (3) $10\sqrt{35} \text{ ms}^{-2}$ (4) $60\sqrt{17} \text{ ms}^{-2}$

For Problems 19–21

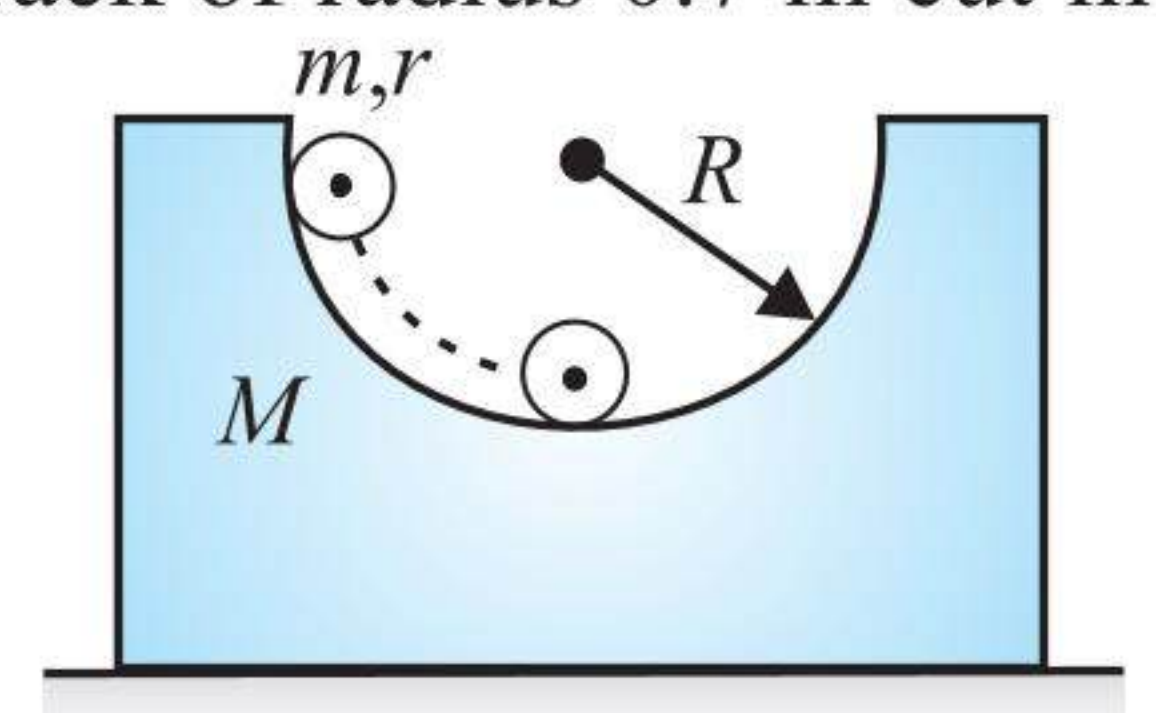
A gardener presses the grasscutter with a force F at an angle θ . Assume the motion of grasscutter as pure rolling. Find the



19. Friction between the grasscutter (assumed as a disc) and ground.
 (1) $\frac{F \sin \theta}{3}$ (2) $\frac{F \cos \theta}{3}$
 (3) $\frac{2F}{3}$ (4) $\frac{2F \cos \theta}{3}$
20. Acceleration of CM of the roller (disc). Assume that the mass of the disc is m and neglect the mass of the connecting rod.
 (1) $\frac{2F \cos \theta}{3m}$ (2) $\frac{F \sin \theta}{3m}$
 (3) $\frac{2F \sin \theta}{3m}$ (4) $\frac{F \cos \theta}{3m}$
21. Maximum force F for no relative sliding if the coefficient of friction between the roller and ground is μ .
 (1) $\frac{\mu mg \cos \theta}{\cos \theta + 3\mu \sin \theta}$ (2) $\frac{\mu mg \sin \theta}{\cos \theta + 3\mu \sin \theta}$
 (3) $\frac{3\mu mg}{\cos \theta - 3\mu \sin \theta}$ (4) $\frac{3\mu mg}{\cos \theta + 3\mu \sin \theta}$

For Problems 22–24

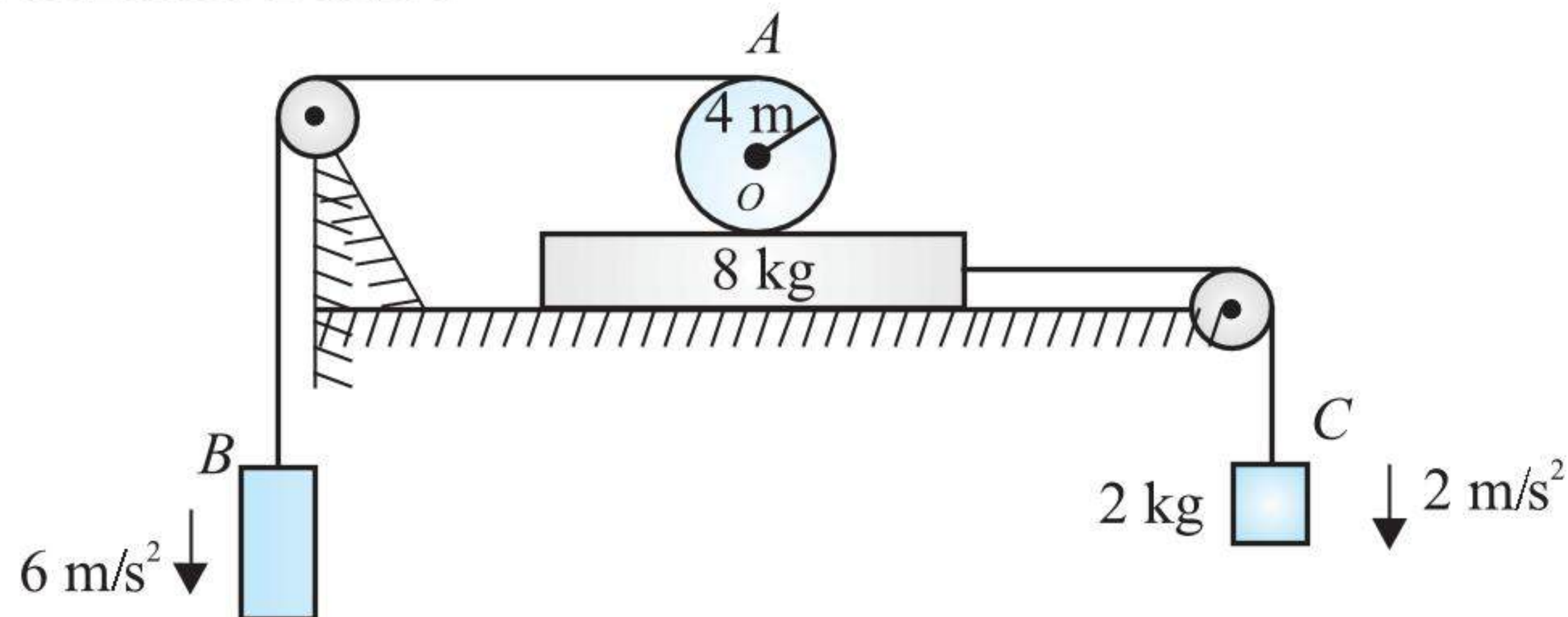
A uniform solid cylinder of mass 2 kg and radius 0.2 m is released from rest at the top of a semicircular track of radius 0.7 m cut in a block of mass $M = 3 \text{ kg}$ as shown in figure. The block is resting on a smooth horizontal surface and the cylinder rolls down without slipping. Based on the above information, answer the following questions:



22. The distance moved by the block when the cylinder reaches the bottom of the track is
 (1) 0.3 m (2) 0.5 m
 (3) 0.7 m (4) 0.2 m
23. The speed of the cylinder when it reaches the bottom of the track is
 (1) $\frac{2}{\sqrt{11}} \text{ m/s}$ (2) $\frac{8}{\sqrt{11}} \text{ m/s}$
 (3) $\frac{4}{\sqrt{11}} \text{ m/s}$ (4) $\frac{6}{\sqrt{11}} \text{ m/s}$
24. The speed of the point of contact of the cylinder with the block w.r.t. ground, when the cylinder reaches the bottom of the track is
 (1) $\frac{2}{\sqrt{11}} \text{ m/s}$ (2) $\frac{8}{\sqrt{11}} \text{ m/s}$
 (3) $\frac{4}{\sqrt{11}} \text{ m/s}$ (4) $\frac{6}{\sqrt{11}} \text{ m/s}$

For Problems 25–27

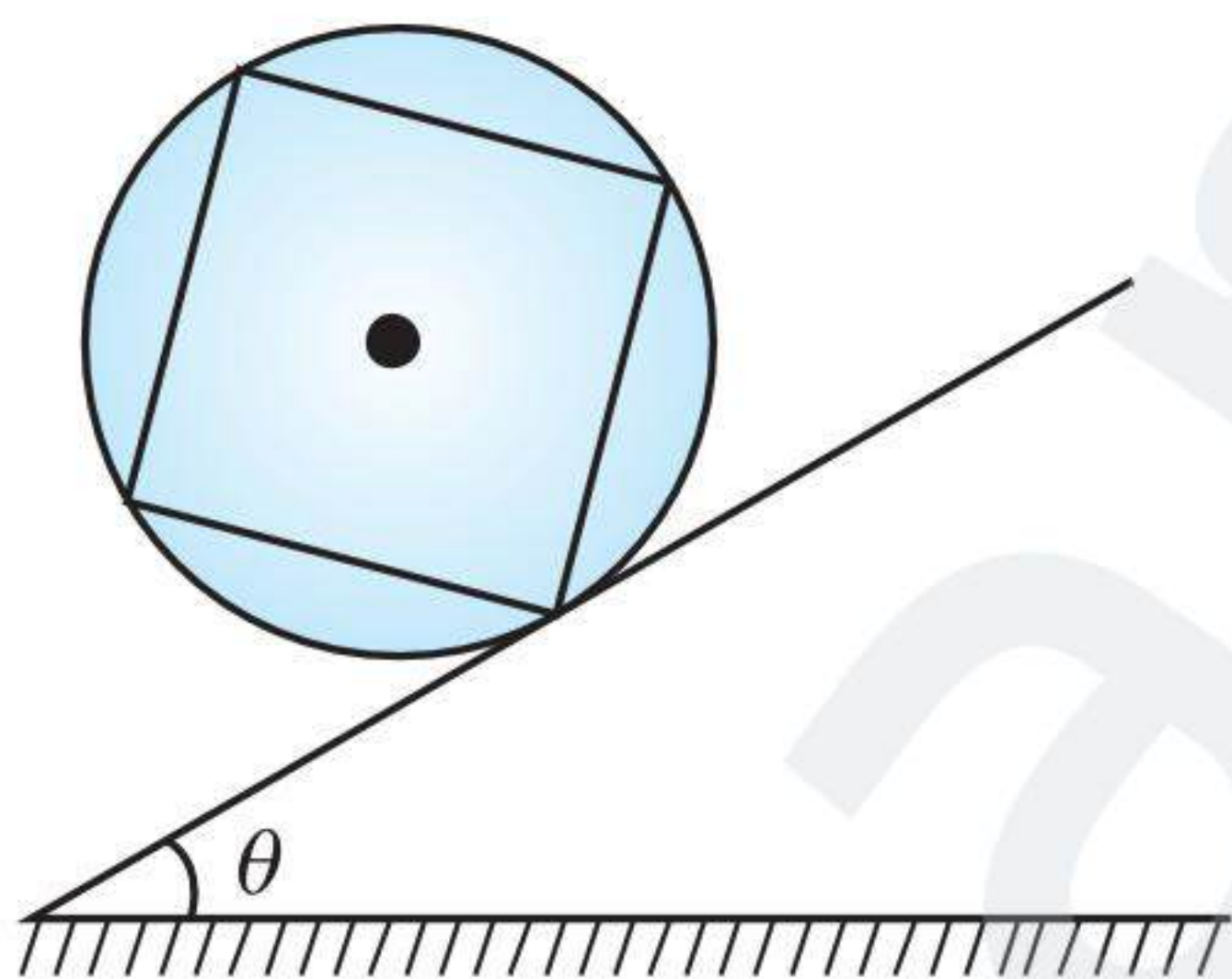
Figure shows a uniform smooth solid cylinder A of radius 4 m rolling without slipping on the 8 kg plank which, in turn, is supported by a fixed smooth surface. Block B is known to accelerate down with 6 m/s^2 and block C moves down with acceleration 2 m/s^2 .



25. What is the angular acceleration of the cylinder?
- (1) $\frac{4}{5} \text{ rad s}^{-2}$ (2) $\frac{6}{5} \text{ rad s}^{-2}$
 (3) 2 rad s^{-2} (4) 1 rad s^{-2}
26. What is the ratio of the mass of the cylinder to the mass of block B ?
- (1) 1 (2) 2
 (3) 3 (4) 4
27. If unwrapped length of the thread between the cylinder and block B is 20 m at the beginning, when the system was released from rest, what would it be 2 s later?
- (1) 28 m (2) 30 m
 (3) 22 m (4) 32.5 m

For Problems 28–30

Four identical rods of mass $M = 6 \text{ kg}$ each are welded at their ends to form a square and then welded to a massive ring having mass $m = 4 \text{ kg}$ and radius $R = 1 \text{ m}$. If the system is allowed to roll down the incline of inclination $\theta = 30^\circ$, determine the minimum value of the coefficient of static friction that will prevent slipping.



28. The moment of inertia of the system about the centre of ring will be
- (1) 20 kg m^2 (2) 40 kg m^2
 (3) 10 kg m^2 (4) 60 kg m^2
29. The acceleration of the system will be
- (1) $\frac{g}{2}$ (2) $\frac{g}{4}$
 (3) $\frac{7g}{24}$ (4) $\frac{g}{8}$
30. The minimum value of friction coefficient to prevent slipping is
- (1) $\frac{5}{7}$ (2) $\frac{5}{12\sqrt{3}}$
 (3) $\frac{5\sqrt{3}}{7}$ (4) $\frac{7}{5\sqrt{3}}$

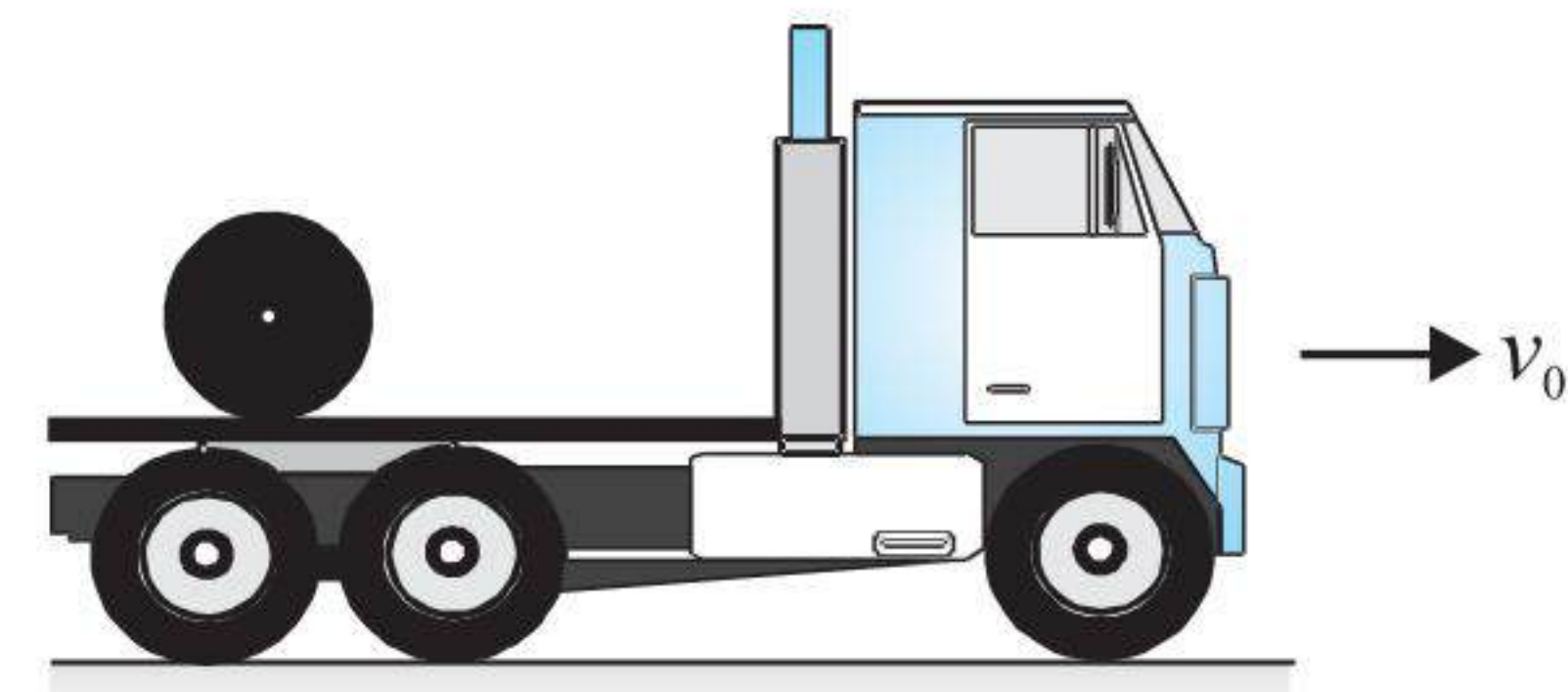
For Problems 31–33

A man of mass 100 kg stands at the rim of a turntable of radius 2 m and moment of inertia 4000 kg m^2 mounted on a vertical frictionless shaft at its centre. The whole system is initially at rest. The man now walks along the outer edge of the turntable (anticlockwise) with a velocity of 1 m/s relative to the earth.

31. With what angular velocity and in what direction does the turntable rotate?
- (1) The table rotates anticlockwise (in the direction of the man motion) with angular velocity 0.05 rad/s .
 (2) The table rotates clockwise (opposite to the man) with angular velocity 0.1 rad/s .
 (3) The table rotates clockwise (opposite to the man) with angular velocity 0.05 rad/s .
 (4) The table rotates anticlockwise (in the direction of the man motion) with angular velocity 0.1 rad/s .
32. Through what angle will it have rotated when the man reaches his initial position on the turntable?
- (1) The table rotates through $2\pi/11$ radians anticlockwise
 (2) The table rotates through $4\pi/11$ radians clockwise
 (3) The table rotates through $4\pi/11$ radians anticlockwise
 (4) The table rotates through $2\pi/11$ radians clockwise
33. Through what angle will it have rotated when the man reaches his initial position relative to earth?
- (1) 36° in clockwise direction
 (2) 36° in anticlockwise direction
 (3) 72° in clockwise direction
 (4) 72° in anticlockwise direction

For Problems 34–35

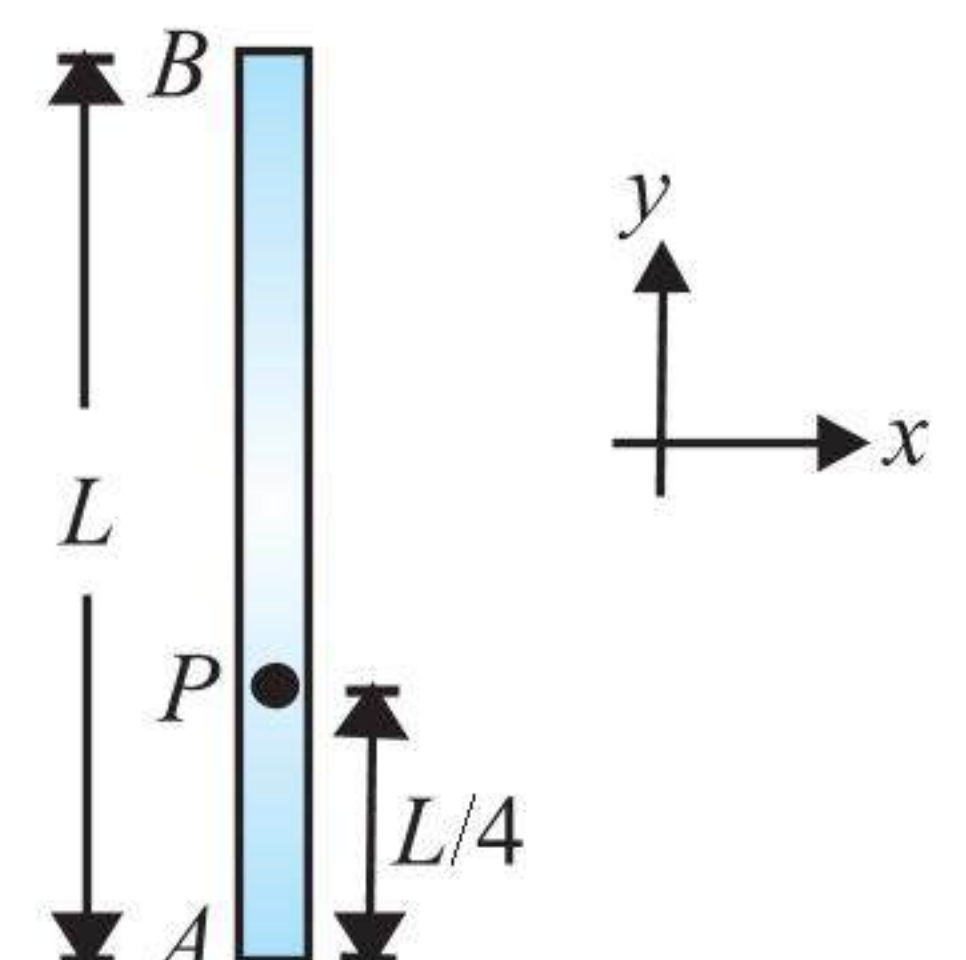
A solid sphere of mass M and radius R is initially at rest. Solid sphere is gradually lowered onto a truck moving with constant velocity v_0 .



34. What is the final speed of the sphere's centre of mass in ground frame when eventually pure rolling sets in
- (1) $\frac{5}{7} v_0$ (2) $\frac{2}{7} v_0$
 (3) $\frac{7}{5} v_0$ (4) $\frac{7}{2} v_0$
35. If a sphere with twice the radius and four times the mass had been used; what would have been its final speed?
- (1) $\frac{5}{7} v_0$ (2) $\frac{2}{7} v_0$
 (3) $\frac{7}{5} v_0$ (4) $\frac{7}{2} v_0$

For Problems 36–38

A uniform rod AB hinged about a fixed point P is initially vertical. A rod is released from vertical position. When rod is in horizontal position.



36. The acceleration of the centre of mass of the rod is

- (1) $-\frac{6g}{7}\hat{i} - \frac{3g}{7}\hat{j}$ (2) $-\frac{12g}{7}\hat{i} - \frac{6g}{7}\hat{j}$
 (3) $-\frac{3g}{7}\hat{i} - \frac{9g}{7}\hat{j}$ (4) $-\frac{9g}{7}\hat{i} - \frac{3g}{7}\hat{j}$

37. The acceleration of end B of the rod is

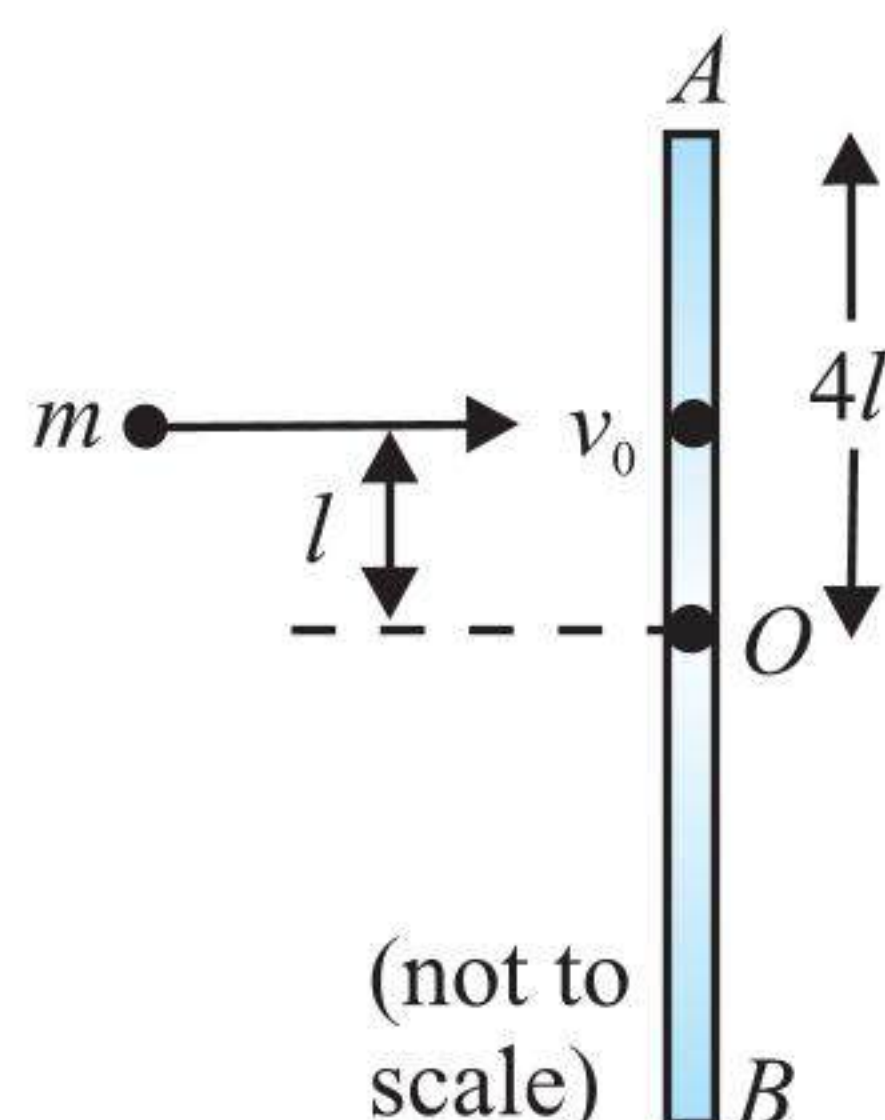
- (1) $-\frac{6g}{7}\hat{i} - \frac{12g}{7}\hat{j}$ (2) $-\frac{5g}{7}\hat{i} - \frac{9g}{7}\hat{j}$
 (3) $-\frac{18g}{7}\hat{i} - \frac{5g}{7}\hat{j}$ (4) $-\frac{18g}{7}\hat{i} - \frac{9g}{7}\hat{j}$

38. The reaction at hinge P is

- (1) $-\frac{8mg}{7}\hat{i} - \frac{12mg}{7}\hat{j}$ (2) $-\frac{3mg}{7}\hat{i} - \frac{9mg}{7}\hat{j}$
 (3) $-\frac{6mg}{7}\hat{i} + \frac{4mg}{7}\hat{j}$ (4) $-\frac{12mg}{7}\hat{i} - \frac{6mg}{7}\hat{j}$

For Problems 39–42

A rod AB of mass M and length $8l$ lies on a smooth horizontal surface. A particle of mass ' m ' and velocity v_0 strikes the rod perpendicular to its length, as shown in figure. As a result of the collision, the centre of mass of rod attains a speed of $v_0/8$ and the particle rebounds back with a speed of $v_0/4$. Find the following.



39. The ratio $\frac{M}{m}$,

- (1) $\frac{M}{m} = 10$ (2) $\frac{M}{m} = 4$
 (3) $\frac{M}{m} = 8$ (4) $\frac{M}{m} = 5$

40. The angular velocity of the rod about O,

- (1) $\frac{5v_0}{128l}$ (2) $\frac{3v_0}{128l}$
 (3) $\frac{v_0}{128l}$ (4) $\frac{7v_0}{128l}$

41. The coefficient of restitution ' e ' for the collision,

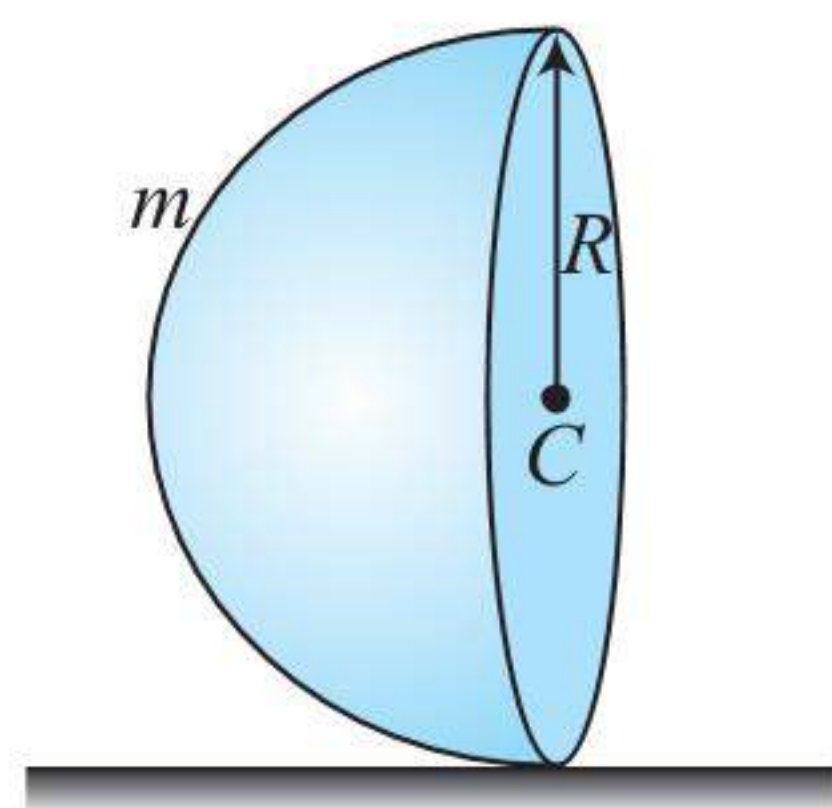
- (1) $\frac{51}{128}$ (2) $\frac{61}{128}$
 (3) $\frac{21}{128}$ (4) $\frac{31}{128}$

42. The velocities of the ends 'A' and 'B' of the rod, namely v_A and v_B , respectively.

- (1) $v_A = \frac{5}{32}v_0$; $v_B = \frac{1}{32}v_0$ (2) $v_A = \frac{7}{32}v_0$; $v_B = \frac{3}{32}v_0$
 (3) $v_A = \frac{7}{32}v_0$; $v_B = \frac{1}{32}v_0$ (4) $v_A = \frac{5}{32}v_0$; $v_B = \frac{3}{32}v_0$

For Problems 43–44

Figure shows a hemispherical bowl of uniform mass density placed on a rough horizontal surface. Mass of the bowl is ' m ' and its radius is ' R '. Initially the geometrical centre ' C ' of the hemispherical bowl is vertically above the point of contact with the ground. Now the bowl is released from rest in the position shown. On the basis of above information, answer the following questions.



43. The minimum coefficient of static friction $\mu_{\min}$ between hemispherical bowl and surface to prevent initial slipping of the bowl, is

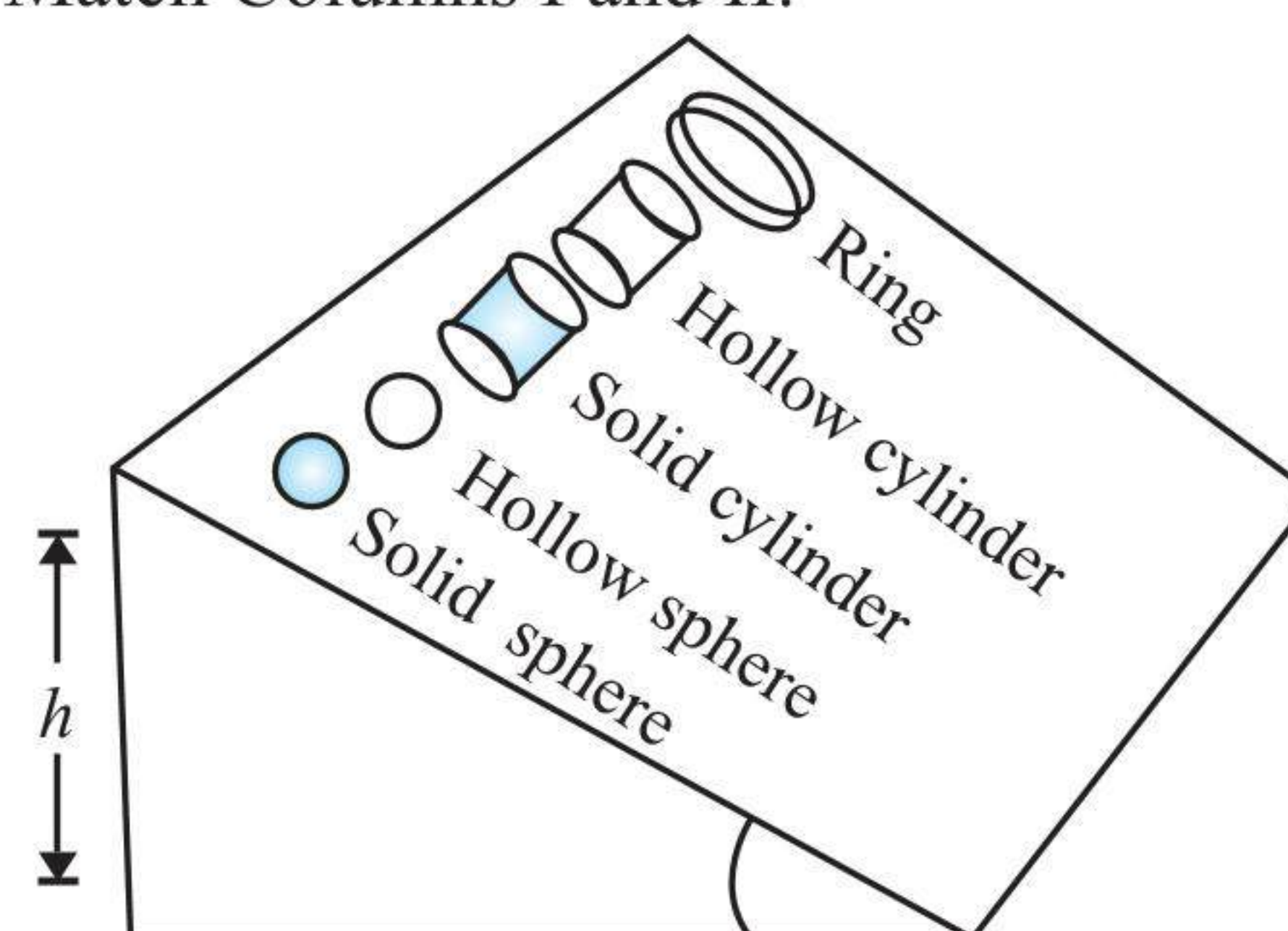
- (1) $\frac{3}{17}$ (2) $\frac{6}{17}$
 (3) 2 (4) $\frac{1}{2}$

44. If coefficient of static friction $\mu_s = \frac{\mu_{\min}}{2}$, then the normal force acting on the bowl, is

- (1) $\frac{85}{118}mg$ (2) $\frac{85}{59}mg$
 (3) $\frac{17}{59}mg$ (4) $\frac{17}{118}mg$

Matrix Match Type

1. A solid sphere, a thin-walled hollow sphere, a solid cylinder, a thin-walled hollow cylinder and a ring, each of mass m and radius R , are simultaneously released at rest from the top of an inclined plane, as shown in figure. The objects roll down the plane without slipping. Also we may consider the objects and the surface on which they roll to be perfectly rigid. Match Columns I and II.

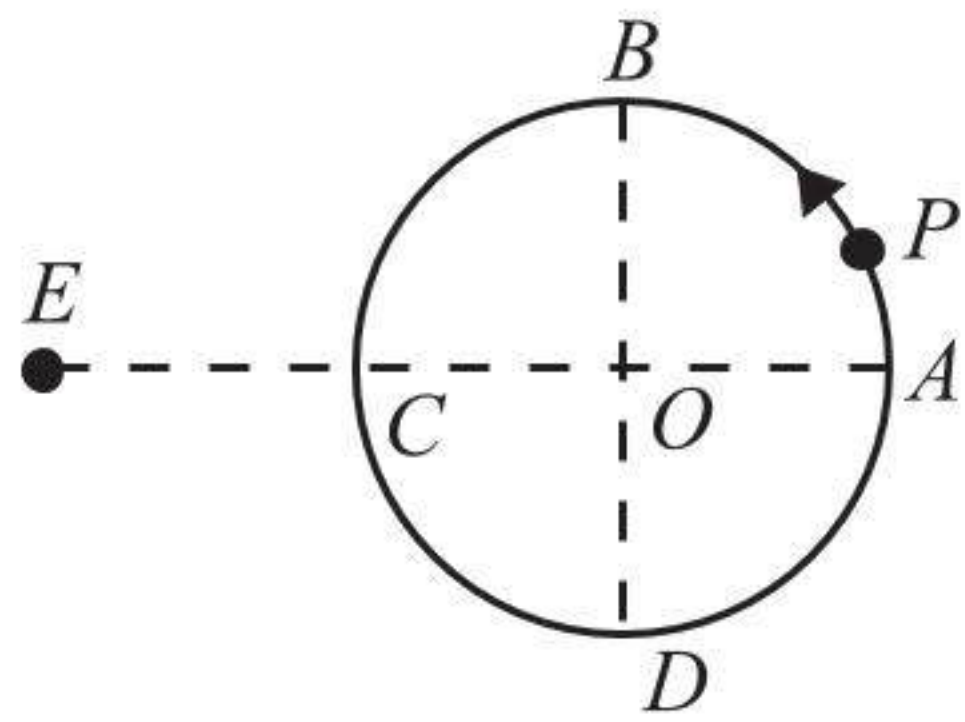


Column I	Column II
i. Time taken to reach the bottom is maximum for	a. Solid sphere
ii. Angular acceleration is maximum for	b. Hollow cylinder
iii. Kinetic energy at the bottom is the same for	c. Hollow sphere
iv. Rotational kinetic energy is maximum for	d. Ring

2. Column I gives the bodies which are released one by one from the top of an inclined plane. Assume each body rolls purely.

Column I	Column II
i. Wooden solid cylinder	a. KE_{rot} is maximum at the bottom
ii. Iron solid cylinder	b. KE_{trans} is minimum at the bottom
iii. Thin iron cylindrical shell	c. Takes minimum time to reach the bottom
iv. Thin wooden cylindrical shell	d. Takes maximum time to reach the bottom

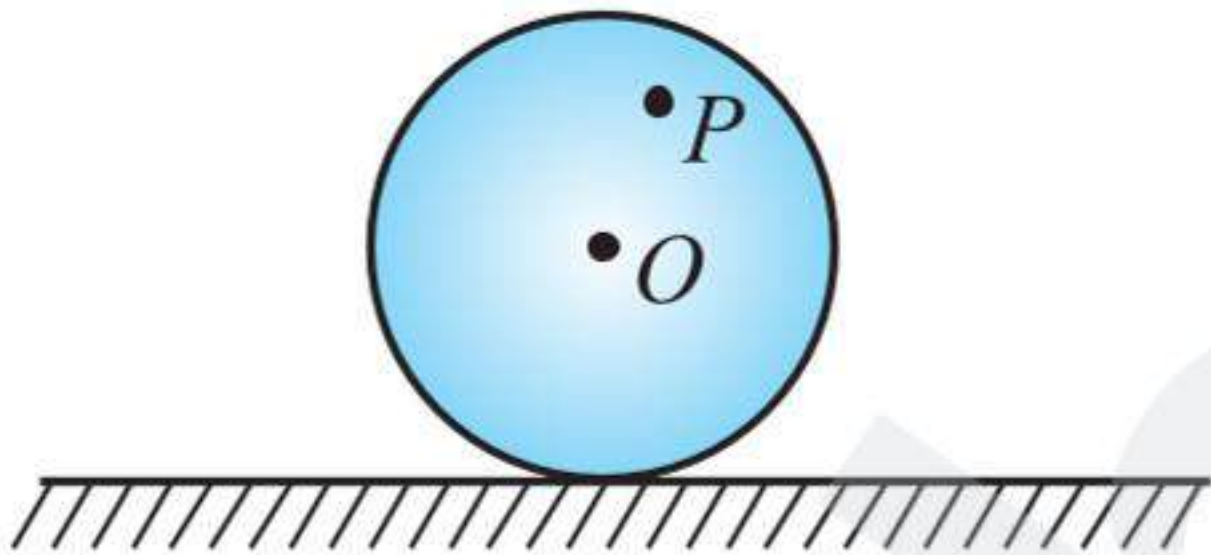
3. A particle P moves with constant speed on a circle in anticlockwise direction as shown in figure.



Match Column I with Column II:

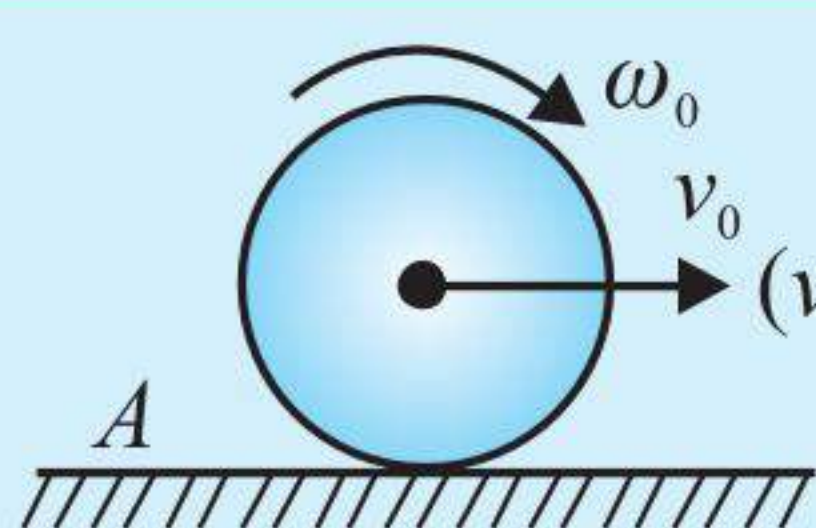
Column I	Column II
i. Angular momentum of the particle about O	a. is minimum when the particle is at A
ii. Angular momentum of the particle about E	b. is maximum when the particle is at A
iii. Angular velocity of the particle about O	c. does not remain constant
iv. Angular velocity of the particle about E	d. remains constant

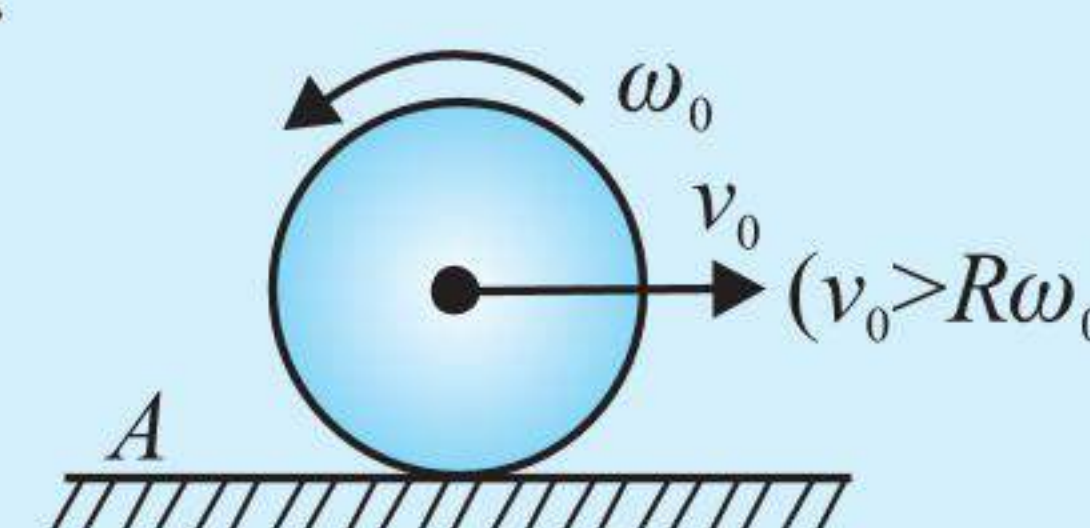
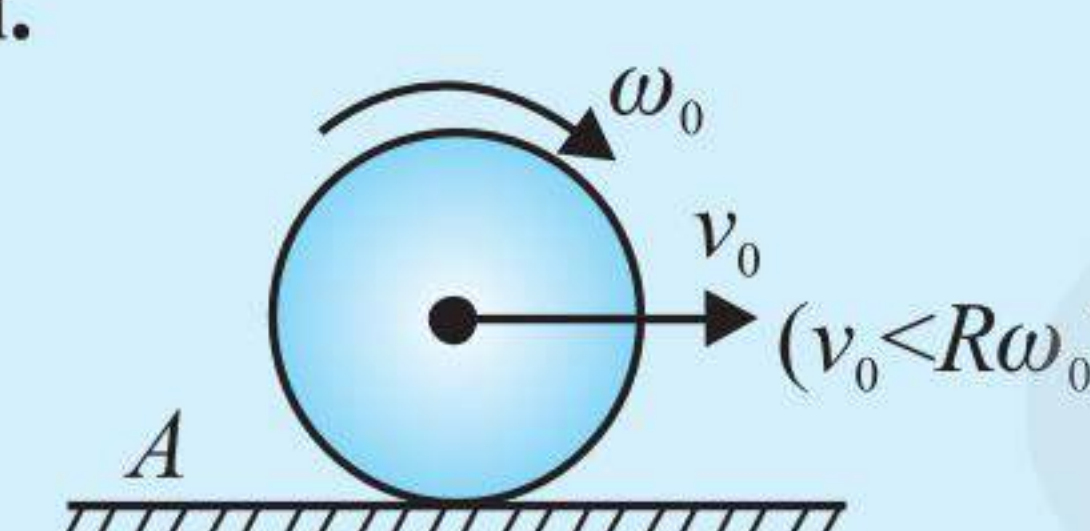
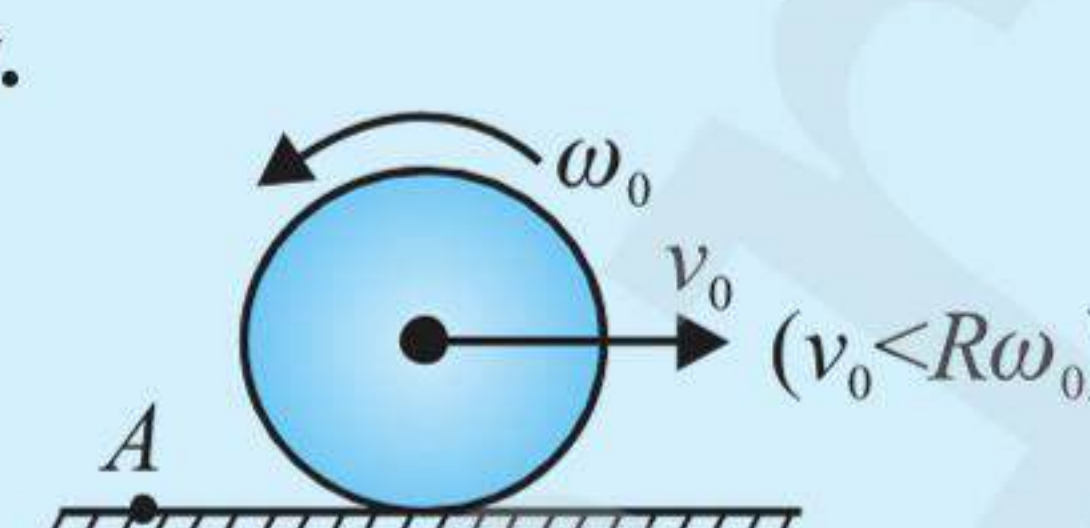
4. A uniform disc rolls without slipping on a rough horizontal surface with uniform angular velocity. Point O is the centre of disc and P is a point on disc as shown in figure. In each situation of Column I a statement is given and the corresponding results are given in Column II. Match the statements in Column I with the results in Column II.



Column I	Column II
i. The velocity of point P on the disc	a. changes in magnitude with time
ii. The acceleration of point P on the disc	b. always directed from that point (the point on disc given in Column I) towards the centre of disc.
iii. The tangential acceleration of point P on the disc	c. is always zero
iv. The acceleration of the point on the disc which is in contact with rough horizontal surface	d. is non-zero and remains constant in magnitude

5. In each situation of Column I, a uniform disc of mass m and radius R rolls on a rough fixed horizontal surface as shown. At $t = 0$ (initially) the angular velocity of the disc is ω_0 and velocity of the centre of mass of the disc is v_0 (in horizontal direction). The relation between v_0 and ω_0 for each situation and also initial sense of rotation is given for each situation in Column I. Then match the statements in Column I with the corresponding results in Column II.

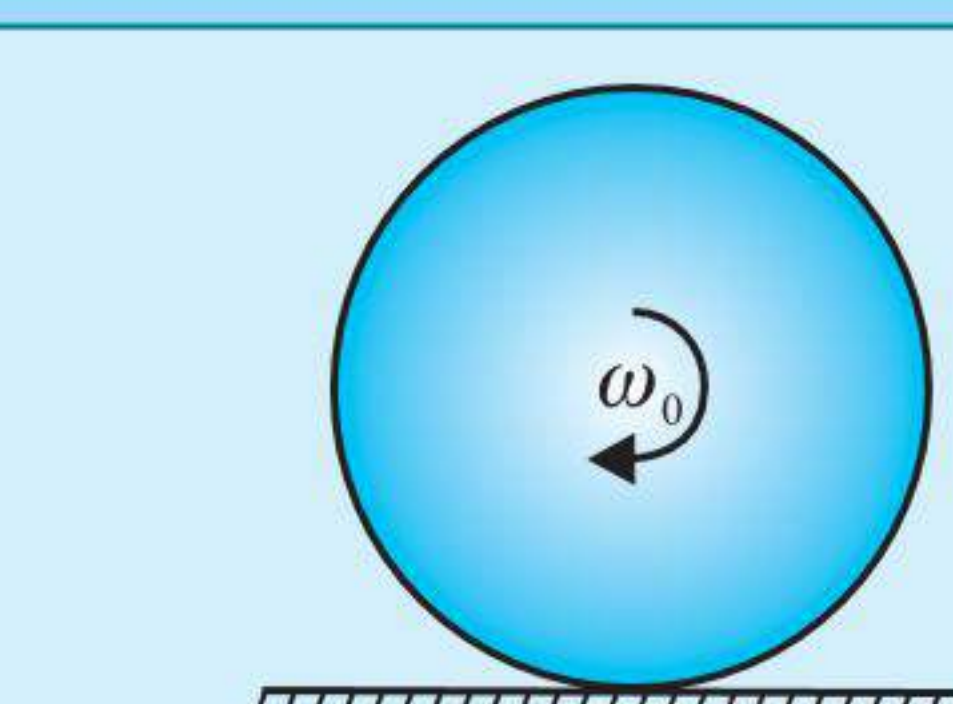
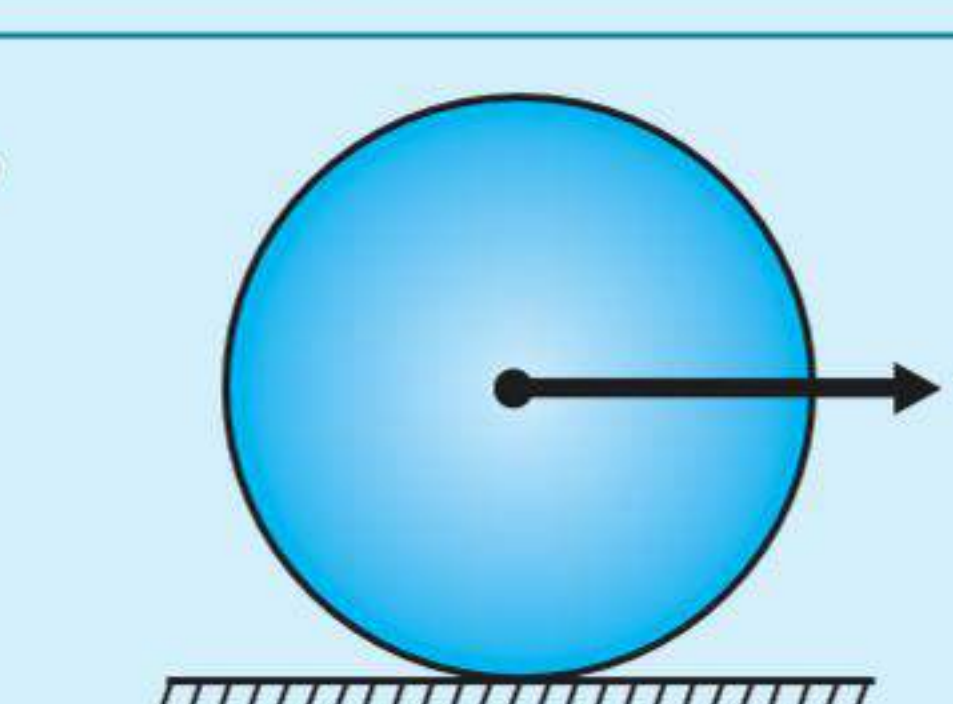
Column I	Column II
i. 	a. The angular momentum of the disc about point A (as shown in the figure) remains conserved.

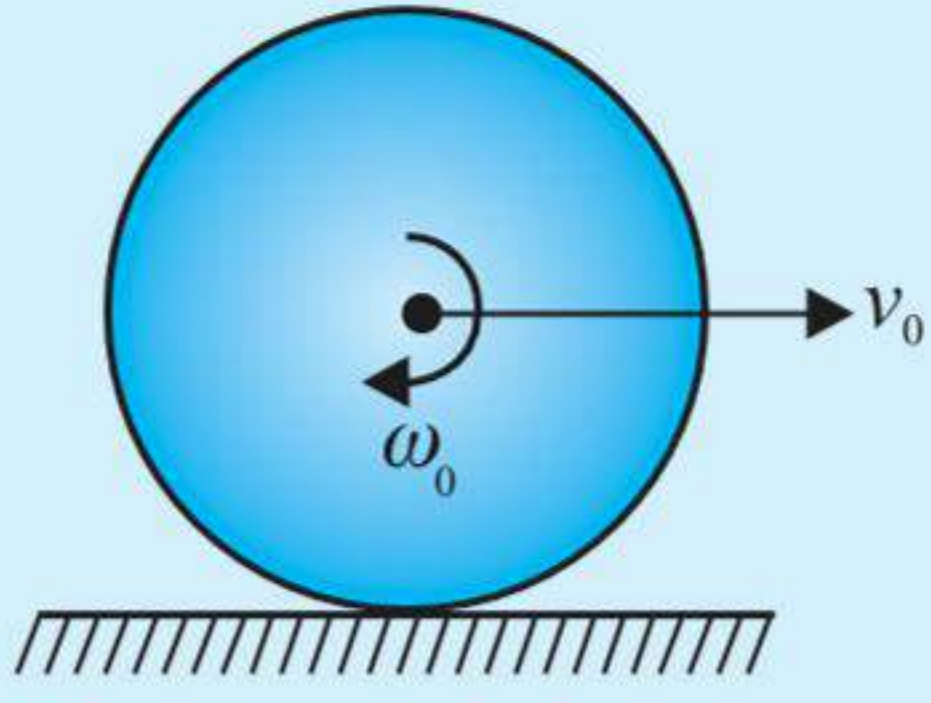
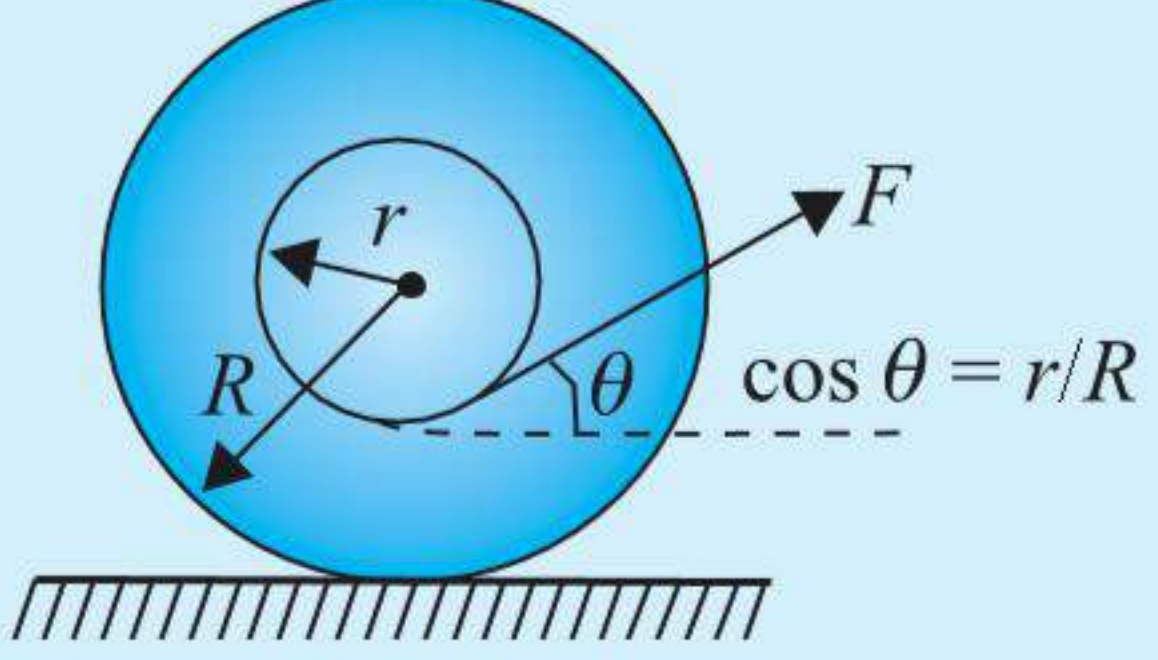
ii. 	b. The kinetic energy of the disc after it starts rolling without slipping is less than its initial kinetic energy.
iii. 	c. In the duration disc rolls with slipping, the friction acts on the disc towards the left.
iv. 	d. In the duration disc rolls with slipping, the friction acts on the disc for some time to the right and for some time to the left.

6. Match the statements in Column I with those in Column II. One or more matching is possible.

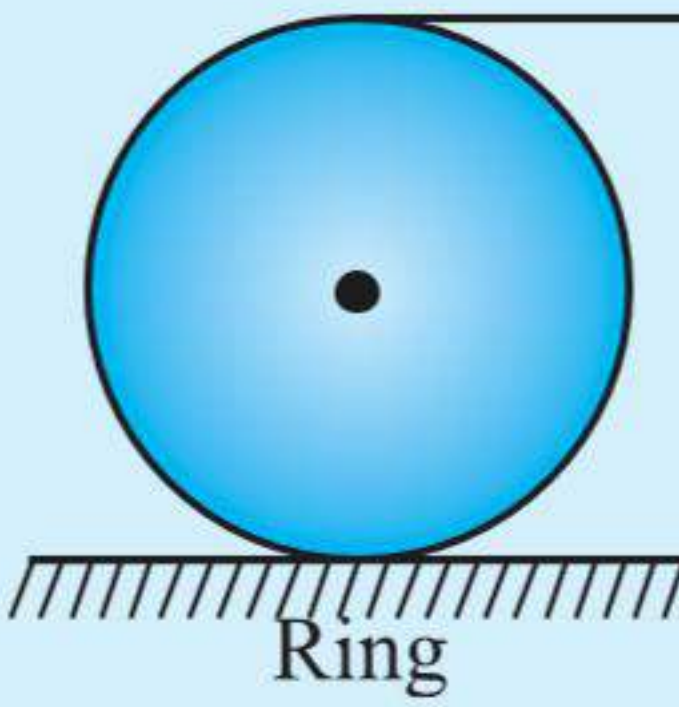
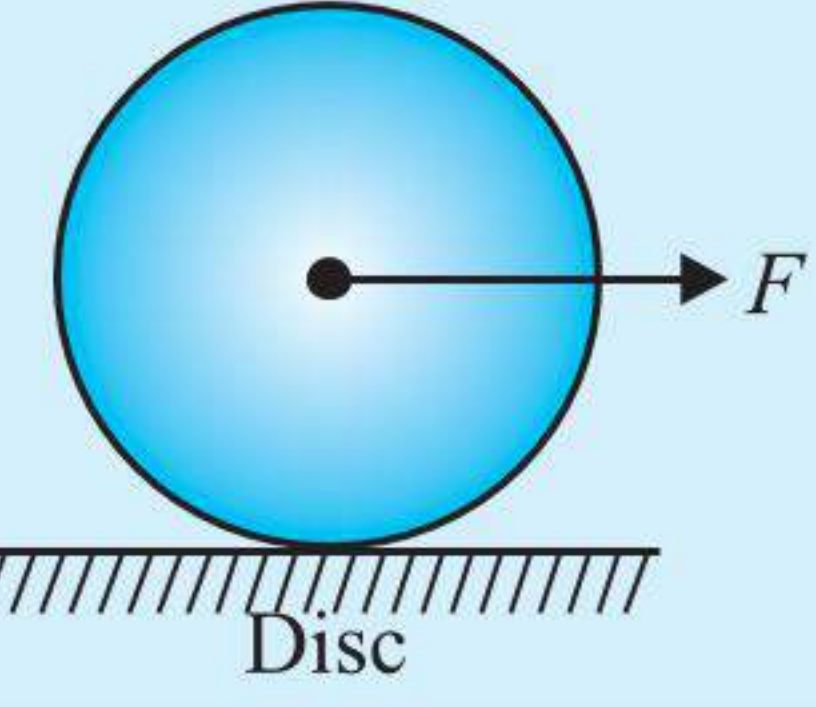
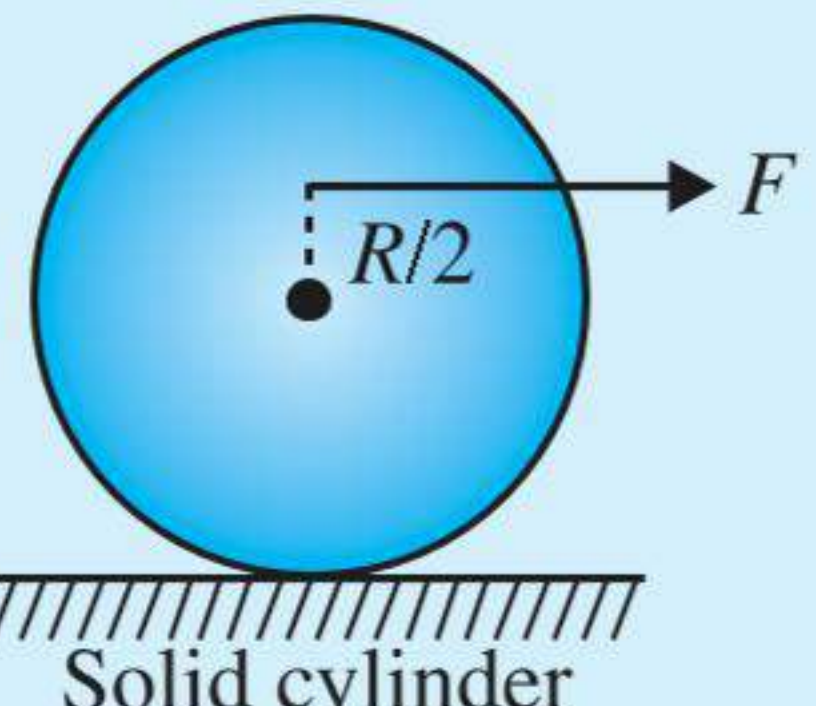
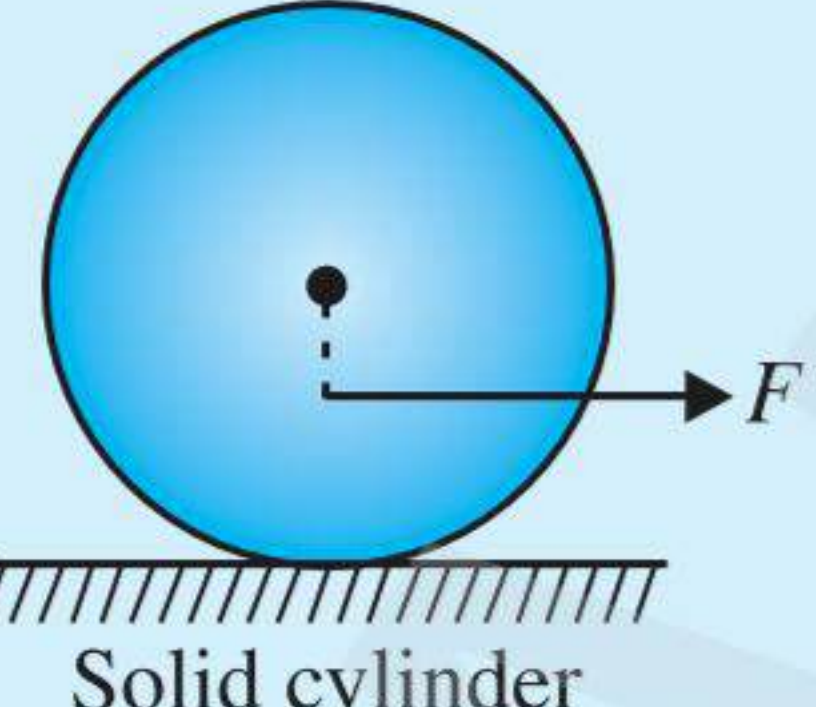
Column I	Column II
i. Solid sphere rolling with slipping on a rough horizontal surface	a. Total kinetic energy is conserved
ii. Solid sphere in pure rolling on a rough horizontal surface and no other force is acting	b. Angular momentum about the CM is conserved
iii. Solid sphere in pure rolling on a smooth horizontal surface	c. Angular momentum about a contact point on the contact surface is conserved
iv. Solid sphere in pure rolling on a rough incline	d. Momentum is conserved
	e. Total mechanical energy is conserved

7. Match the columns.

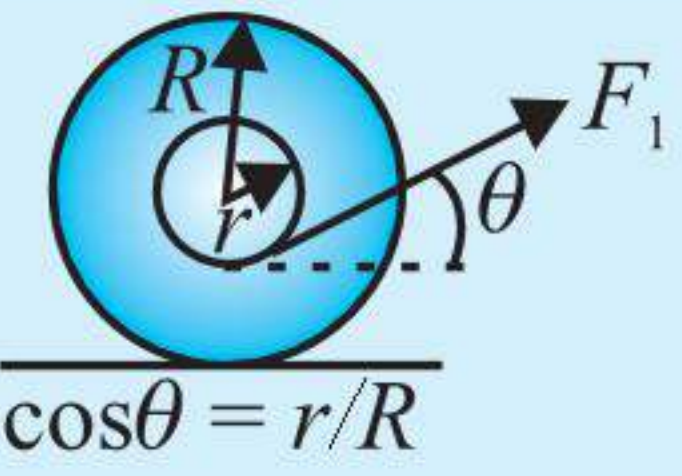
Column I	Column II
i.  A spinning ball lowered on a rough surface	a. Net work done by friction force is negative
ii.  A ball is projected with velocity v_0 on a rough surface	b. Angular momentum about the CM is conserved

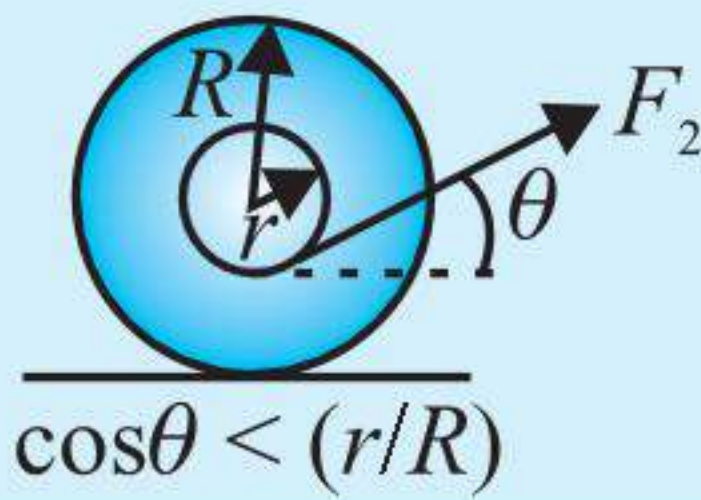
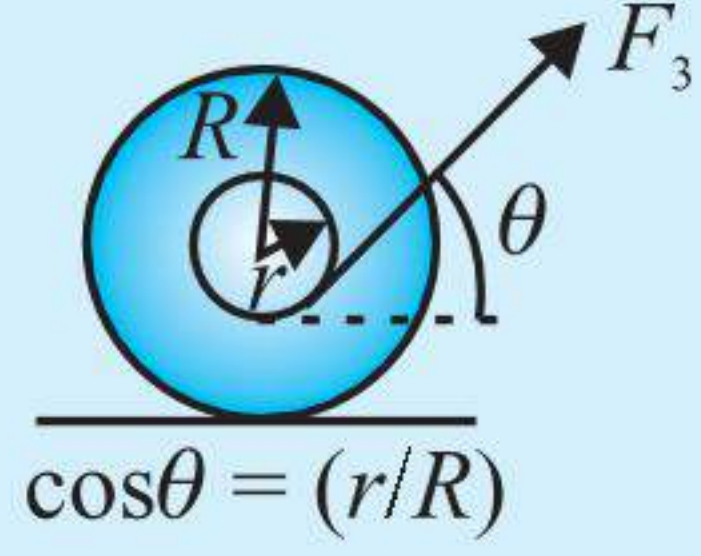
iii.  A ball projected with velocity $v_0 = R\omega_0$ projected on a smooth horizontal surface	c. Translational work done by friction is positive
iv.  A spool pulled by a force F , rolling without slipping on a rough surface	d. Net work done by friction is zero

8. Assume sufficient friction to prevent slipping.

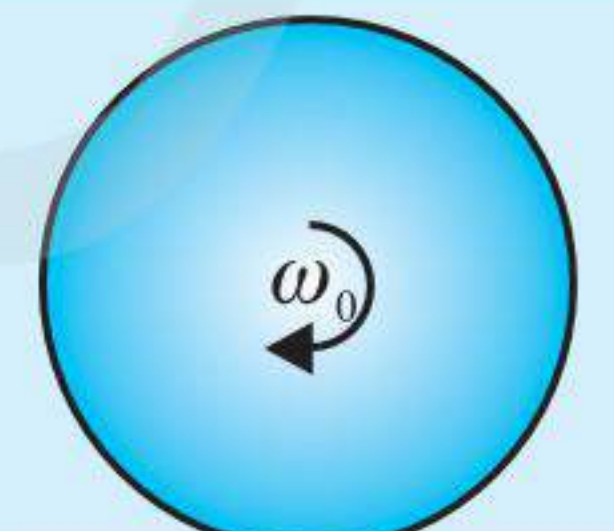
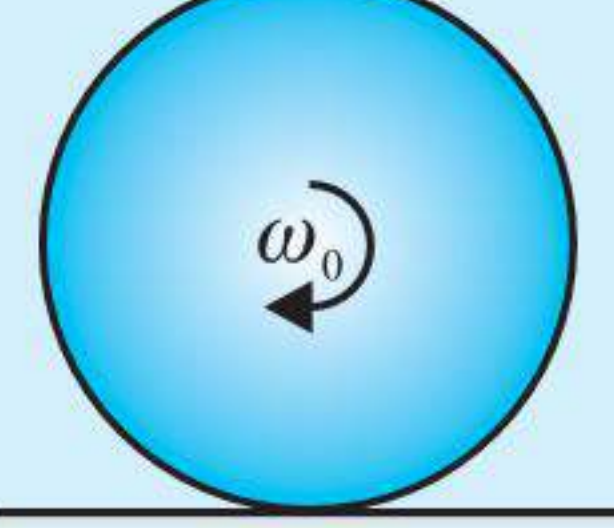
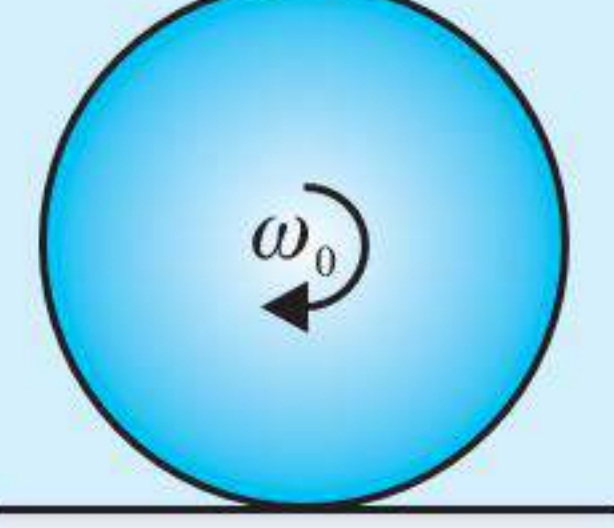
Column I	Column II
i.  Ring	a. Body accelerates forward
ii.  Disc	b. Rotation about the centre of mass is clockwise
iii.  Solid cylinder	c. Friction force acts backward
iv.  Solid cylinder	d. No friction acts

9. A uniform yo-yo (longer radius R , small radius r) is resting on a perfectly rough horizontal table. This equation should be attempted after forces F_1 , F_2 , F_3 and F_4 are applied separately as shown.

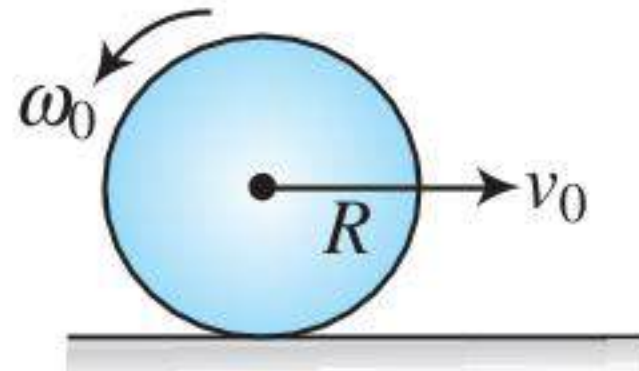
Column I	Column II
i.  $\cos \theta = r/R$	a. Centre of mass of the yo-yo accelerates towards the left

ii.  $\cos \theta < (r/R)$	b. Centre of mass of the yo-yo accelerates towards the right
iii.  $\cos \theta = (r/R)$	c. Friction acts on the yo-yo towards the left
	d. Friction acts on the yo-yo towards the right

10. Match the columns

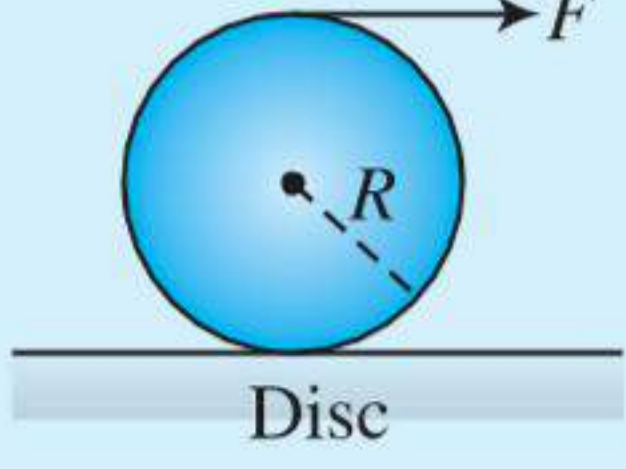
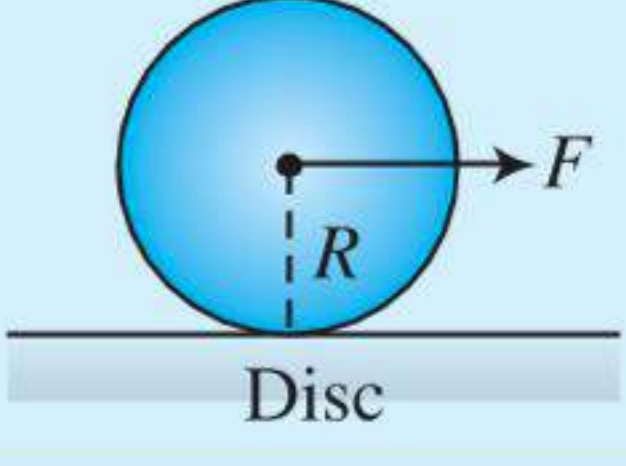
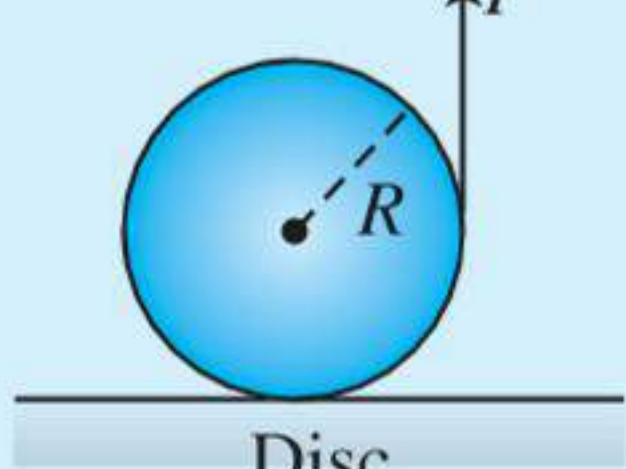
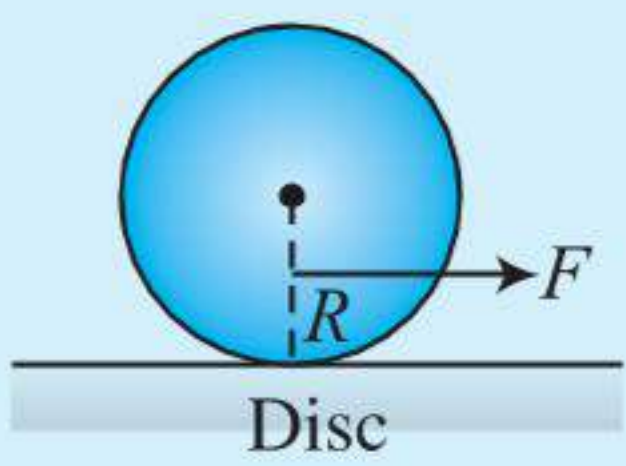
Column I	Column II
i.  Ring	a. Rotational work done by the friction is negative till pure rolling begins
ii.  Cylinder	b. Translational work done is positive till pure rolling starts
iii.  Solid sphere	c. When pure rolling begins velocity of centre of mass is minimum
iv.  Hollow sphere	d. Takes maximum time for pure rolling to begin

11. A solid cylinder of mass M and Radius R is given a linear velocity v_0 and angular velocity ω_0 as shown and then it is released gently on a rough horizontal surface. Match the columns appropriately.

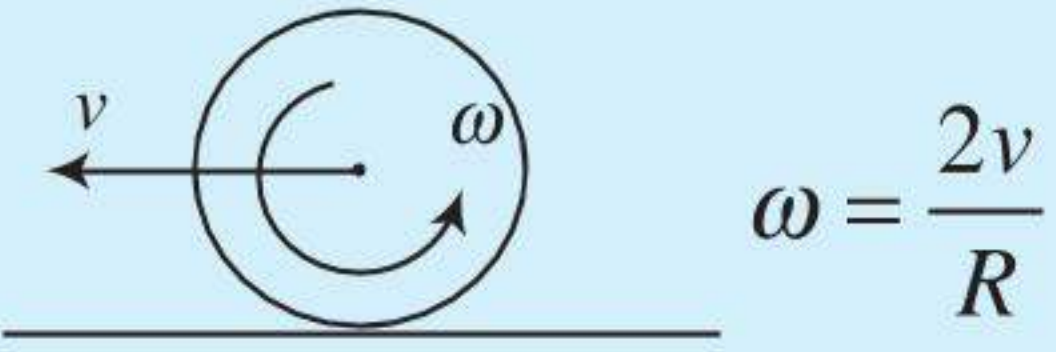
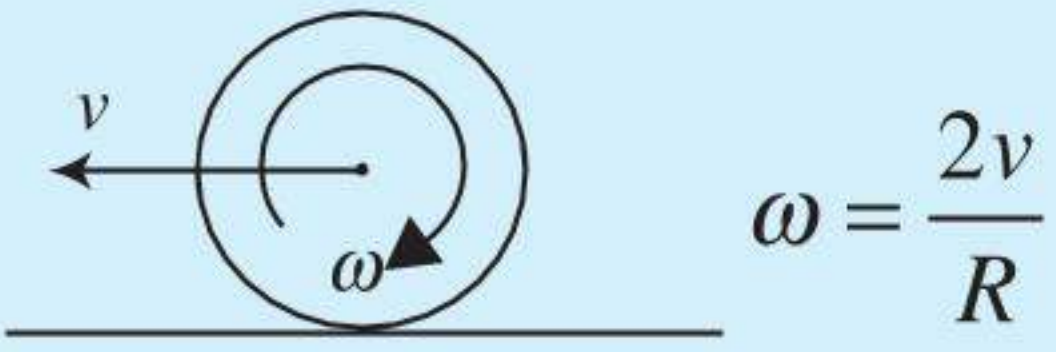
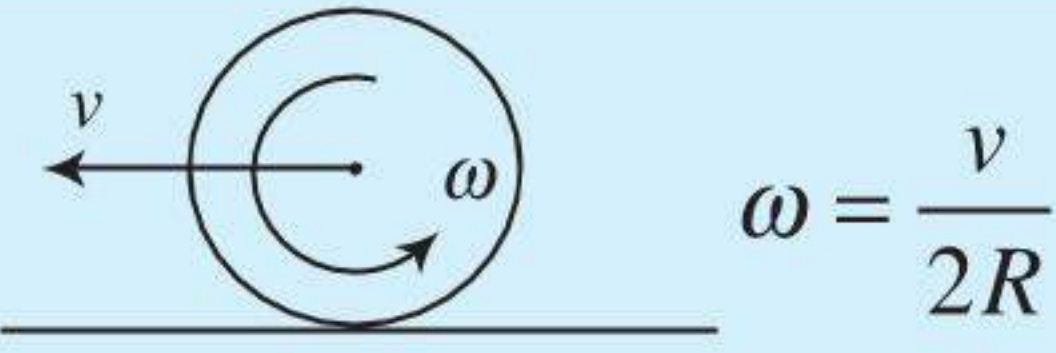
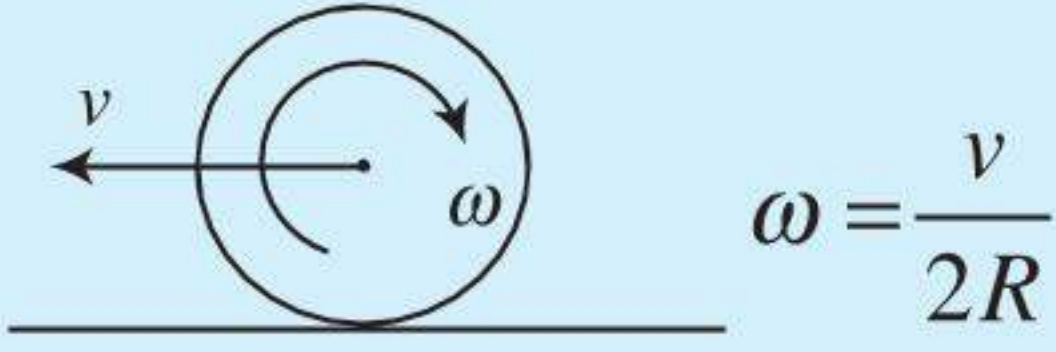


Column I	Column II
i. Finally cylinder moves in forward direction	a. may be possible
ii. Finally cylinder moves in backward direction	b. not possible at all
iii. Finally cylinder stops	c. is conserved about some point
iv. Angular momentum of cylinder	d. is not conserved about any point

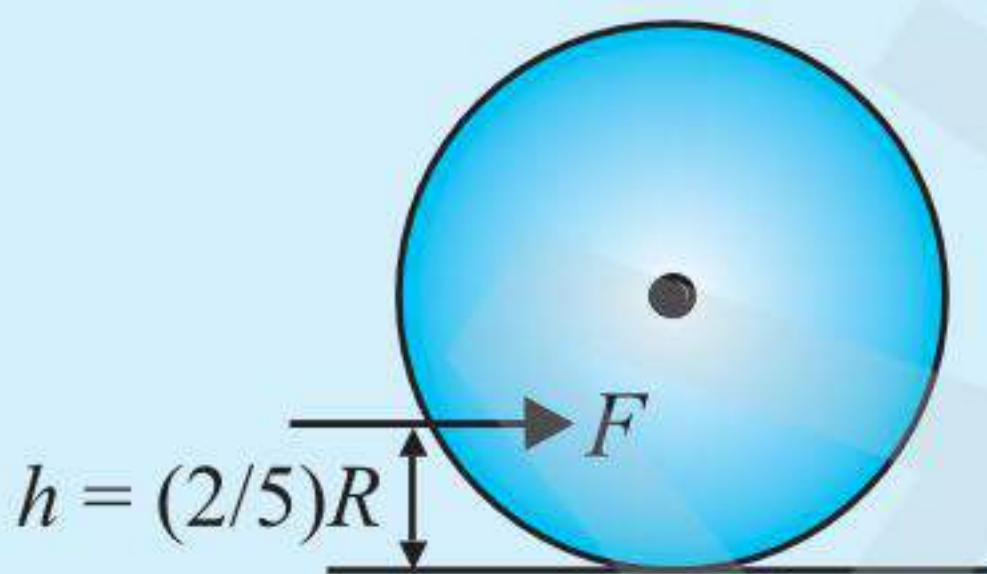
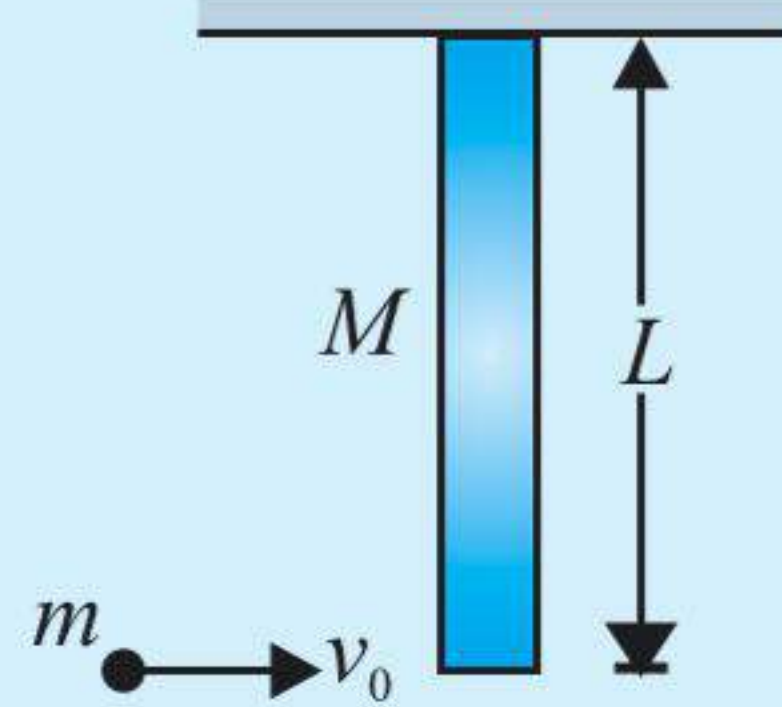
12. In each case, there is sufficient friction for regular rigid uniform body to undergo pure rolling on a rigid horizontal surface. Now match the column I and II

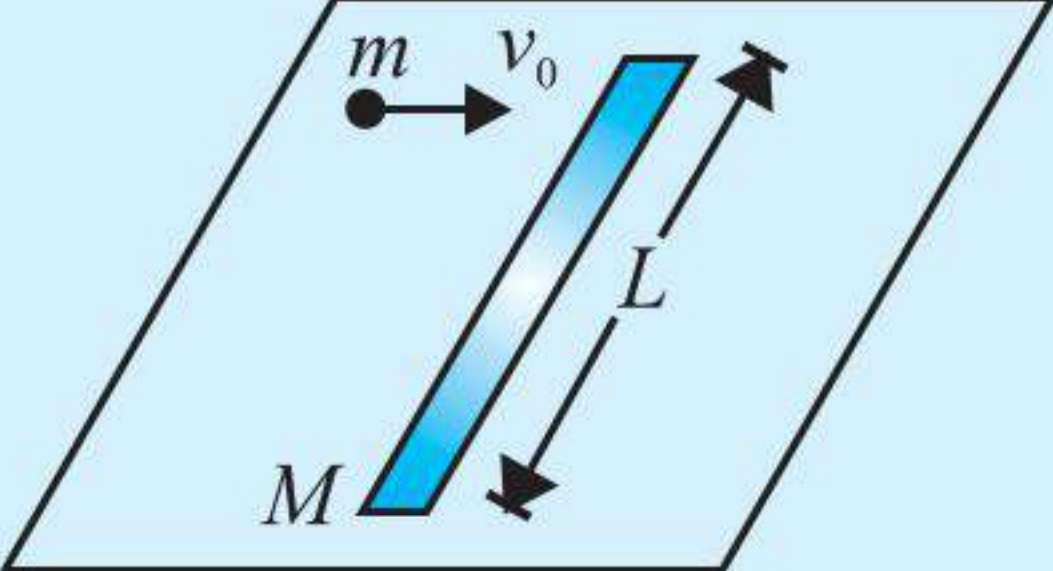
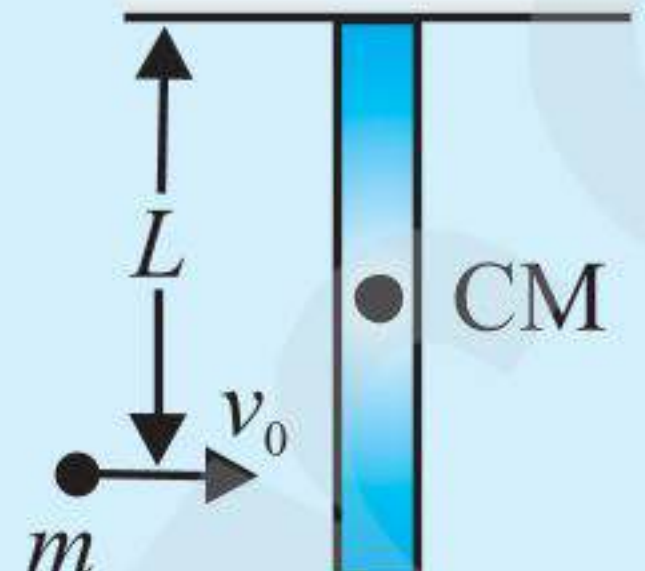
Column I	Column II
i. 	a. The direction of static friction on the disc is towards right.
ii. 	b. The direction of static friction on the disc is towards left.
iii. 	c. The angular acceleration will be clockwise
iv. 	d. Acceleration of the centre mass is in the direction of F.

13.

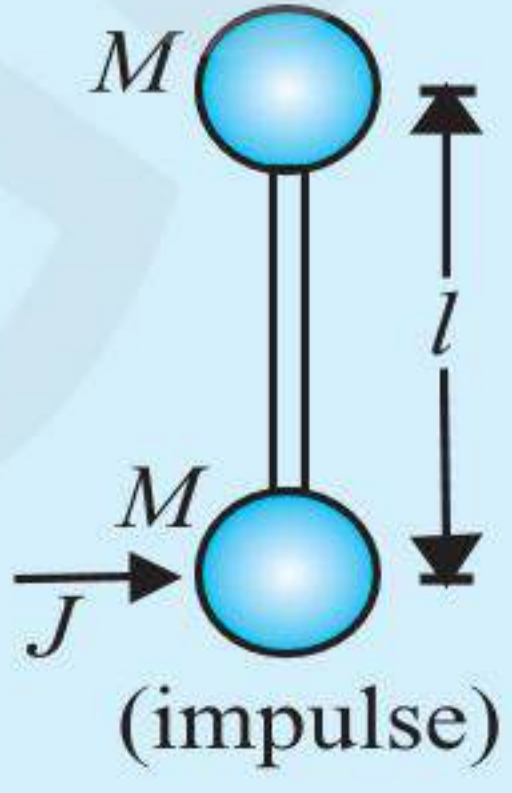
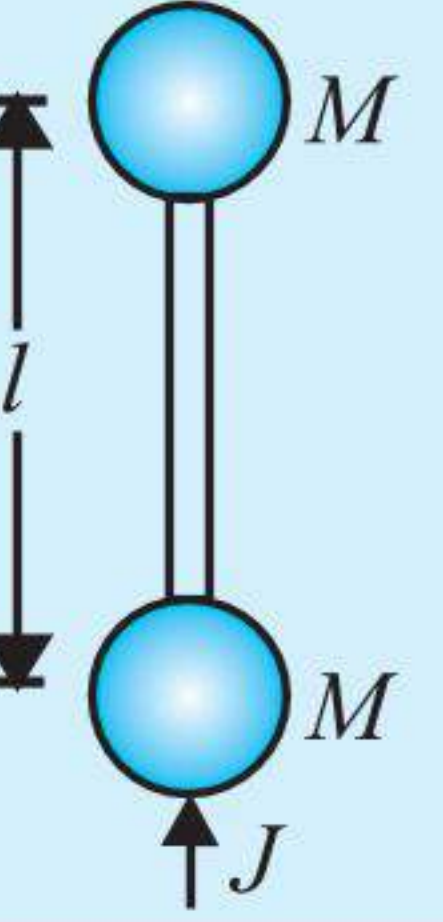
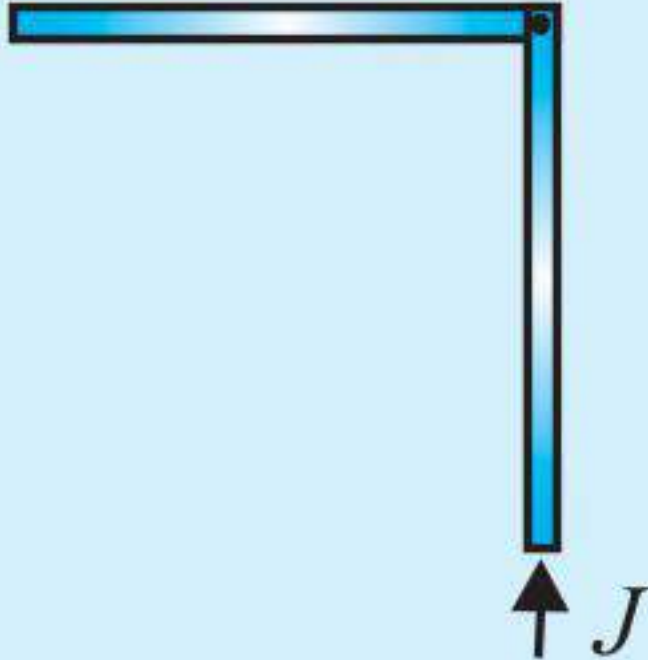
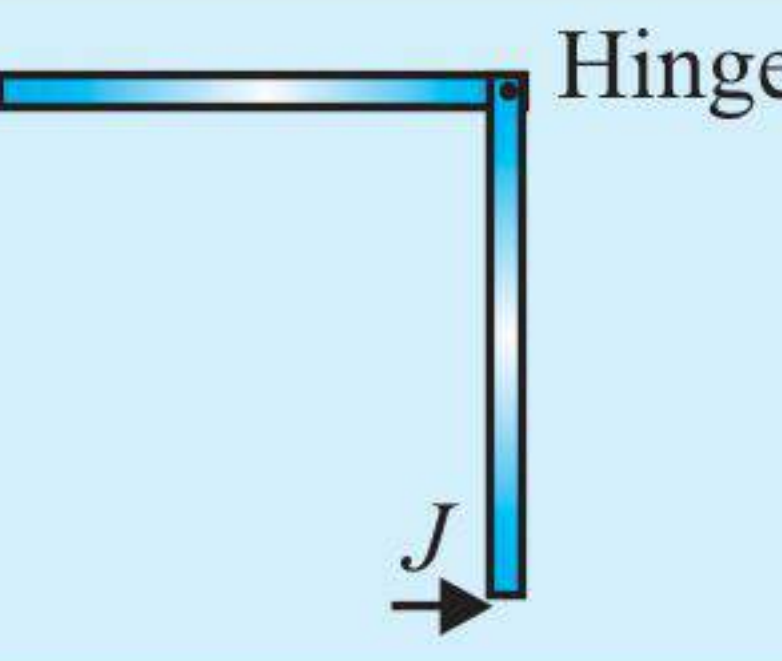
Column I (Initially)	Column II (When rolling without slipping begins)
i. 	a. v_{cm} is towards left in case of uniform ring
ii. 	b. v_{cm} is towards left in case of solid uniform sphere
iii. 	c. v_{cm} is towards right in case of uniform ring
iv. 	d. v_{cm} is towards right in case of solid uniform sphere

14. Match the entries in Column I with those in Column II.

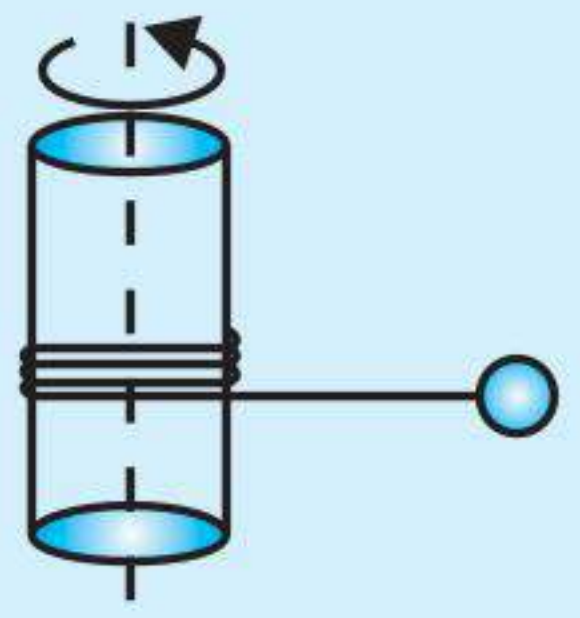
Column I	Column II
i.  An impulsive force acts at a height $h = (2/5)R$ on a solid sphere, lying on a rough surface.	a. Momentum is conserved
ii.  A small particle of mass m moving horizontally collides with a vertical hinged rod inelastically; $e = 0$.	b. Angular momentum is conserved

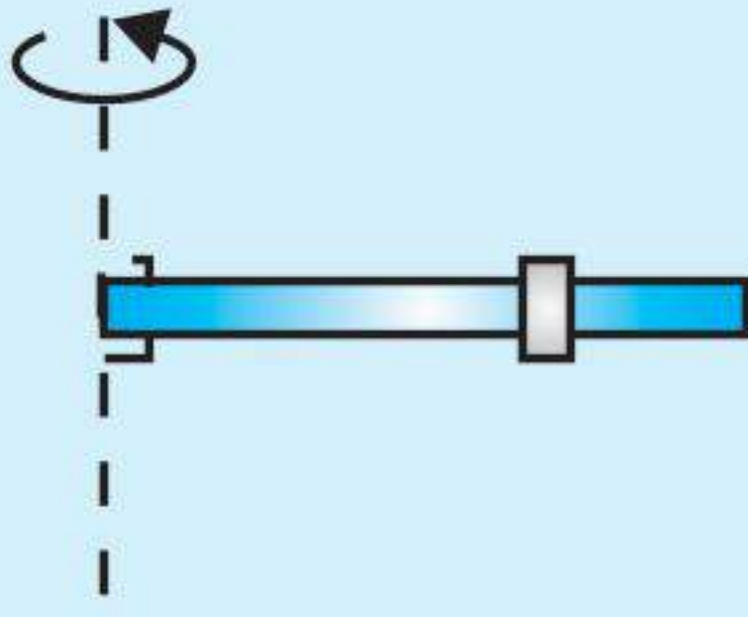
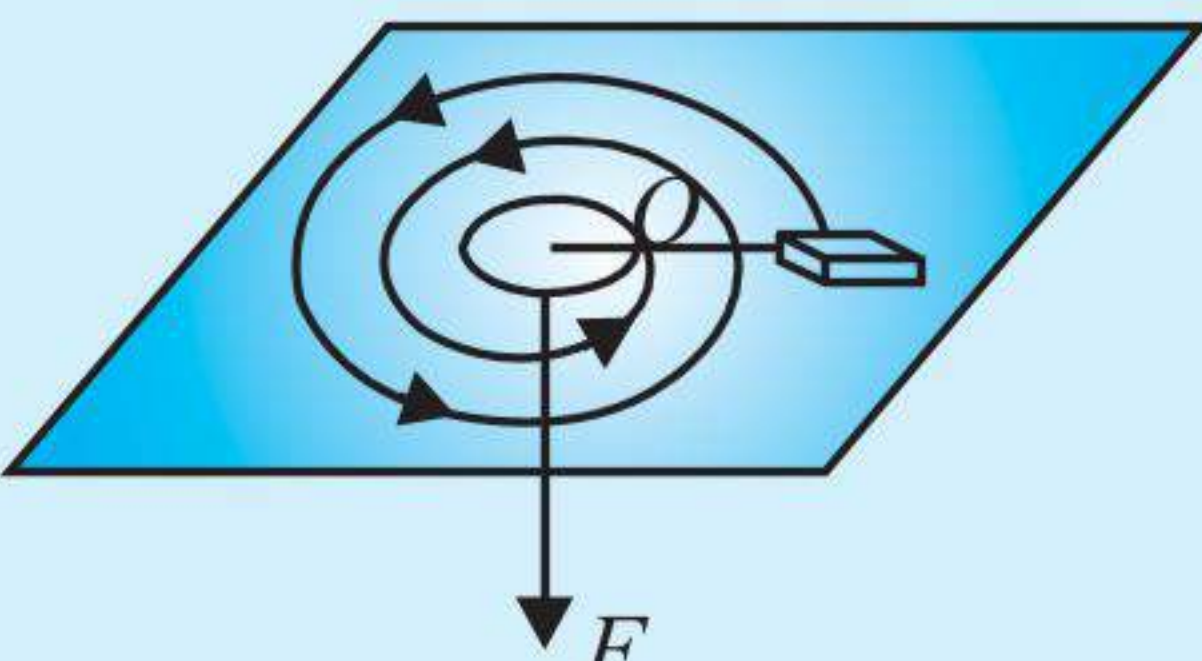
iii.  A small particle of mass m strikes elastically at the end of a horizontal rod kept on a smooth surface	c. Energy is conserved
iv.  A small particle of mass m collides with a vertical rod elastically	d. Momentum increases

15.

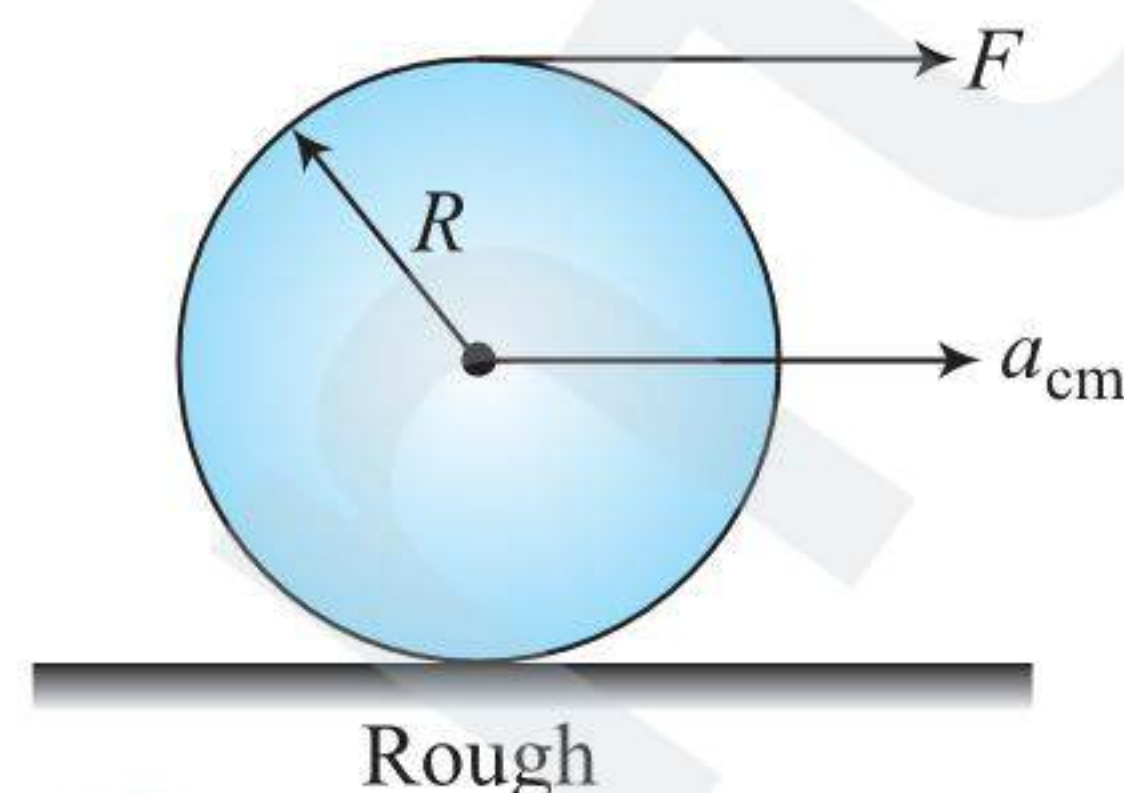
Column I	Column II
i.  Rod is massless, dumb-bell is placed on a smooth horizontal surface	a. Translation occurs
ii. 	b. Rotation occurs
iii. 	c. Angular momentum increases
iv. 	d. Linear momentum increases

16.

Column I	Column II
i.  A small particle of mass m is given an initial velocity in horizontal plane and winds its cord around the fixed vertical shaft of radius a .	a. Conservation of angular momentum

ii.  A smooth rod rotates with some angular velocity. A small sleeve starts sliding along the rod.	b. Conservation of kinetic energy
iii. Two ice-skaters approach each other at equal speeds along the parallel path separated by some distance. They link hands as they pass by and pull each other to reduce their separation.	c. Conservation of total mechanical energy
iv.  A small body tied to a non-stretchable thread of mass m is lying over a smooth horizontal plane. Other end of the thread is being drawn into a hole O.	d. Work is done by internal forces.

17. A uniform rigid body of mass m and radius R is placed at rest on a rough horizontal surface. A constant horizontal force F is applied at the topmost point of the body. The body starts rolling without slipping. Different shapes of bodies are given in the column I and based on this problem some physical quantities related to them are given in column II.



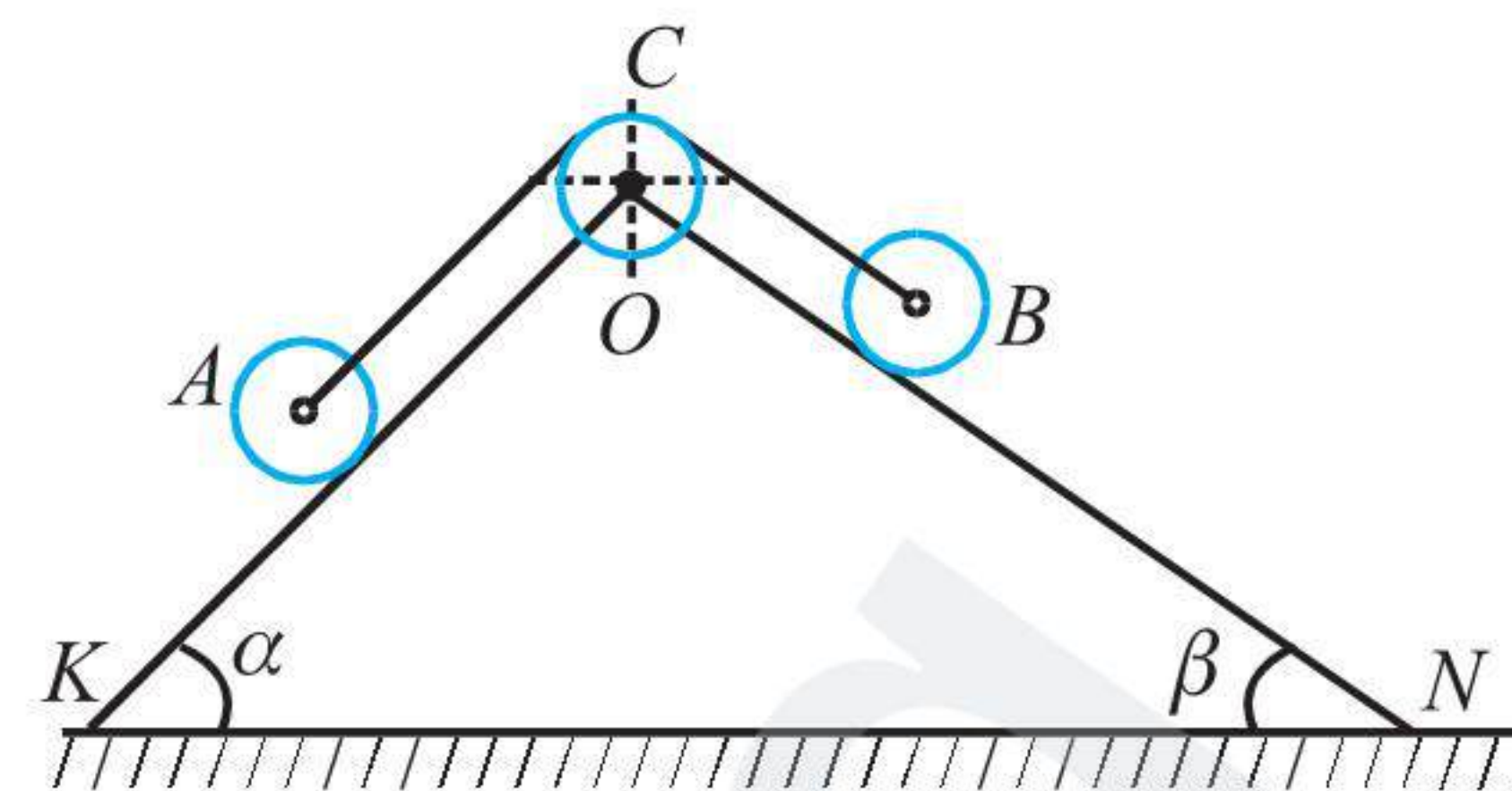
Column I	Column II
i. Solid sphere	a. Friction force is zero
ii. Ring	b. Magnitude of friction force is maximum
iii. Hollow sphere	c. Acceleration of the centre of mass is $4F/3m$
iv. Disc	d. Magnitude of friction force is $F/5$

- (1) i – b, ii – a, iii – c, iv – c
 (2) i – b, ii – a, iii – d, iv – c
 (3) i – d, ii – b, iii – a, iv – c
 (4) i – c, ii – a, iii – d, iv – b

Numerical Value Type

1. A wheel A is connected to a second wheel B by means of inextensible string, passing over a pulley C , which rotates about a fixed horizontal axle O , as shown in the figure. The system is released from rest. The wheel A rolls down

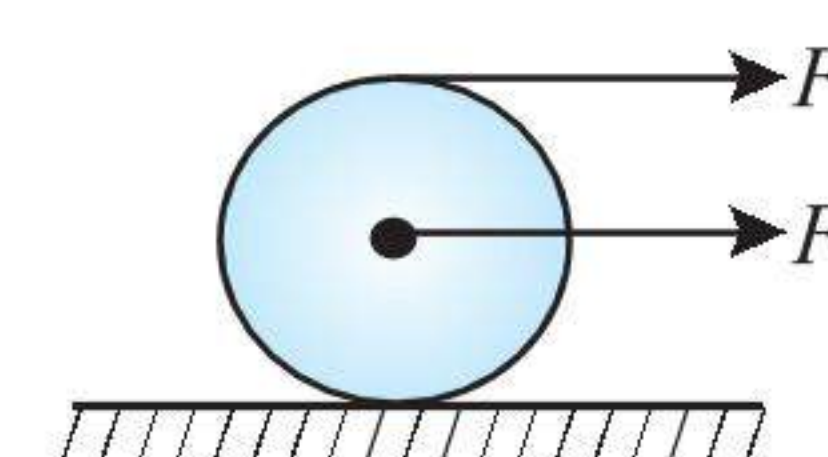
the inclined plane OK thus pulling up wheel B which rolls along the inclined plane ON .



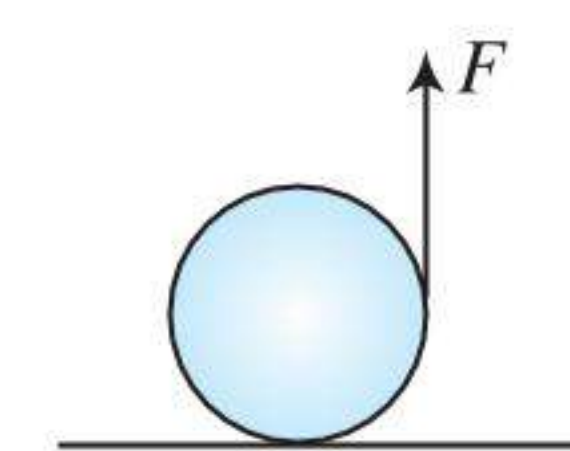
Determine the velocity (in m/s) of the axle of wheel A , when it has travelled a distance $s = 3.5$ m down the slope. Both wheels and the pulley are assumed to be homogeneous disks of identical weight and radius. Neglect the weight of the string. The string does not slip over C .

[Take $\alpha = 53^\circ$ and $\beta = 37^\circ$]

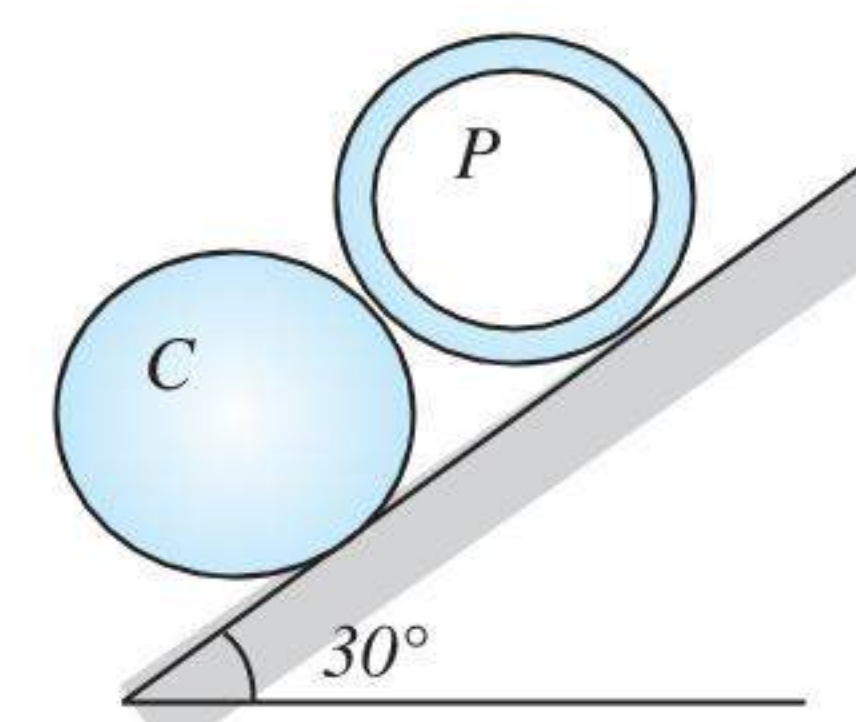
2. Two forces of magnitude F are acting on a uniform disc kept on a horizontal rough surface as shown in the figure. Friction force by the horizontal surface on the disc is nF . Find the value of n .



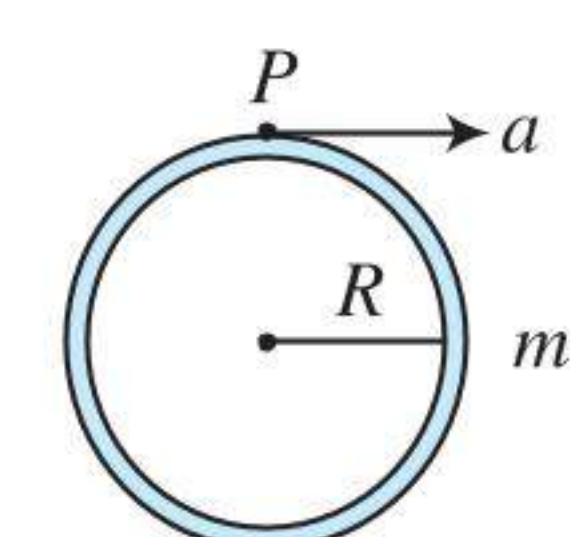
3. We apply a force of 10 N on a cord wrapped around a cylinder of mass 2 kg. The cylinder rolls without slipping on the floor. What is its kinetic energy (in joule) when cylinder has moved by a distance of $3/5$ m.



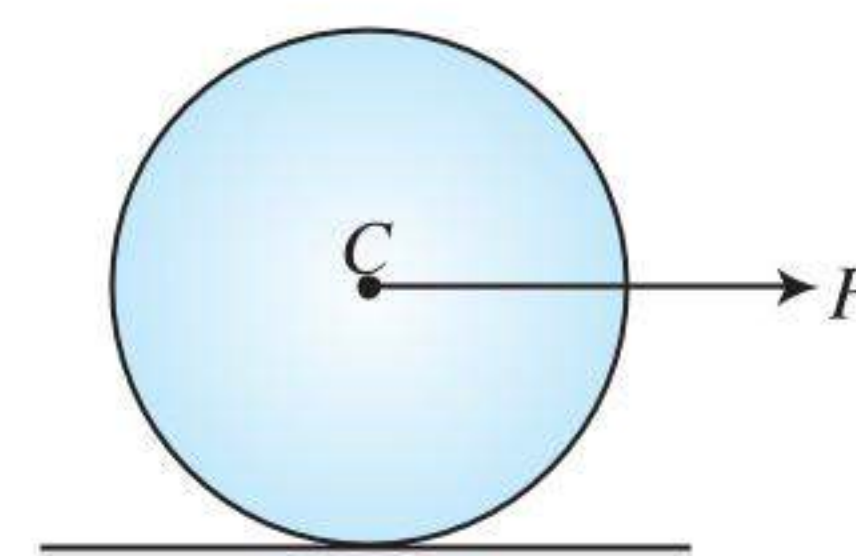
4. A solid cylinder C and a hollow pipe P of same diameter are in contact when they are released from rest as shown in the figure on a long incline plane. Cylinder C and pipe P roll without slipping. Determine the clear gap (in m) between them after $2\sqrt{3}$ seconds.



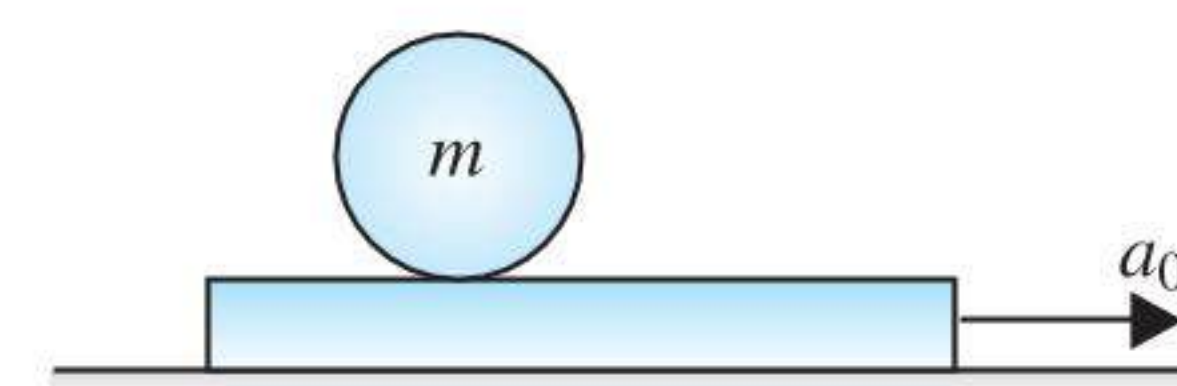
5. A smooth ring of mass m and radius $R = 1$ m is pulled at P with a constant acceleration $a = 4 \text{ m s}^{-2}$ on a horizontal surface such that the plane of the ring lies on the surface. Find the angular acceleration of the ring at the given position. (in rad/s^2)



6. A horizontal force $F = 14$ N acts at the centre of mass of a sphere of mass $m = 1$ kg. If the sphere rolls without sliding, find the frictional force (in N)



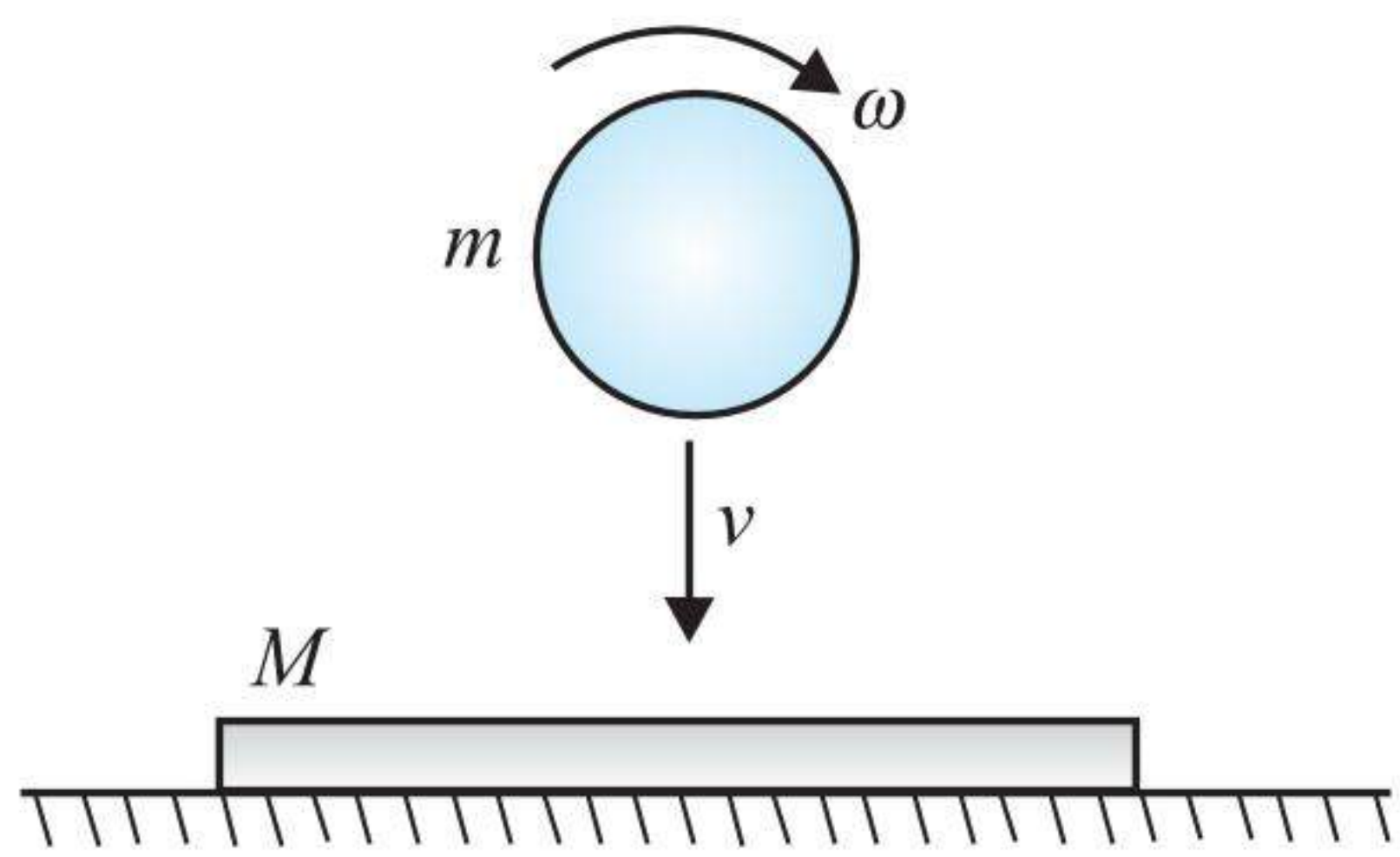
7. A rolling body of mass $m = 4$ kg, radius R and radius of gyration $k = R/\sqrt{3}$ is placed as a plank which moves with an acceleration $a_0 = 1 \text{ ms}^{-2}$. Find the frictional force acting on the body if it rolls without sliding. (in N).



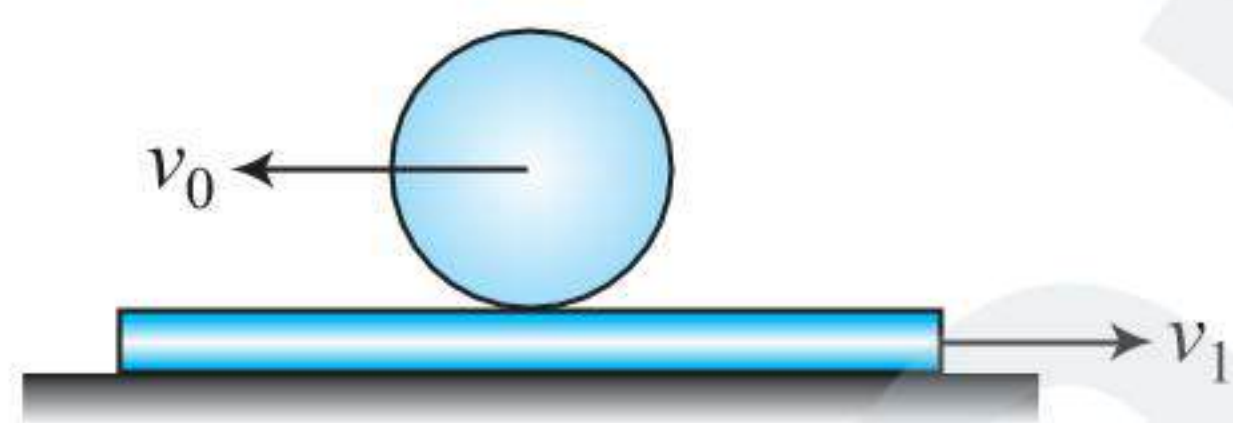
8. A solid sphere rolls on a smooth horizontal surface at 10 m/s and then rolls up a smooth inclined plane of inclination 30° with horizontal. The mass of the sphere is 2 kg. Find the height attained by the sphere before it stops (in m).
9. A loop and a disc roll without slipping with same linear velocity v . The mass of the loop and the disc is same. If the

total kinetic energy of the loop is 8 J, find the kinetic energy of the disc (in J).

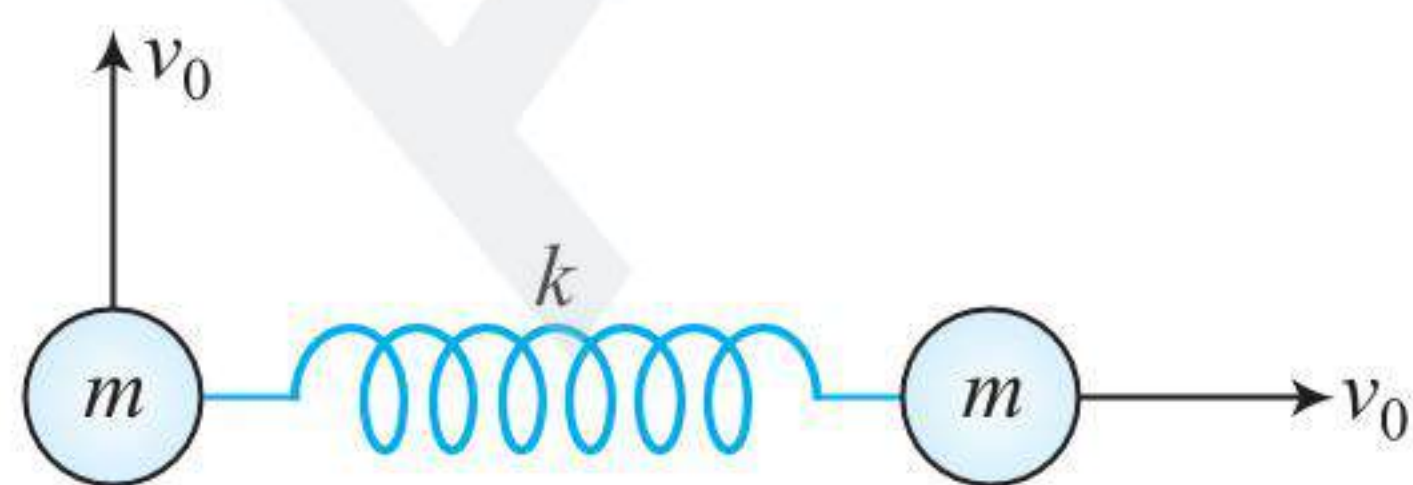
10. A ring of mass 3 kg is rolling without slipping with linear velocity 1 ms^{-1} on a smooth horizontal surface. A rod of same mass is fitted along its one diameter. Find total kinetic energy of the system (in J).
11. A solid sphere of mass 3 kg is kept on a horizontal surface. The coefficient of static friction between the surfaces in contact is $2/7$. What maximum force can be applied at the highest point in the horizontal direction so that the sphere does not slip on the surface? (in N)
12. A solid ball of mass m and radius r spinning with angular velocity ω falls on a horizontal slab of mass M with rough upper surface (coefficient of friction μ) and smooth lower surface. Immediately after collision the normal component of velocity of the ball remains half of its value just before collision and it stops spinning. Find the velocity of the sphere in horizontal direction immediately after the impact (given: $R\omega = 5$).



13. A sphere of radius r and mass m has a linear velocity $v_0 \text{ m/s}$ directed to the left and no angular velocity as it is placed on a horizontal platform moving to the right with a constant velocity 10 m/s . If after sliding on the platform the sphere is to have no linear velocity relative to the ground as it starts rolling on the platform without sliding. The coefficient of kinetic friction between the sphere and the platform is μ_k . Determine the required value of v_0 in m/s .

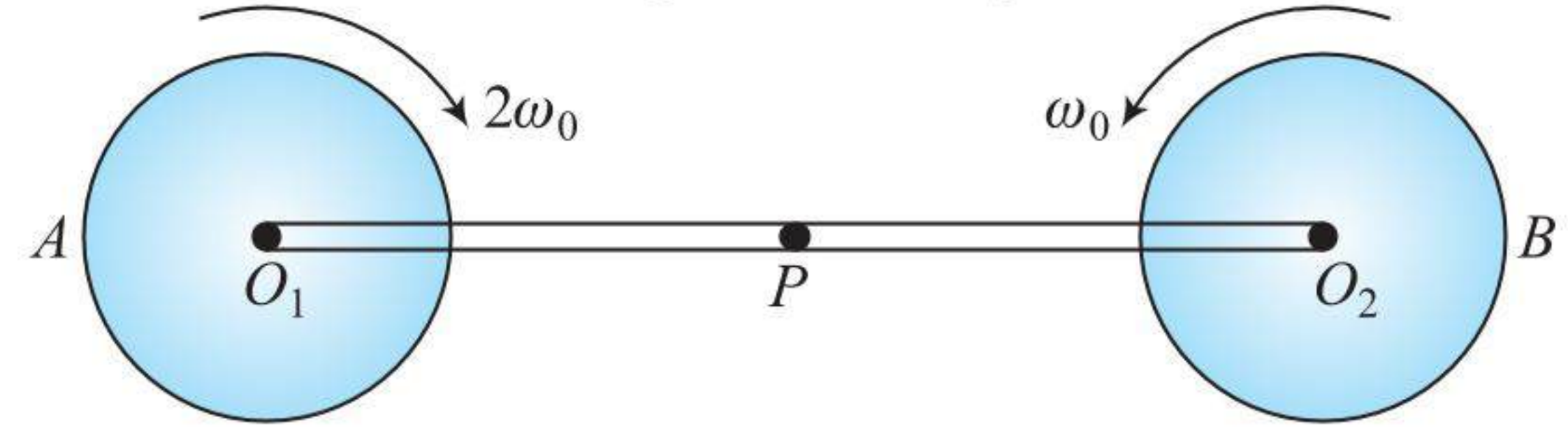


14. Two small spheres of mass 1 kg each are connected to each other by means of an unstretched spring of force constant $k = 100 \text{ N/m}$ and natural length 2.4 m. The system is placed on a smooth horizontal surface and the two spheres are given velocities v_0 , as shown in the figure. The maximum elongation in the spring is found to be 0.4 m during the subsequent motion. Calculate v_0 in m/s . Take $\sqrt{31} = 5.6$

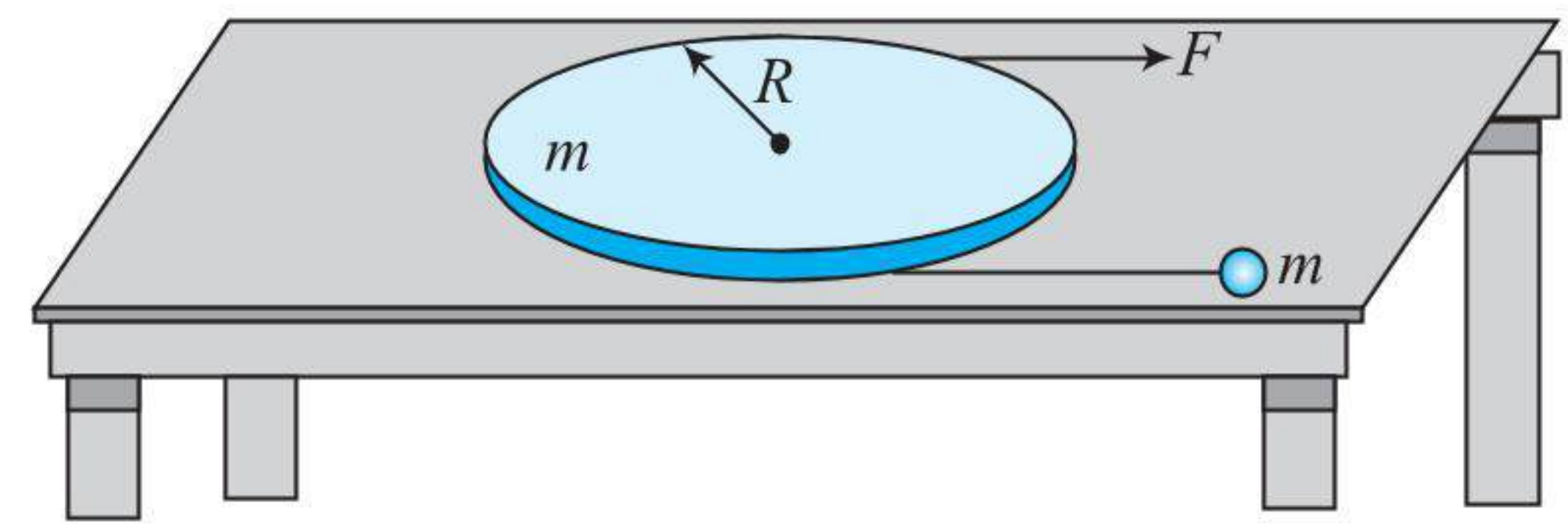
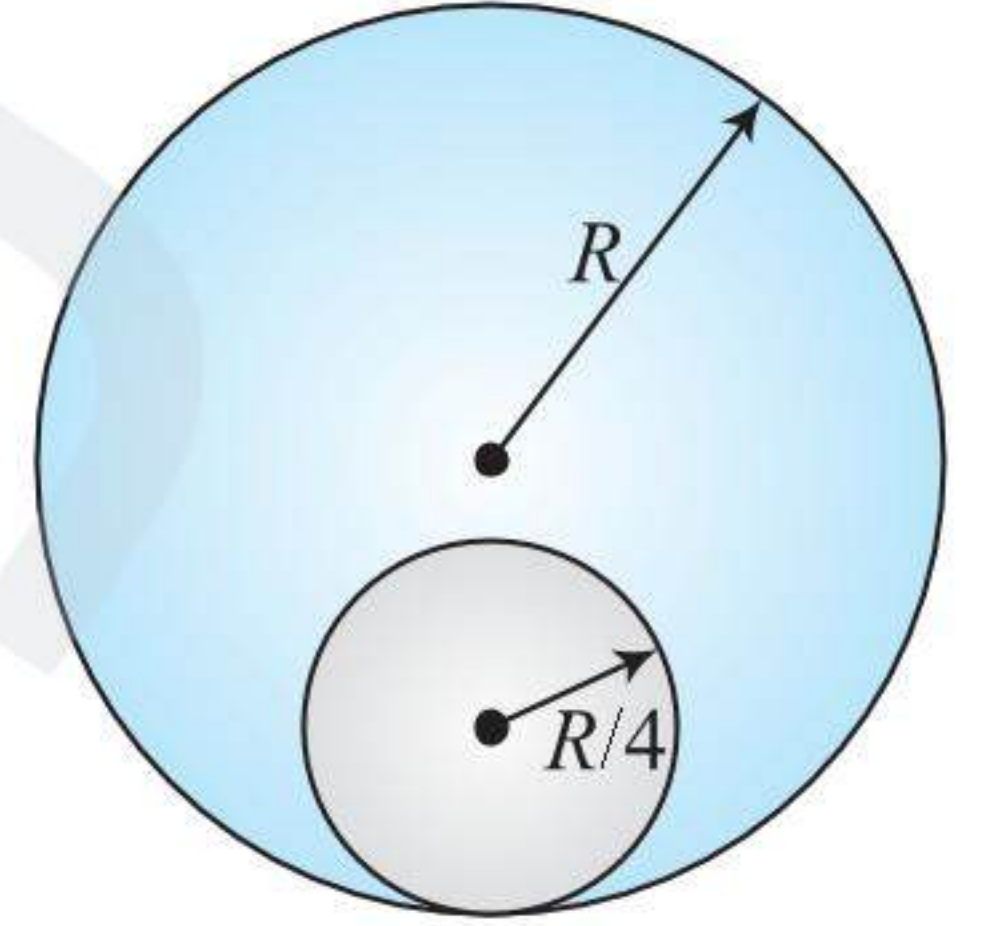


15. A rod of mass ' M ' and length $6R$ is pivoted at its centre (point P) to rotate in vertical plane as shown. Two discs of same radius ' R ' are joined at the ends O_1 and O_2 of the rod. Mass of the discs A and B are ' M ' and ' $2M$ ' respectively. Initial angular velocities of discs are shown in figure. The system is released from rest from horizontal position. Find

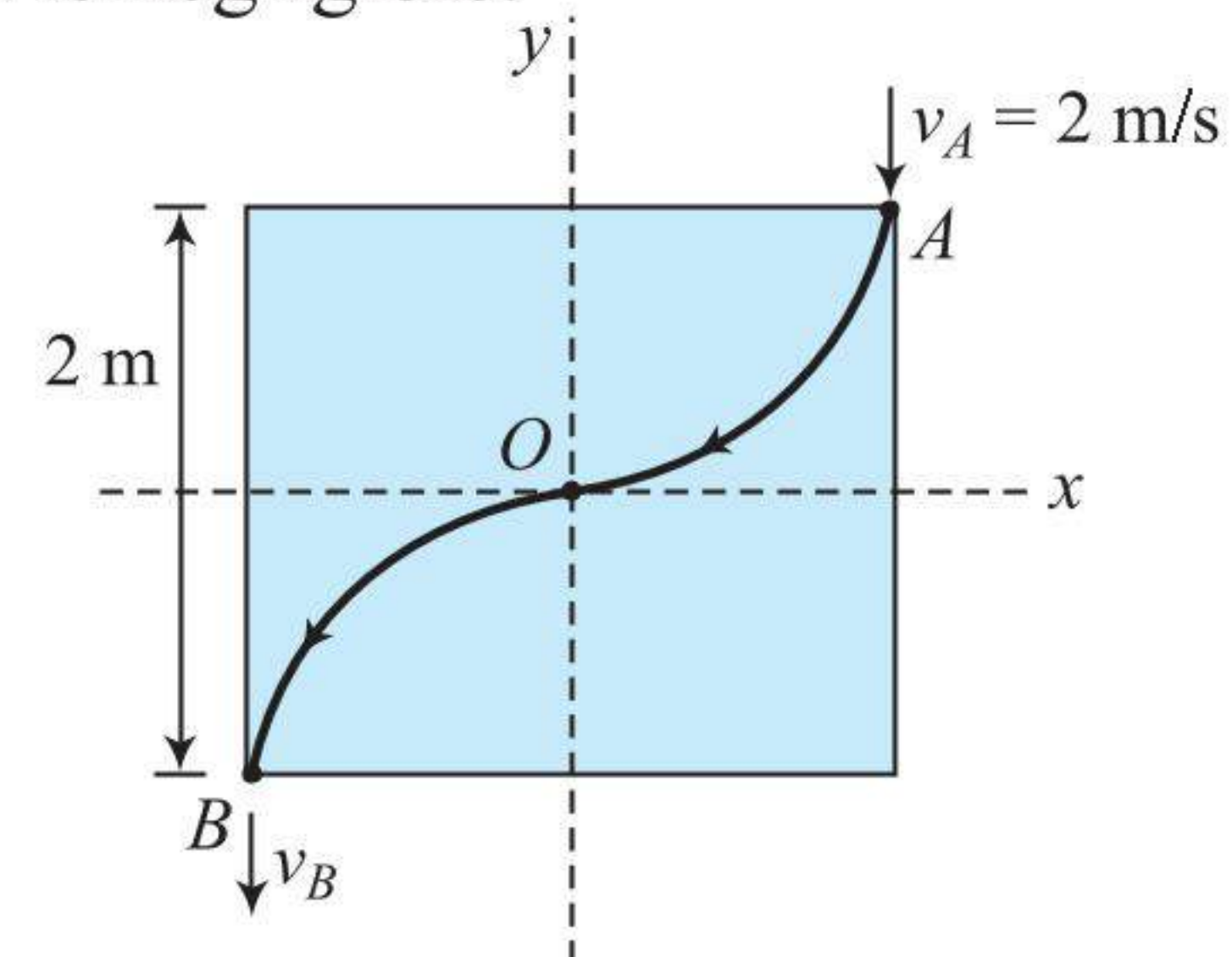
the angular velocity of the rod (in rad/sec) when it is vertical. (Given $R = 0.5 \text{ m}$ and $g = 10 \text{ m/s}^2$)



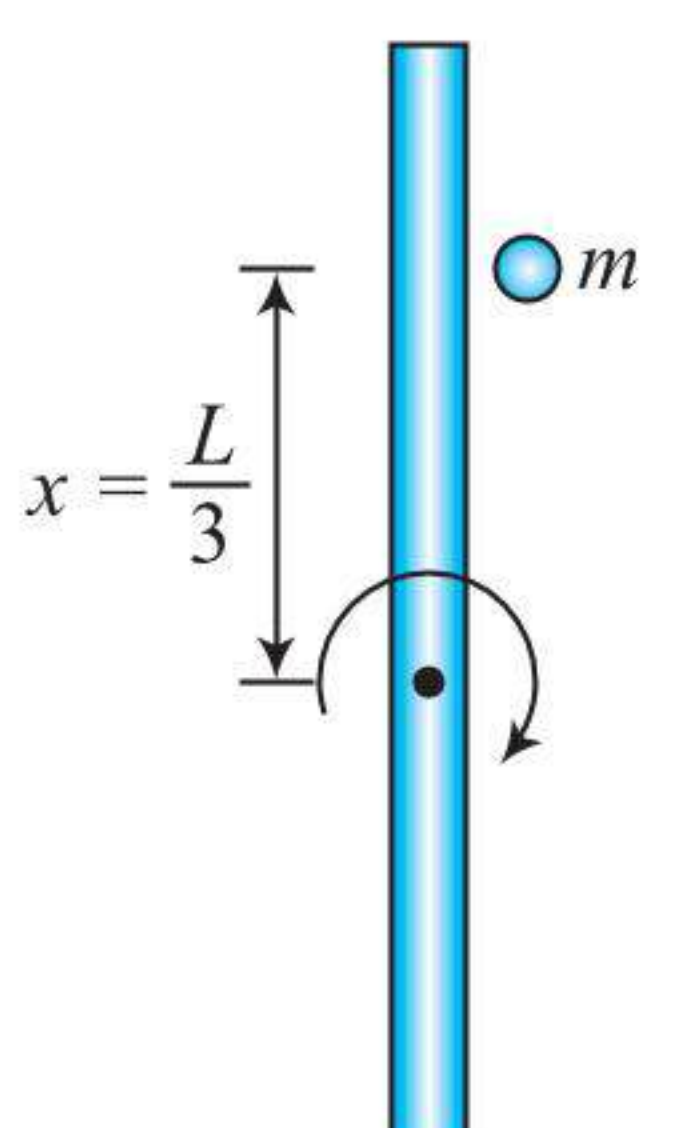
16. A ring of mass m and radius $R/4$ rolls inside a fixed hoop of radius R such that at the highest point of its trajectory the normal reaction becomes zero. The friction on the hoop is sufficient to ensure pure rolling. Find the angular velocity of the ring in rad/s . (Given $R = 0.3 \text{ m}$ and $g = 10 \text{ m/s}^2$)
17. A uniform disc of mass $m = 1 \text{ kg}$ and radius $R = 20 \text{ cm}$ lies flat on a smooth horizontal table. A mass less string runs halfway around it as shown in figure. One end of the string is attached to a small block of mass m and the other end is being pulled with a force $F = 20 \text{ N}$. The circumference of the disc is sufficiently rough so that the string does not slip over it. Find acceleration of the small block (in m/s^2).



18. A metal plate which is square in shape of side length $l = 2 \text{ m}$ has a groove made in the shape of two quarter circles joining at the centre of the plate. The plate is free to rotate about vertical axis passing through its centre. The moment of inertia of the plate about this axis is $I_0 = 4 \text{ kg-m}^2$. A particle of mass $m = 1 \text{ kg}$ enters the groove at end A travelling with a velocity of $v_A = 2 \text{ m/s}$ parallel to the side of the square plate. The particle moves along the frictionless groove of the horizontal plate and comes out at the other end B with speed v . Find the magnitude of v (in m/s) assuming that width of the groove is negligible.



19. A uniform thin stick of mass $M = 24 \text{ kg}$ and length L rotates on a frictionless horizontal plane, with its centre of mass stationary. A particle of mass m is placed on the plane at a distance $x = L/3$ from the centre of the stick. This stick hits the particle elastically. Find the value of m/M so that after the collision, there is no rotational motion of the stick.



Archives

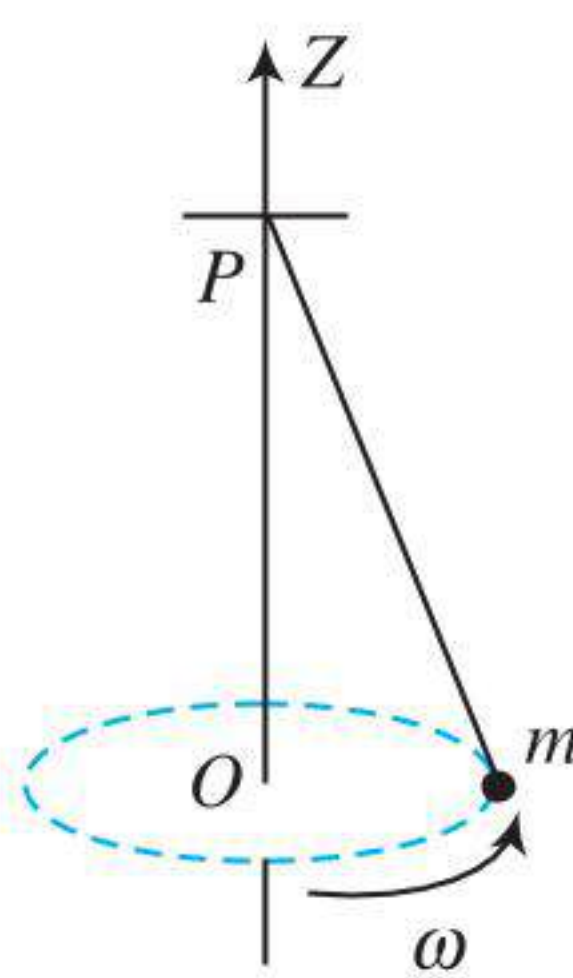
JEE ADVANCED

Single Correct Answer Type

- If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that
 - linear momentum of the system does not change in time
 - kinetic energy of the system does not change in time
 - angular momentum of the system does not change in time
 - potential energy of the system does not change in time

(IIT-JEE 2009)

- A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x - y plane with centre at O and constant angular speed ω . If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



- $\vec{L}_O$ and $\vec{L}_P$ do not vary with time.
- $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.
- $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time.
- $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

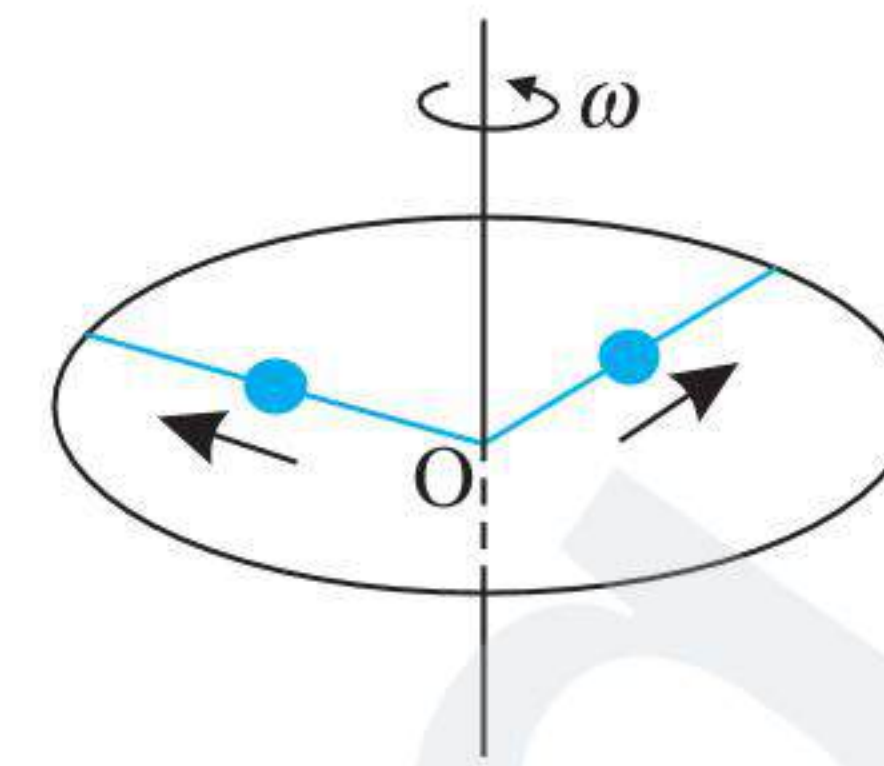
(IIT-JEE 2012)

- Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is/are correct?

- Both cylinders P and Q reach the ground at the same time
- Cylinder P has larger linear acceleration than cylinder Q
- Both cylinders reach the ground with same translational kinetic energy.
- Cylinder Q reaches the ground with larger angular speed.

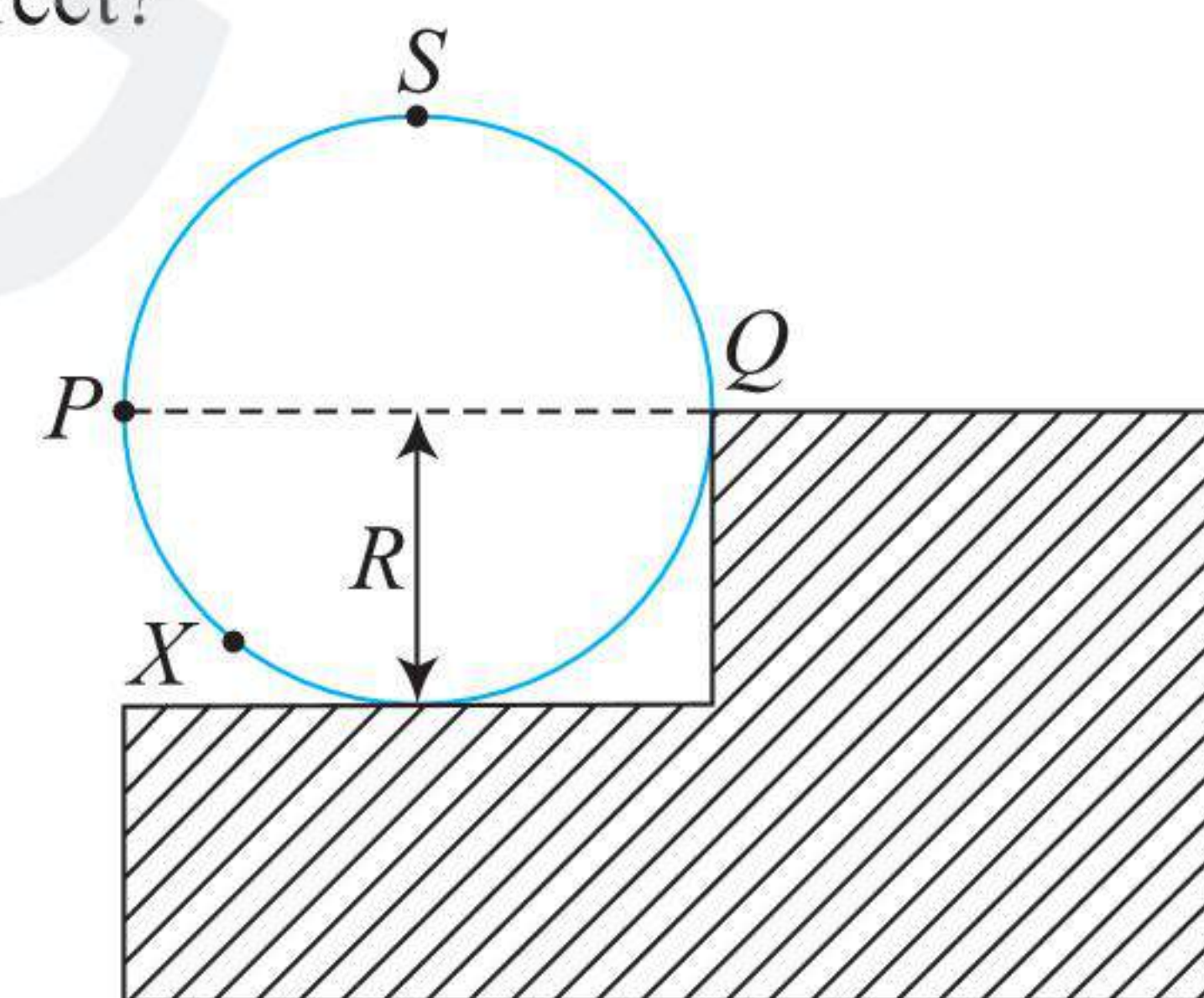
(IIT-JEE 2012)

- A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O . These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O . At this instant the distance of the other mass from O is



- $\frac{2}{3}R$
- $\frac{1}{3}R$
- $\frac{3}{5}R$
- $\frac{4}{5}R$ (JEE Advanced 2015)

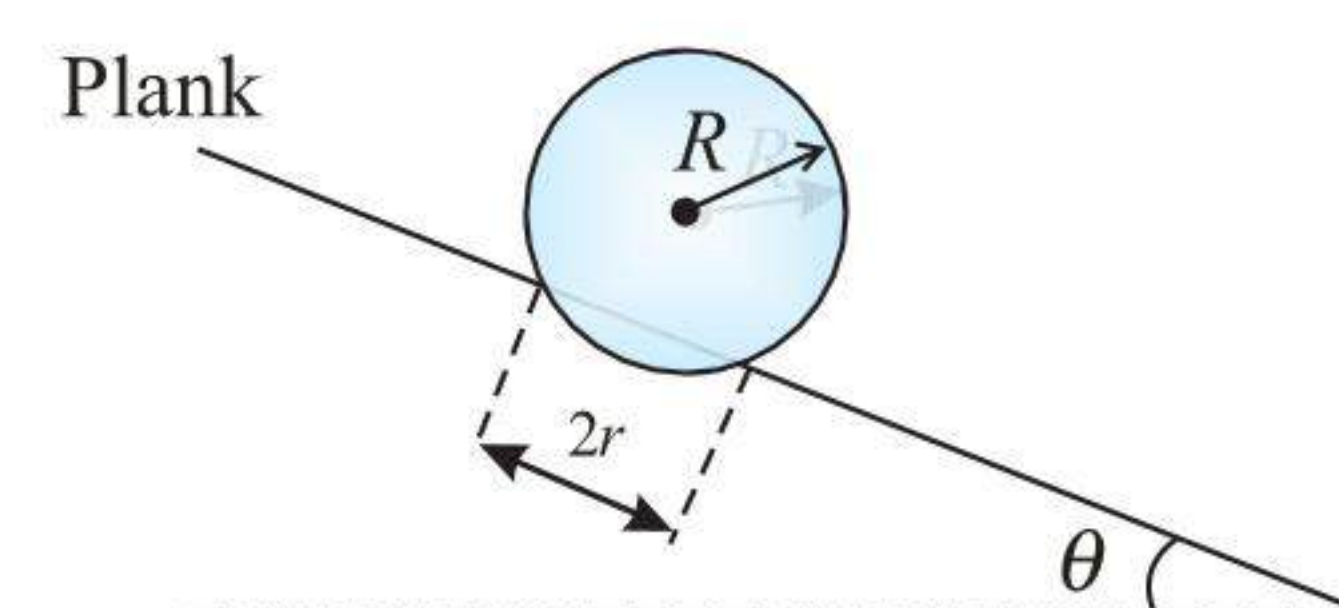
- A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q . Which of the following options is/are correct?



- If the force is applied normal to the circumference at point X then τ is constant
- If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step
- If the force is applied normal to the circumference at point P then τ is zero
- If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

(JEE Advanced 2017)

- A football of radius R is kept on a hole of radius r ($r < R$) made on a plank kept horizontally. One end of the plank is now lifted so that it gets tilted making an angle θ from the horizontal as shown in the figure below. The maximum value of θ so that the football does not start rolling down the plank satisfies (figure is schematic and not drawn to scale)

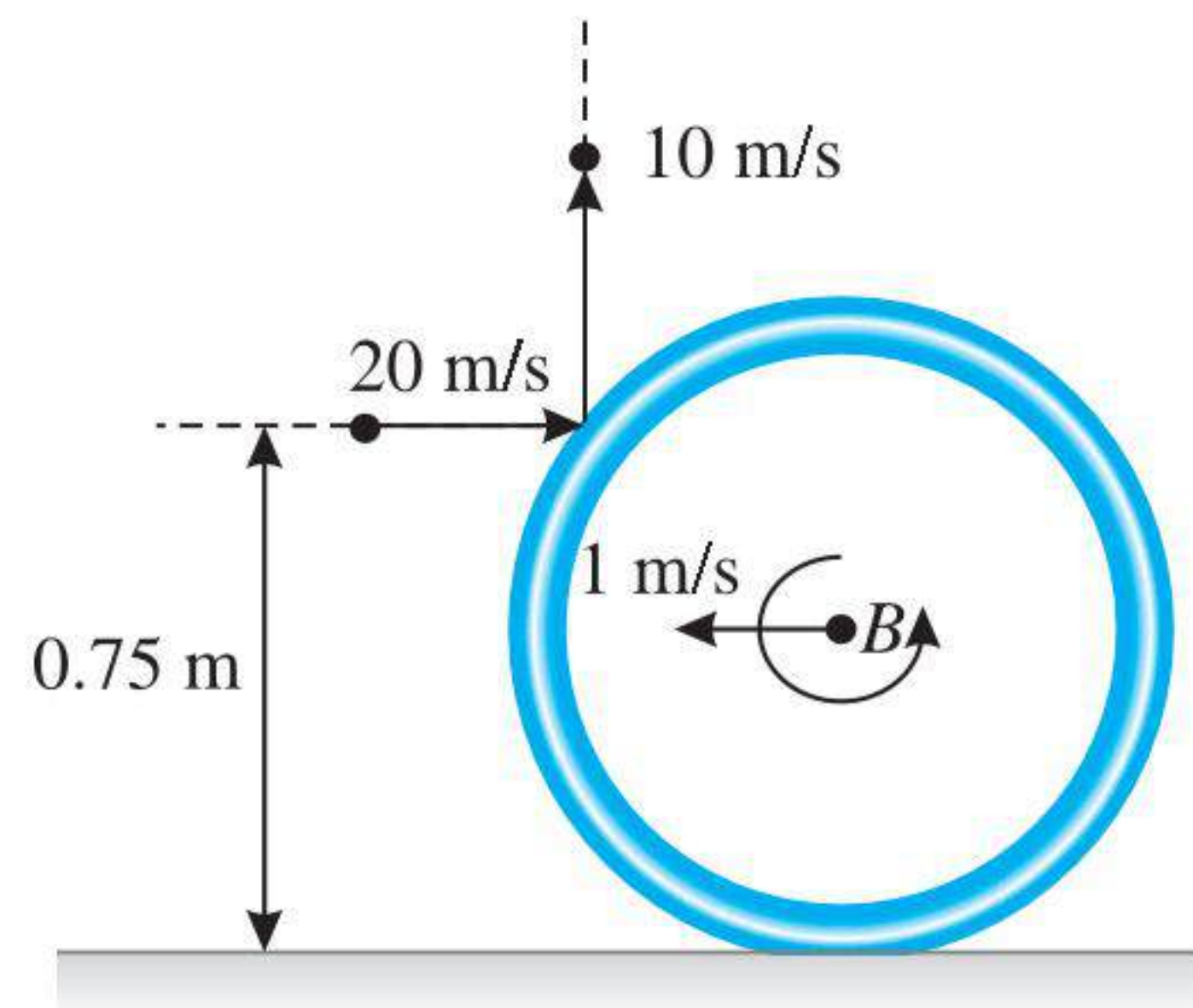


- $\sin \theta = \frac{r}{R}$
- $\tan \theta = \frac{r}{R}$
- $\sin \theta = \frac{r}{2R}$
- $\cos \theta = \frac{r}{2R}$

(JEE Advanced 2020)

Multiple Correct Answers Type

1. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity



10 m/s. Immediately after the collision

- (1) the ring has pure rotation about its stationary
- (2) the ring comes to a complete stop
- (3) friction between the ring and the ground is to the left
- (4) there is no friction between the ring and the ground

(IIT-JEE 2011)

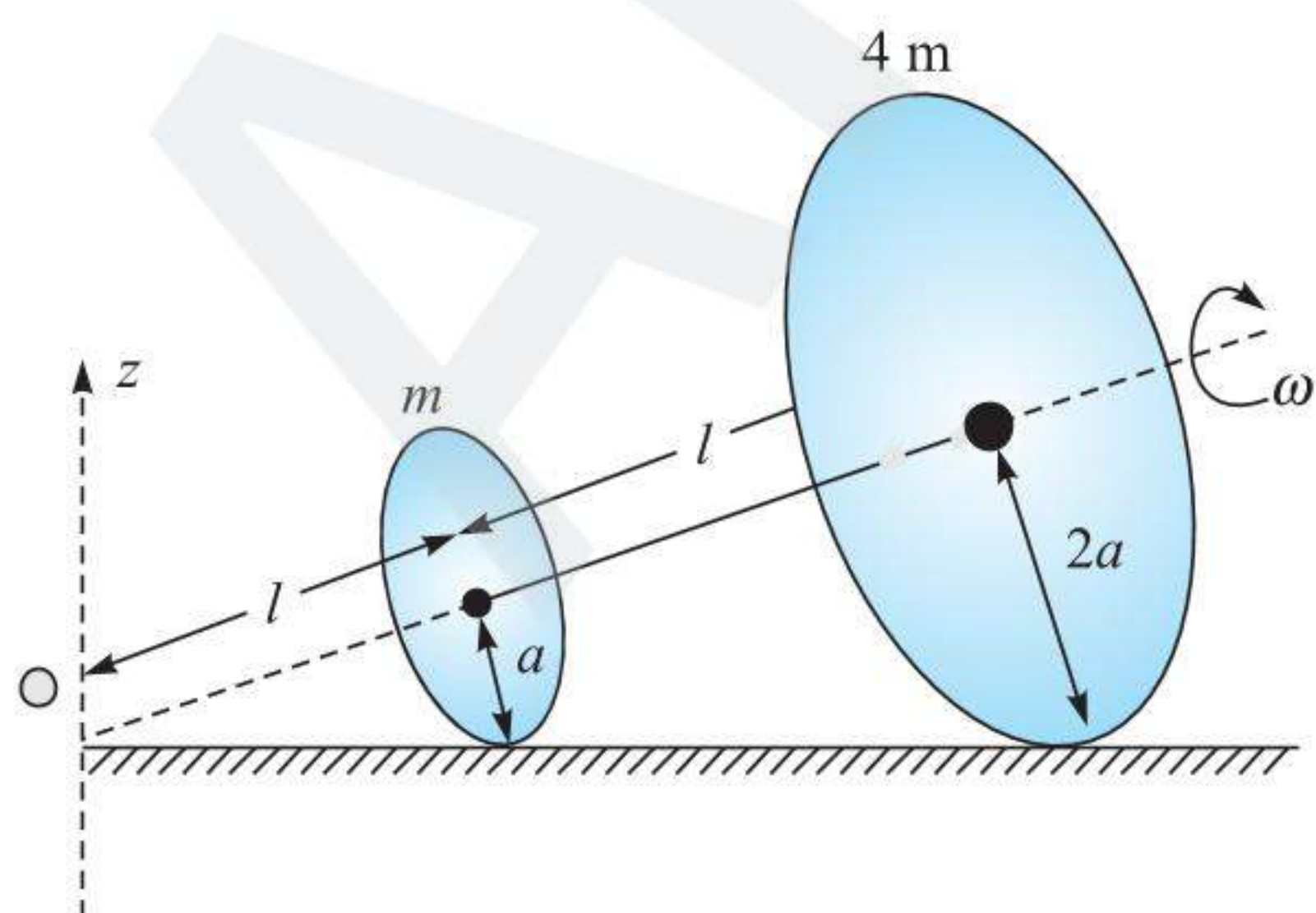
2. The position vector $\vec{r}$ of a particle of mass m is given by the following equation $\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$, where $\alpha = \frac{10}{3} \text{ ms}^{-3}$, $\beta = 5 \text{ ms}^{-2}$ and $m = 0.1 \text{ kg}$. At $t = 1 \text{ s}$, which of the following statement(s) is(are) true?

- (1) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j}) \text{ ms}^{-1}$
- (2) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -\left(\frac{5}{3}\right) \hat{k} \text{ Nms}$

- (3) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j}) \text{ N}$
- (4) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -\left(\frac{20}{3}\right) \hat{k} \text{ Nm}$

(JEE Advanced 2016)

3. Two thin circular discs of mass m and $4m$, having radii of a and $2a$, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}a$ through their centers. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω . The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement (s) is (are) true?

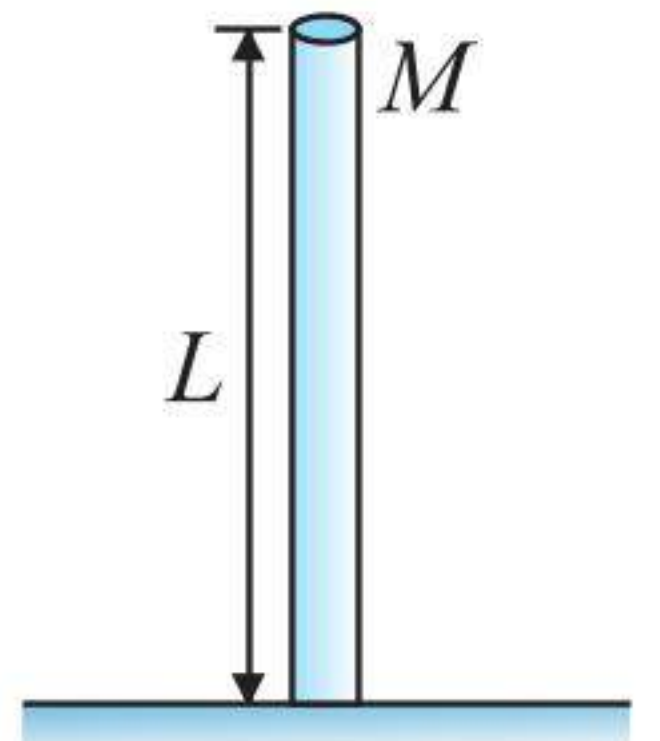


- (1) The magnitude of the z-component of $\vec{L}$ is $55 ma^2 \omega$.
- (2) The magnitude of angular momentum of the assembly about its centre of mass is $17 ma^2 \frac{\omega}{2}$.

- (3) The magnitude of angular momentum of centre of mass of the assembly about the point O is $81 ma^2 \omega$.
- (4) The centre of mass of the assembly rotates about the z-axis with an angular speed of $\frac{\omega}{5}$.

(JEE Advanced 2016)

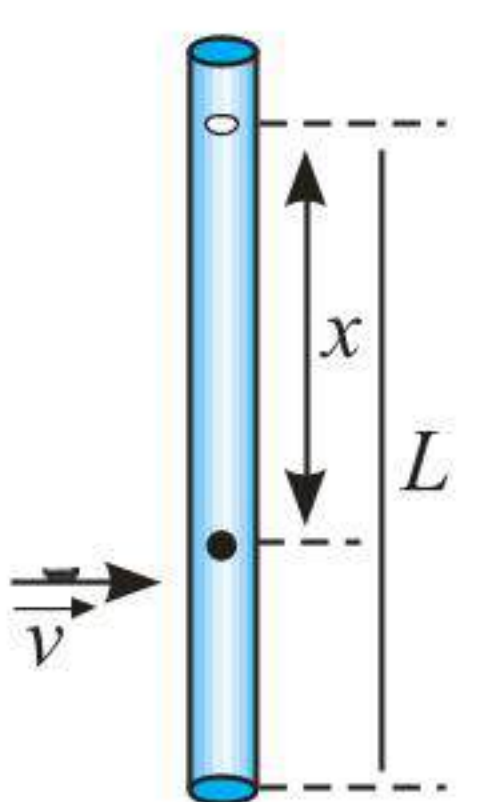
4. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical? [g is the acceleration due to gravity]



- (1) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$
- (2) The angular acceleration of the rod will be $\frac{2g}{L}$
- (3) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$
- (4) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

(JEE Advanced 2019)

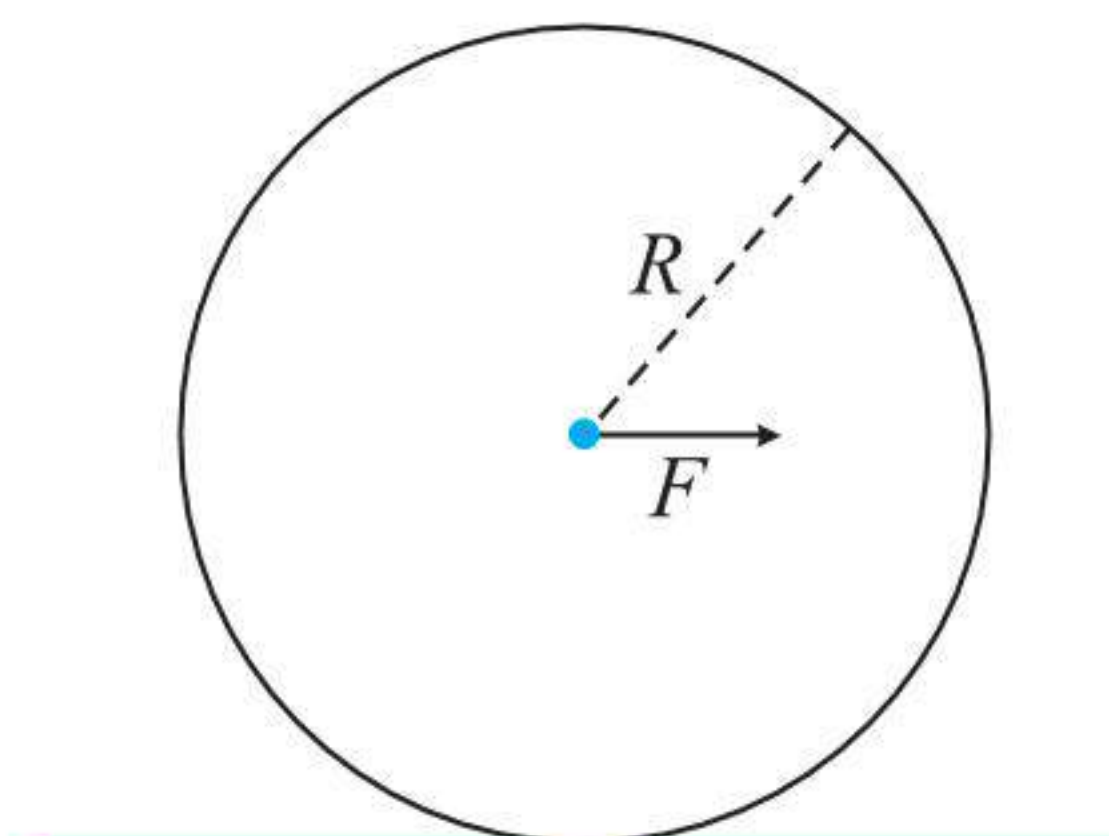
5. A rod of mass m and length L , pivoted at one of its ends, is hanging vertically. A bullet of the same mass moving at speed v strikes the rod horizontally at a distance x from its pivoted end and gets embedded in it. The combined system now rotates with angular speed ω about the pivot. The maximum angular speed ω_M is achieved for $x = x_M$. Then



- (1) $\omega = \frac{3vx}{L^2 + 3x^2}$
- (2) $\omega = \frac{12vx}{L^2 + 12x^2}$
- (3) $x_M = \frac{L}{\sqrt{3}}$
- (4) $\omega_M = \frac{v}{2L} \sqrt{3}$

(JEE Advanced 2020)

6. A horizontal force F is applied at the center of mass of a cylindrical object of mass m and radius R , perpendicular to its axis as shown in the figure. The coefficient of friction between the object and the ground is μ . The center of mass of the object has an acceleration a . The acceleration due to gravity is g . Given that the object rolls without slipping, which of the following statement(s) is(are) correct?



- (1) For the same F , the value of a does not depend on whether the cylinder is solid or hollow
- (2) For a solid cylinder, the maximum possible value of a is $2\mu g$
- (3) The magnitude of the frictional force on the object due to the ground is always μmg
- (4) For a thin-walled hollow cylinder, $a = \frac{F}{2m}$

(JEE Advanced 2021)

7. A particle of mass $M = 0.2$ kg is initially at rest in the xy -plane at a point $(x = -l, y = -h)$, where $l = 10$ m and $h = 1$ m. The particle is accelerated at time $t = 0$ with a constant acceleration $a = 10$ m/s² along the positive x -direction. Its angular momentum and torque with respect to the origin, in SI units, are represented by $\vec{L}$ and $\vec{\tau}$, respectively. $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the positive x, y and z -directions, respectively. If $\hat{k} = \hat{i} \times \hat{j}$ then which of the following statement(s) is(are) correct?

- (1) The particle arrives at the point $(x = l, y = -h)$ at time $t = 2$ s.
- (2) $\vec{\tau} = 2\hat{k}$ when the particle passes through the point $(x = l, y = -h)$
- (3) $\vec{L} = 4\hat{k}$ when the particle passes through the point $(x = l, y = -h)$
- (4) $\vec{\tau} = \hat{k}$ when the particle passes through the point $(x = 0, y = -h)$

(JEE Advanced 2021)

Linked Comprehension Type

For Problems 1–2

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r . The finger rotates with an angular velocity ω_0 . The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g .

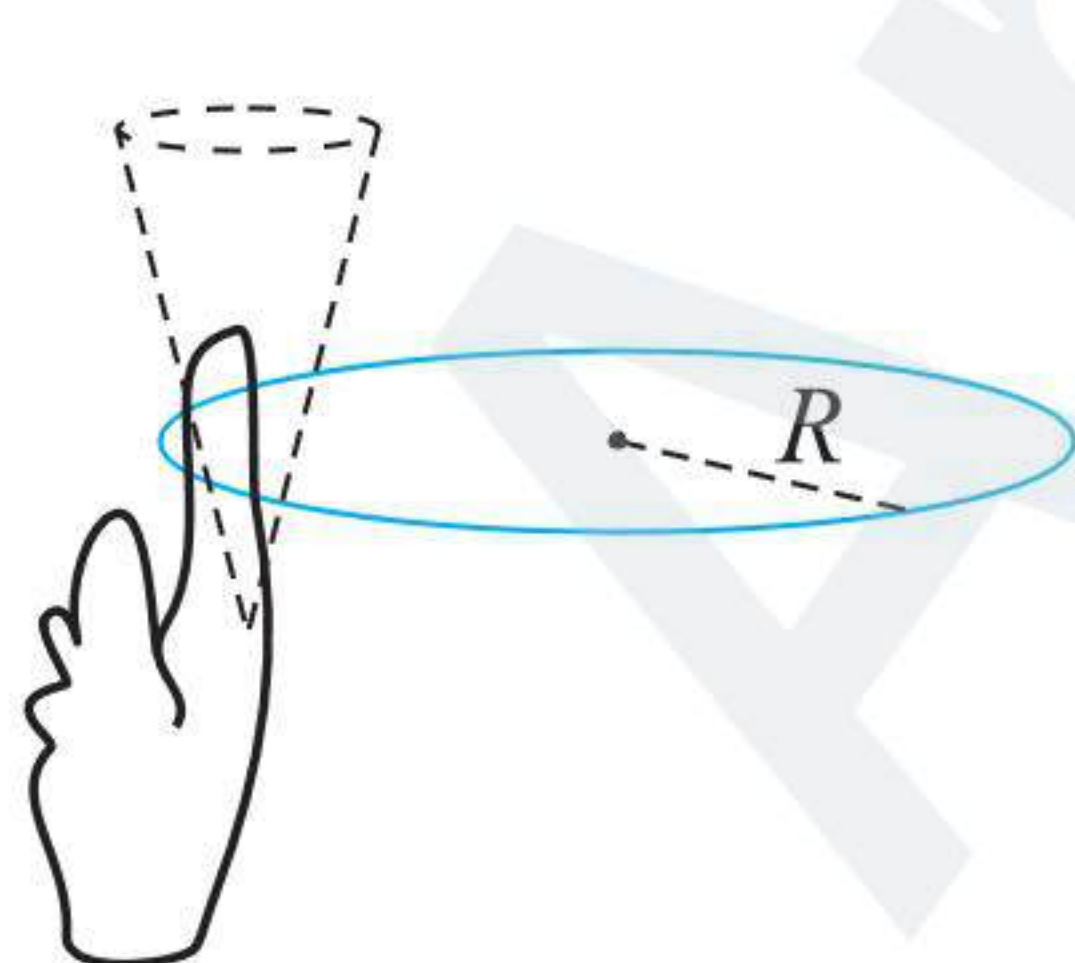


Figure 1

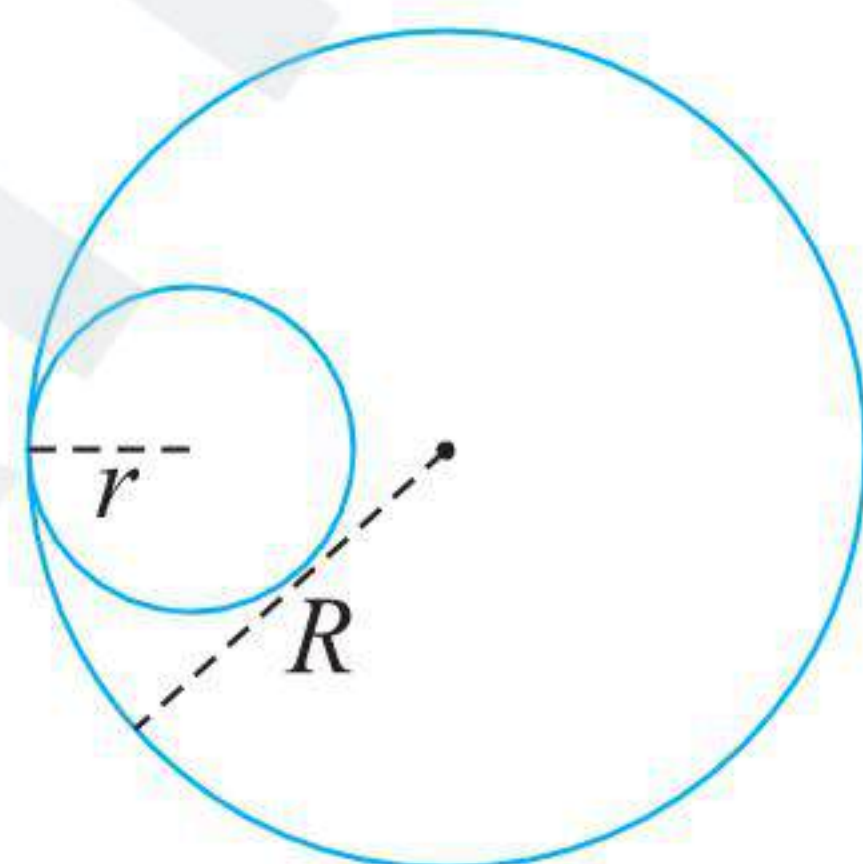


Figure 2

(JEE Advanced 2017)

1. The total kinetic energy of the ring is:

- (1) $M\omega_0^2 R^2$
- (2) $M\omega_0^2 (R-r)^2$
- (3) $\frac{1}{2} M\omega_0^2 (R-r)^2$
- (4) $\frac{3}{2} M\omega_0^2 (R-r)^2$

2. The minimum value of ω_0 below which the ring will drop down is:

- (1) $\sqrt{\frac{3g}{2\mu(R-r)}}$
- (2) $\sqrt{\frac{g}{\mu(R-r)}}$
- (3) $\sqrt{\frac{2g}{\mu(R-r)}}$
- (4) $\sqrt{\frac{2g}{2\mu(R-r)}}$

Matrix Match Type

1. In the Column I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq \beta$.

In each case, the force acting on the particle is either zero or conservative. In Column II, five physical quantities of the particle are mentioned; $\vec{p}$ is the linear momentum, $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy.

Match each path in Column I with those quantities in Column II, which are conserved for that path.

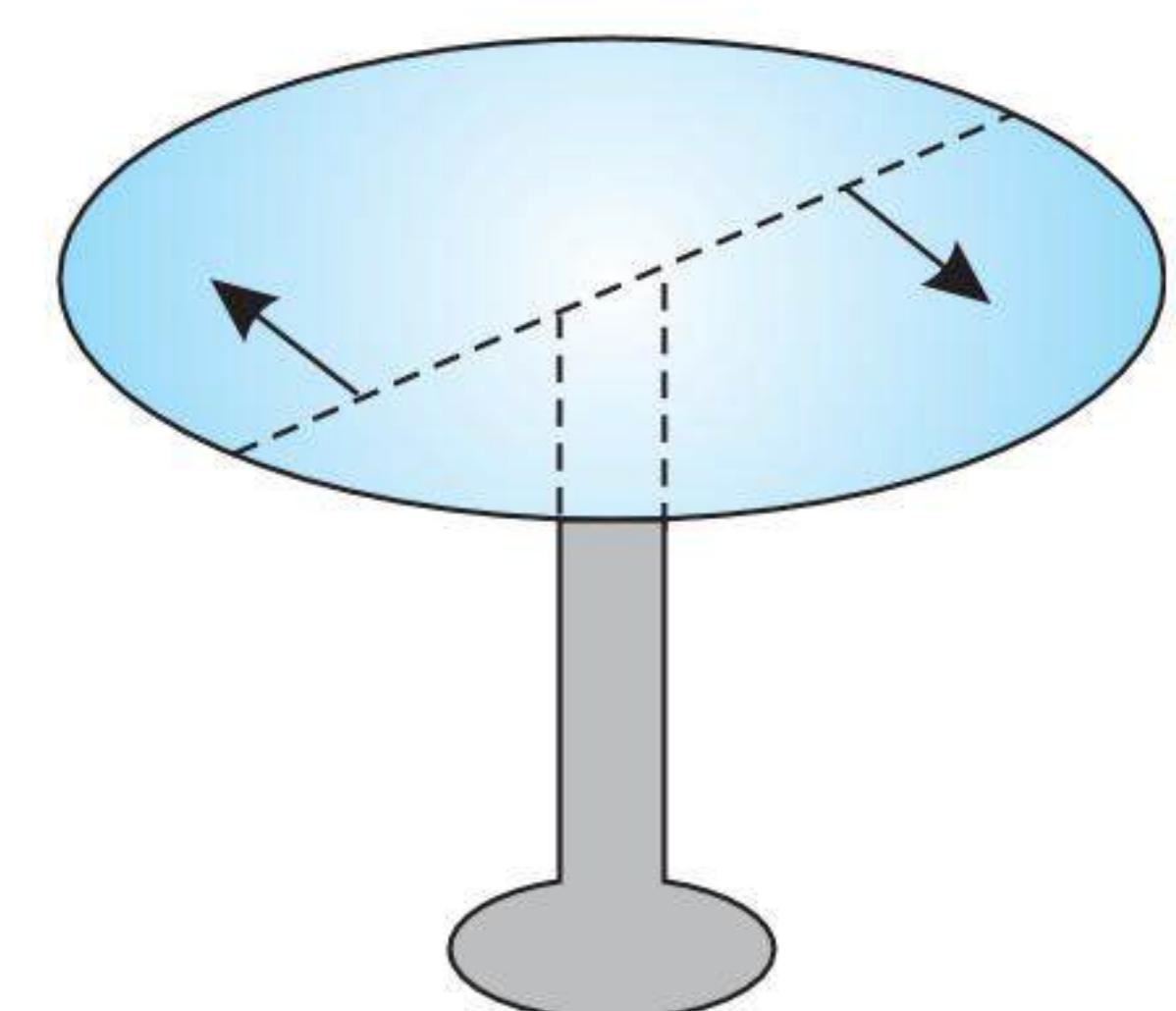
(JEE Advanced 2018)

Column I		Column II	
i.	$\vec{r}(t) = \alpha t \hat{i} + \beta t \hat{j}$	a.	$\vec{p}$
ii.	$\vec{r}(t) = \alpha \cos \omega t \hat{i} + \beta \sin \omega t \hat{j}$	b.	$\vec{L}$
iii.	$\vec{r}(t) = \alpha (\cos \omega t \hat{i} + \sin \omega t \hat{j})$	c.	K
iv.	$\vec{r}(t) = \alpha t \hat{i} + \frac{\beta}{2} t^2 \hat{j}$	d.	U
		e.	E

- (1) i. $\rightarrow$ a, b, c, d, e; ii. $\rightarrow$ b, e; iii. $\rightarrow$ b, c, d, e; iv. $\rightarrow$ e.
- (2) i. $\rightarrow$ a, b, c, d, e; ii. $\rightarrow$ c, e; iii. $\rightarrow$ b, c, d, e; iv. $\rightarrow$ b, e.
- (3) i. $\rightarrow$ b, c, d; ii. $\rightarrow$ e; iii. $\rightarrow$ a, b, d; iv. $\rightarrow$ b, e.
- (4) i. $\rightarrow$ a, b, c, e; ii. $\rightarrow$ b, e; iii. $\rightarrow$ b, c, d, e; iv. $\rightarrow$ b, e.

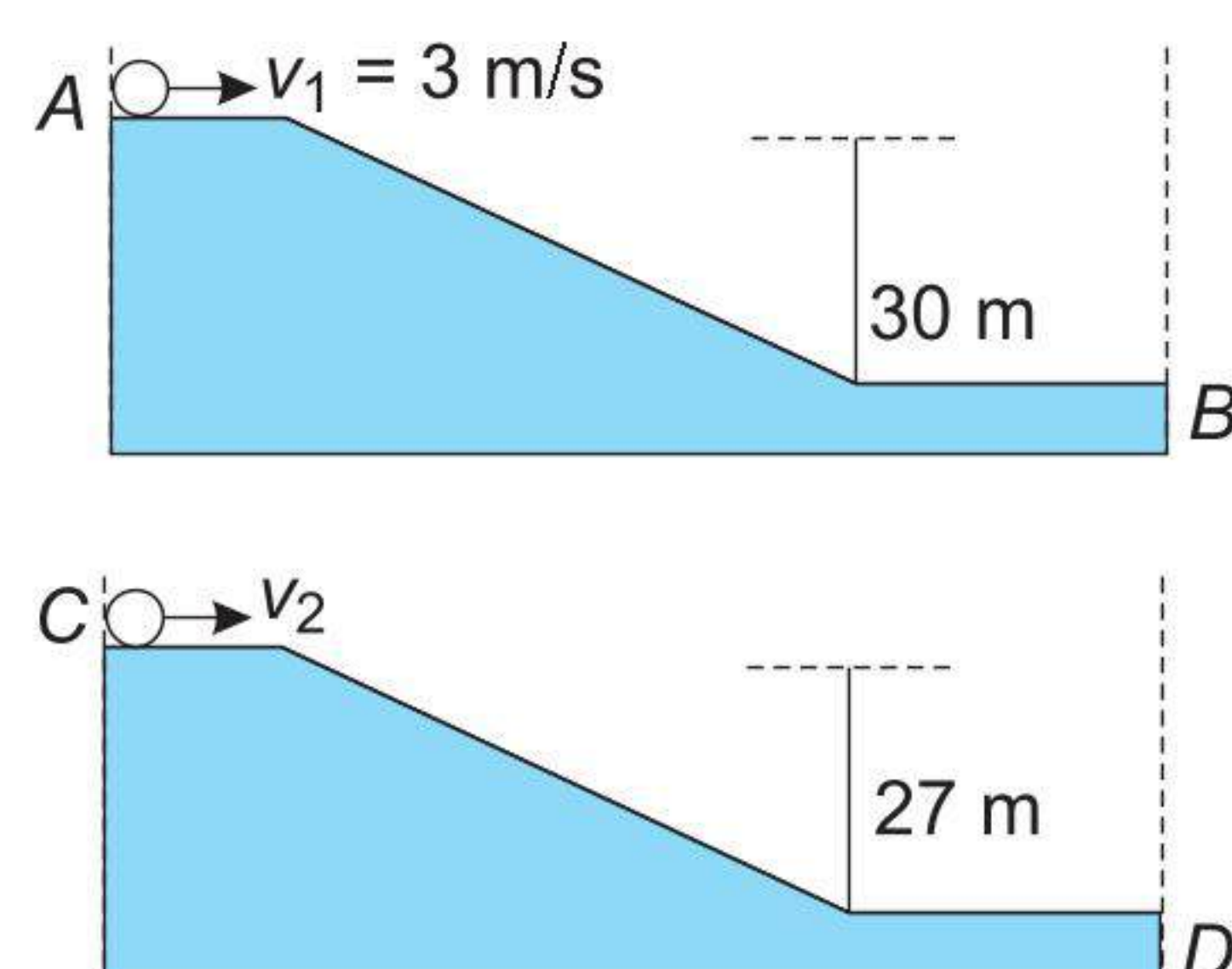
Numerical Value Type

1. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms⁻¹ with respect to the ground. The rotational speed of the platform in rad s⁻¹ after the balls leave the platform is



(JEE Advanced 2014)

2. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2 , respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3 \text{ m/s}$, then v_2 in m/s is _____. ($g = 10 \text{ m/s}^2$)



(JEE Advanced 2015)

3. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B , respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is:

(JEE Advanced 2015)

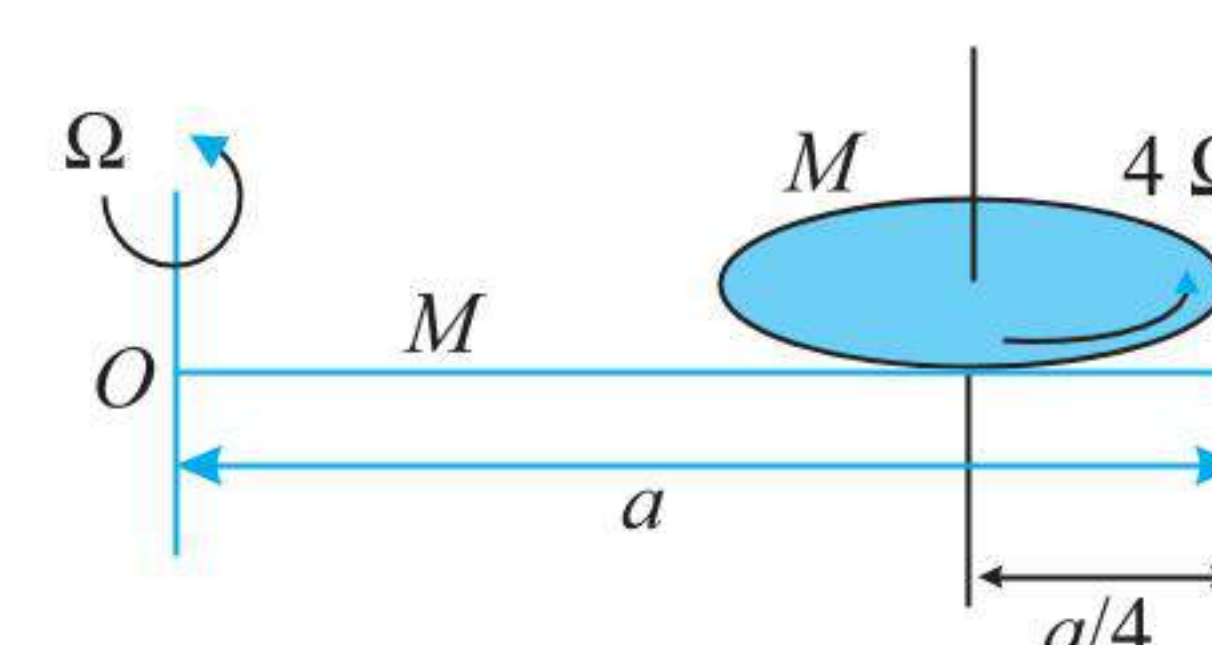
4. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the

horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $\frac{(2-\sqrt{3})}{\sqrt{10}} \text{ s}$, then the height of the top of the inclined plane,

in meters, is _____. Take $g = 10 \text{ ms}^{-2}$.

(JEE Advanced 2018)

5. A thin rod of mass M and length a is free to rotate in horizontal plane about a fixed vertical axis passing through point O . A thin circular disc of mass M and of radius $a/4$ is pivoted on this rod with its center at a distance $a/4$ from the free end so that it can rotate freely about its vertical axis, as shown in the figure. Assume that both the rod and the disc have uniform density and they remain horizontal during the motion. An outside stationary observer finds the rod rotating with an angular velocity Ω and the disc rotating about its vertical axis with angular velocity 4Ω . The total angular momentum of the system about the point O is $\left(\frac{Ma^2\Omega}{48}\right)n$. The value of n is _____.



(JEE Advanced 2021)

Answers Key

EXERCISES

Single Correct Answer Type

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (2) | 2. (4) | 3. (1) | 4. (2) | 5. (2) |
| 6. (1) | 7. (3) | 8. (4) | 9. (2) | 10. (4) |
| 11. (4) | 12. (4) | 13. (2) | 14. (2) | 15. (1) |
| 16. (1) | 17. (2) | 18. (4) | 19. (4) | 20. (3) |
| 21. (4) | 22. (2) | 23. (4) | 24. (3) | 25. (2) |
| 26. (4) | 27. (3) | 28. (2) | 29. (3) | 30. (1) |
| 31. (4) | 32. (3) | 33. (1) | 34. (3) | 35. (4) |
| 36. (1) | 37. (4) | 38. (2) | 39. (3) | 40. (4) |
| 41. (1) | 42. (3) | 43. (4) | 44. (4) | 45. (2) |
| 46. (2) | 47. (2) | 48. (2) | 49. (2) | 50. (1) |
| 51. (4) | 52. (2) | 53. (1) | 54. (2) | 55. (4) |
| 56. (3) | 57. (2) | 58. (2) | 59. (2) | 60. (3) |
| 61. (1) | 62. (2) | 63. (4) | 64. (1) | 65. (3) |
| 66. (2) | 67. (2) | 68. (3) | 69. (3) | 70. (1) |
| 71. (4) | 72. (2) | 73. (4) | 74. (2) | 75. (3) |
| 76. (1) | 77. (3) | 78. (3) | 79. (1) | 80. (3) |
| 81. (2) | 82. (2) | 83. (1) | 84. (1) | 85. (4) |
| 86. (1) | 87. (1) | 88. (2) | | |

Multiple Correct Answers Type

- | | | |
|--------------------|-----------------|---------------------|
| 1. (1),(2),(3),(4) | 2. (3),(4) | 3. (3),(4) |
| 4. (2),(4) | 5. (1),(2) | 6. (1),(4) |
| 7. (1),(4) | 8. (1),(2),(4) | 9. (2),(4) |
| 10. (1),(2),(3) | 11. (1),(2),(4) | 12. (2),(3),(4) |
| 13. (3),(4) | 14. (1),(3),(4) | 15. (1),(2),(3),(4) |
| 16. (1),(3),(4) | 17. (1),(2),(3) | 18. (1),(2),(3),(4) |
| 19. (1),(2),(4) | 20. (2),(3),(4) | 21. (1),(3) |
| 22. (2),(3),(4) | 23. (1),(4) | 24. (2),(4) |
| 25. (2),(3),(4) | 26. (1),(4) | 27. (1),(4) |

Linked Comprehension Type

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (1) | 2. (3) | 3. (3) | 4. (2) | 5. (2) |
| 6. (1) | 7. (4) | 8. (2) | 9. (3) | 10. (4) |
| 11. (1) | 12. (4) | 13. (3) | 14. (4) | 15. (4) |
| 16. (1) | 17. (2) | 18. (1) | 19. (2) | 20. (1) |
| 21. (3) | 22. (4) | 23. (4) | 24. (4) | 25. (4) |
| 26. (2) | 27. (1) | 28. (1) | 29. (3) | 30. (2) |
| 31. (3) | 32. (4) | 33. (1) | 34. (2) | 35. (2) |
| 36. (1) | 37. (4) | 38. (3) | 39. (1) | 40. (2) |
| 41. (1) | 42. (3) | 43. (2) | 44. (1) | |

Matrix Match Type

- i. $\rightarrow$ b., d.; ii. $\rightarrow$ a.; iii. $\rightarrow$ a., b., c., d.; iv. $\rightarrow$ b., d.
- i. $\rightarrow$ c.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a., b., c.; iv. $\rightarrow$ a., b., c.
- i. $\rightarrow$ d.; ii. $\rightarrow$ b., c.; iii. $\rightarrow$ d.; iv. $\rightarrow$ a., c.
- i. $\rightarrow$ a.; ii. $\rightarrow$ b., d.; iii. $\rightarrow$ a.; iv. $\rightarrow$ b., d.
- i. $\rightarrow$ a., b., c.; ii. $\rightarrow$ a., b., c.; iii. $\rightarrow$ a., b.; iv. $\rightarrow$ a., b., c.
- i. $\rightarrow$ c.; ii. $\rightarrow$ a., b., c., d., e.; iii. $\rightarrow$ a., b., c., d., e.; iv. $\rightarrow$ e.
- i. $\rightarrow$ a., b., c.; ii. $\rightarrow$ a.; iii. $\rightarrow$ d.; iv. $\rightarrow$ d.
- i. $\rightarrow$ a., b., d.; ii. $\rightarrow$ a., b., c.; iii. $\rightarrow$ a., b., d.; iv. $\rightarrow$ a., b., c.
- i. $\rightarrow$ b., c.; ii. $\rightarrow$ a., c.; iii. $\rightarrow$ c.
- i. $\rightarrow$ a., b., d.; ii. $\rightarrow$ a., b.; iii. $\rightarrow$ a., b., c.; iv. $\rightarrow$ a., b.
- i. $\rightarrow$ a.; ii. $\rightarrow$ a.; iii. $\rightarrow$ a.; iv. $\rightarrow$ c.
- i. $\rightarrow$ a., c., d.; ii. $\rightarrow$ b., c., d.; iii. $\rightarrow$ b.; iv. $\rightarrow$ b., c., d.
- i. $\rightarrow$ a., b.; ii. $\rightarrow$ b., c.; iii. $\rightarrow$ a., b.; iv. $\rightarrow$ a., b.
- i. $\rightarrow$ d.; ii. $\rightarrow$ b., d.; iii. $\rightarrow$ a., b., c.; iv. $\rightarrow$ b., c., d.
- i. $\rightarrow$ a., b., c., d.; ii. $\rightarrow$ a., d.; iii. $\rightarrow$ a., b., c., d.; iv. $\rightarrow$ b., c., d.
- i. $\rightarrow$ b., c.; ii. $\rightarrow$ a., b., c.; iii. $\rightarrow$ a., d.; iv. $\rightarrow$ a., d.
- (2)

Numerical Value Type

- | | | | | |
|-------------|------------|------------|------------|------------|
| 1. (2) | 2. (0) | 3. (6) | 4. (5) | 5. (2) |
| 6. (4) | 7. (1) | 8. (7) | 9. (6) | 10. (5) |
| 11. (20) | 12. (2) | 13. (4.00) | 14. (5.00) | 15. (2.00) |
| 16. (20.00) | 17. (5.00) | 18. (1.20) | 19. (3.00) | |

ARCHIVES

JEE Advanced

Single Correct Answer Type

- | | | | | |
|--------|--------|--------|--------|--------|
| 1. (1) | 2. (3) | 3. (4) | 4. (4) | 5. (3) |
| 6. (1) | | | | |

Multiple Correct Answers Type

- | | | |
|----------------|----------------|------------|
| 1. (1),(3) | 2. (1),(2),(4) | 3. (2),(4) |
| 4. (1),(3),(4) | 5. (1),(3),(4) | 6. (2),(4) |
| 7. (1),(2),(3) | | |

Linked Comprehension Type

- | | |
|--------|--------|
| 1. (2) | 2. (2) |
|--------|--------|

Matrix Match Type

- | |
|--------|
| 1. (1) |
|--------|

Numerical Value Type

- | | | | | |
|--------|--------|--------|-----------|---------|
| 1. (4) | 2. (7) | 3. (6) | 4. (0.75) | 5. (49) |
|--------|--------|--------|-----------|---------|

Chapter 4

Concept Application Exercises

Exercise 4.1

1. $\vec{L} = \vec{r} \times \vec{p}$

The magnitude of $\vec{L}$ is given by $L = rp \sin \theta (\hat{k})$

In this example, velocity of the particle at any time

$$v = gt; \quad r \sin \theta = x_0 \text{ and } p = mv = mgt$$

$$\therefore \vec{L} = mgx_0 t (\hat{k}) \quad \dots(i)$$

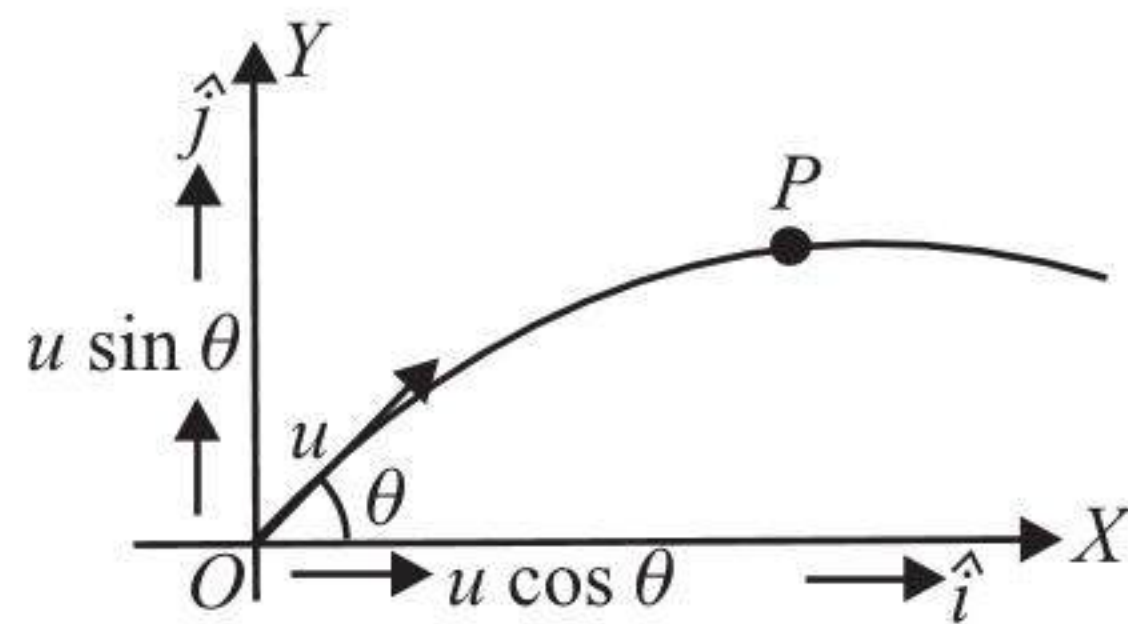
The right hand rule shows that $\vec{L}$ is parallel to torque, i.e., directed perpendicular to paper inwards.

2. Initial velocity is $\vec{u} = u(\cos \theta \hat{i} + \sin \theta \hat{j})$

$$\therefore \text{Velocity after time } t \text{ will be } \vec{v} = \vec{u} + \vec{a}t$$

$$\begin{aligned} \therefore \vec{v} &= u(\cos \theta \hat{i} + \sin \theta \hat{j}) - gt \hat{j} \\ &= u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j} \end{aligned}$$

If $\vec{r}$ be the radius vector of the particle P at time t (as shown in figure), then from $\vec{s} = \vec{u}t + \vec{a}t^2/2$



$$\begin{aligned} \therefore \vec{r} &= ut(\cos \theta \hat{i} + \sin \theta \hat{j}) - \frac{gt^2}{2} \hat{j} \\ &= ut \cos \theta \hat{i} + \left(ut \sin \theta - \frac{gt^2}{2} \right) \hat{j} \end{aligned}$$

$\therefore$ Angular momentum $\vec{L}$ of the particle at time t will be

$$\vec{L} = m(\vec{r} \times \vec{v})$$

$$\begin{aligned} \text{i.e., } \vec{L} &= m \left[ut \cos \theta \hat{i} + \left(ut \sin \theta - \frac{gt^2}{2} \right) \hat{j} \right] \\ &\quad \times [ut \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}] \\ &= \frac{-mut^2 g \cos \theta \hat{k}}{2} \end{aligned}$$

3. $\vec{L} = \vec{L}_{\text{cm}} + m(\vec{r}_0 \times \vec{v}_0) \quad \dots(i)$

$$\text{Here, } \vec{L}_{\text{cm}} = I\omega = \left(\frac{1}{2} mR^2 \right) \left(\frac{v_0}{R} \right) = \frac{1}{2} mv_0 R \text{ and } m(\vec{r}_0 \times \vec{v}_0)$$

(perpendicular to the paper inwards)

Since, both the terms of right hand side of Eq. (i) are in the same direction,

$$\therefore |\vec{L}| = \frac{1}{2} mv_0 R + mv_0 R$$

$$\text{or } |\vec{L}| = \frac{3}{2} mv_0 R$$

4. (i) $\vec{L}_0 = \vec{L}_{\text{cm}} + \vec{r} \times m\vec{v}_{\text{cm}}$

$$\text{where, } \vec{L}_{\text{cm}} = \frac{ml^2}{12} \omega (\hat{j})$$

(as rotation about com with same ω)

$$\vec{r} \times m\vec{v}_{\text{cm}} = \left(a\hat{j} + \frac{l}{2}\hat{i} \right) \times \left(m\frac{l}{2}\omega(-\hat{k}) \right) = \frac{mla\omega}{2}(-\hat{i}) + \frac{ml^2}{4}\omega\hat{j}$$

$$\vec{L}_0 = \frac{ml^2}{3}\omega\hat{j} + \frac{mla\omega}{2}(-\hat{i})$$

(ii) $\vec{L}_0 = \vec{r} \times m\vec{v}$ (as it is a particle)

$$\vec{L}_0 = (-l \cos \theta \hat{j} - l \sin \theta \hat{i}) \times (ml \sin \theta \omega \hat{k})$$

$$\vec{L}_0 = 2m\omega l^2 \cos \theta \sin \theta (-\hat{i}) + ml^2 \sin^2 \theta \omega \hat{j}$$

5. We know, that for any body $\vec{L}_0 = \vec{L}_{\text{cm}} + \vec{r} \times m\vec{v}_{\text{cm}}$

So, we will calculate angular momentum of rod and disc about O and vectorially add them.

$$\begin{aligned} \text{For rod: } \vec{L}_0 &= \frac{ml^2\omega_1}{12} (\hat{j}) + \left(\frac{l}{2}\hat{i} + R\hat{j} \right) \times m \left(-\omega_1 \frac{l}{2} \hat{k} \right) \\ &= \frac{ml^2\omega_1}{12} (\hat{j}) + \frac{ml^2\omega_1}{4} (\hat{j}) - \frac{m\omega_1 lR}{2} (\hat{i}) \\ &= \frac{ml^2}{3} \omega_1 (\hat{j}) - \frac{ml}{2} R \omega_1 (\hat{i}) \quad \dots(i) \end{aligned}$$

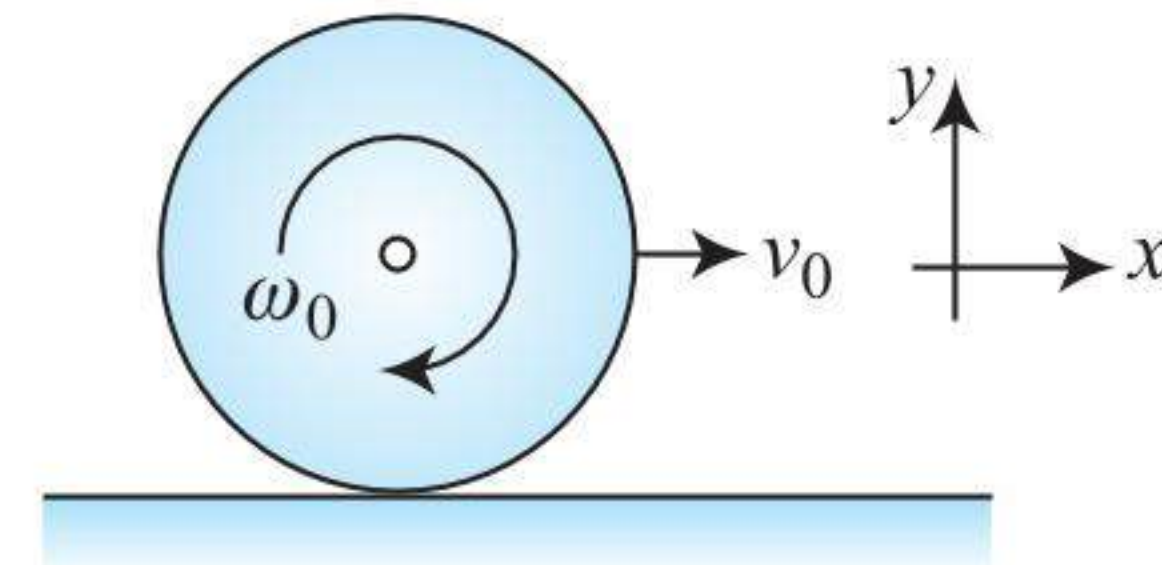
For disc:

$$\begin{aligned} \vec{L}_0 &= \left(\frac{-mR^2}{2} \omega_2 \hat{i} \right) + \left(\frac{mR^2}{4} \omega_1 \hat{j} \right) + (l\hat{i} + R\hat{j}) \times m(-\omega_1 l \hat{k}) \\ &= \left(\frac{-mR^2}{2} \omega_2 \right) \hat{i} + \left(\frac{mR^2}{4} \omega_1 \right) \hat{j} + (ml^2 \omega_1) \hat{j} - mRl \omega_1 (\hat{i}) \quad \dots(ii) \end{aligned}$$

So total angular momentum by adding Eqs. (i) and (ii),

$$\vec{L} = \frac{4ml^2}{3} \omega_1 \hat{j} - \left(\frac{mR^2}{2} \omega_2 - \frac{3}{2} m\omega_1 lR \right) \hat{i} + \frac{mR^2}{4} \omega_1 \hat{j}$$

6. Since $\vec{L}_p = \vec{L}_{\text{cm}} + \vec{r} \times \vec{p}_{\text{cm}} = I_{\text{cm}} \omega_0 (-\hat{k}) + Mv_0 R (-\hat{k})$



Since sphere is in pure rolling motion hence $\omega = v_0/R$

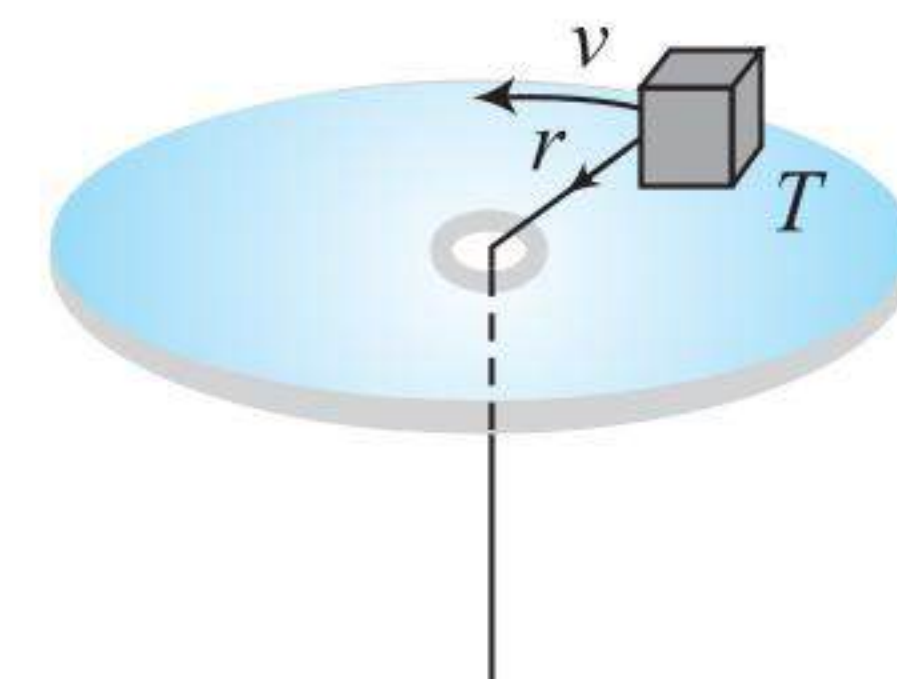
$$\Rightarrow \vec{L}_p = \left[\frac{2}{5} MR^2 \left(\frac{v_0}{R} \right) + Mv_0 R \right] (-\hat{k}) = \frac{7}{5} Mv_0 R (-\hat{k})$$

Exercise 4.2

1. From conservation of angular momentum for the isolated system of two disks:

$$(I_1 + I_2)\omega_f = I_1\omega_i \quad \text{or} \quad \omega_f = \frac{I_1}{I_1 + I_2} \omega_i$$

2. The tension of the rope is the only net force on the block and it does not exert any torque about the axis of rotation. Hence, the angular momentum of the block about the axis should remain conserved.



$$\Rightarrow mvr = \text{constant}$$

$$\text{Let } r_1 = 0.5 \text{ m and } v_1 = 4 \text{ m/s}$$

Let r_2, v_2, T_2 be the radius, velocity and tension, respectively, when the string breaks.

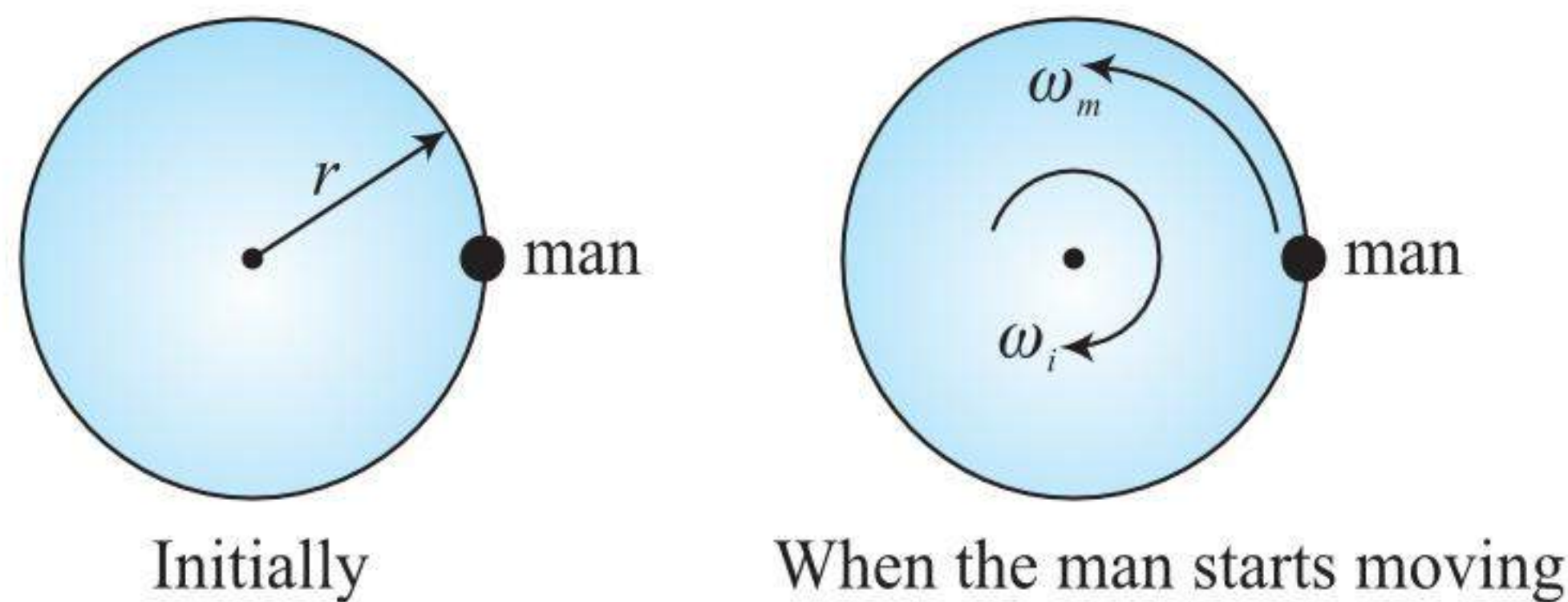
$$\Rightarrow T_2 = 600 \text{ N}$$

$$mv_1 r_1 = mv_2 r_2 \quad \text{and} \quad T_2 = mv_2^2 / r_2$$

$$\Rightarrow mv_1 r_1 = m \sqrt{\frac{r_2 T_2}{m}} r_2$$

$$r_2 = \left(\frac{mv_1^2 r_1^2}{T_2} \right)^{\frac{1}{3}} = \left(\frac{4 \times 16 \times 0.25}{600} \right)^{\frac{1}{3}} = \left(\frac{16}{600} \right)^{\frac{1}{3}} \approx 0.3 \text{ m}$$

3. As for the man and table system, initial angular momentum is zero and it will remain zero as no external torque about rotation axis. When the man starts moving in the anticlockwise direction to make angular momentum zero, the table will start rotating in the clockwise direction.



- (a) By conservation of angular momentum on the man-table system, $L_i = L_f$

$$\Rightarrow 0 + 0 = I_m \omega_m + I_t \omega_t$$

$$\Rightarrow \omega_t = -\frac{I_m \omega_m}{I_t}, \text{ where } \omega_m = \frac{v}{r} = \frac{1}{2} \text{ rad/s}$$

$$= -\frac{100(2)^2 \times \left(\frac{1}{2}\right)}{4000}$$

(negative sign represents clockwise direction)

$$\Rightarrow \omega_t = -\frac{1}{20} \text{ rad/s}$$

Thus, the table rotates clockwise (opposite to the man) with angular velocity 0.05 rad/s.

- (b) If the man completes one revolution relative to the table. Angular displacement of the man w.r.t. table is $\theta_{mt} = 2\pi$.

$$\Rightarrow 2\pi = \theta_m - \theta_t \text{ or } 2\pi = \omega_m t - \omega_t t$$

(where t is the time taken)

$$t = \frac{2\pi}{\omega_m - \omega_t} = \frac{2\pi}{0.5 - (-0.05)} = \frac{2\pi}{0.55} \text{ s}$$

Angular displacement of table is

$$\theta_t = \omega_t t = 0.05 \times \left(\frac{2\pi}{0.55} \right) = \left(\frac{2\pi}{11} \right) \text{ rad}$$

The table rotates through $(2\pi/11)$ rad clockwise.

- (c) If the man completes one revolution relative to the earth, then

$$\theta_m = 2\pi$$

$$\text{Time } t = \frac{2\pi}{\omega_m} = \frac{2\pi}{0.5} \text{ s}$$

During this time, angular displacement of the table

$$\theta_t = \omega_t(t) = 0.05 \times \left(\frac{2\pi}{0.5} \right) = \frac{\pi}{5} \text{ rad} = 36^\circ$$

(in clockwise direction)

4. There will be no torque on the system if we ignore frictional torque at the axle. Therefore, the angular momentum of the system is conserved.

$$\vec{L}_{\text{initial}} = \vec{L}_{\text{final}}$$

$$I_{\text{initial}} \omega_0 = I_{\text{final}} \omega$$

$$4ma^2 \omega_0 = 4m(2a)^2 \omega$$

$$\text{from which we have, } \omega = \frac{\omega_0}{4}$$

5. When the girl lands on the plank, she exerts a force on it, similarly the plank exerts an equal and opposite force on the girl. For the system consisting of girl and plank these forces are internal; therefore angular momentum is conserved.

$$\vec{L}_i = \frac{1}{2} ml \sqrt{2gh}$$

$$\vec{L}_f = \frac{1}{6} m l^2 \omega + \frac{1}{4} m l^2 \omega$$

$$\vec{L}_f = \vec{L}_i$$

$$\frac{1}{2} ml \sqrt{2gh} = \frac{5}{12} m l^2 \omega$$

$$\Rightarrow \omega = \frac{6}{5} \sqrt{\frac{2gh}{l}}$$

6. Initially the system was at rest, thus the initial angular momentum was zero. As child jumps off from the platform, it gains an angular momentum in the opposite direction. This implies that the platform must also gain the same amount of angular momentum in opposite direction.

When the child jumps off, the platform gains an angular velocity ω . Then the net velocity of child with respect to earth is $u - R\omega$. As no external torque is present, the net angular momentum of the system must be finally zero, thus

$$m(u - R\omega) = I\omega$$

$$\omega = \frac{muR}{mR^2 + I}$$

7. Again in this case no external torque is acting on the system so we can equate the angular momentum before catch and after catch. If after catching the ball, platform starts rotating with an angular speed ω , we have

Angular momentum of the ball about centre of platform before catch = Angular momentum of the platform plus child plus ball after catch

$$m_1 u R = (I + mR^2 + m_1 R^2) \omega$$

$$\text{or } \omega = \frac{m_1 u R}{I + mR^2 + m_1 R^2}$$

Here we can conserve angular momentum with respect to the centre pivot of the platform. This is because when the child catches the ball, it has a linear momentum and due to it the platform tends to gain a linear momentum, which develops a normal reaction on the pivot in opposite direction and it will not allow the platform to move translationally. If the platform was resting on a smooth plane without pivot at the centre, after catching the ball, the platform would also move translationally with its rotational motion and in that case linear momentum of system will also remain conserved.

8. As no external torque is present, we can conserve angular momentum before and after starting the walk by the child. As the child walks in opposite direction to the rotation of platform, its angular momentum will be in opposite direction which will tend to decrease the total angular momentum, hence the angular speed of the platform must increase to maintain the angular momentum conservation we have

$$(I + mR^2) \omega_1 = I \omega_2 - m(u - R\omega_2)R$$

The term on left side of the above expression is the angular momentum of platform plus child system when the child was at rest and the first term on right side of the expression is the angular momentum of the platform with increased angular speed (ω_2), when the child starts walking. The second term on right side is the final angular momentum of the child. This term is negative due to his walking in opposite direction and we have taken the velocity of the child as $(u - R\omega_2)$ because u is the relative speed of the child on platform which is rotating with angular speed ω_2 in opposite direction.

Again as we have used in the above problem, be careful, that angular momentum conservation should always be used with respect to earth or some inertial reference frame.

$$\omega_2 = \frac{(I + mR^2) \omega_1 + muR}{I + mR^2}$$

9. For circular motion of a body tied to a string on a horizontal plane

$$\left(\frac{mv^2}{r}\right) = T$$

Here as tension is provided by the hanging mass M , i.e., $T = g$, therefore

$$\text{so, } \left(\frac{mv^2}{r}\right) = Mg$$

According to the given problem,

$$\frac{(mv_1^2/r_1)}{(mv_2^2/r_2)} = \frac{M_1g}{M_2g} = \frac{8}{1}$$

$$\text{or, } \frac{v_1^2 r_2}{v_2^2 r_1} = \frac{8}{1} \quad \dots(i)$$

Now as force T is central, so angular momentum is also conserved, i.e.,

$$mv_1 r_1 = mv_2 r_2 \quad \dots(ii)$$

So substituting the value of v_1/v_2 from Eq. (ii) in Eq. (i),

$$\left[\frac{r_2}{r_1}\right]^2 \times \frac{r_2}{r_1} = \frac{8}{1}$$

$$\text{i.e., } \frac{r_2}{r_1} = 2 \quad \dots(iii)$$

$$\text{so that } \frac{\Delta r}{r_1} = \frac{r_2 - r_1}{r_1} = \frac{r_2}{r_1} - 1 = 2 - 1 = 1$$

Furthermore as in circular motion, $v = r\omega$

$$\text{so, } \frac{\omega_2}{\omega_1} = \frac{v_2}{v_1} \times \frac{r_2}{r_1} = \left[\frac{r_1}{r_2}\right]^2 \quad \left[\text{as from Eq. (ii), } \frac{v_2}{v_1} = \frac{r_1}{r_2}\right]$$

$$\Rightarrow \frac{\omega_2}{\omega_1} = \left[\frac{1}{2}\right]^2 = \frac{1}{4} \quad \left[\text{as from Eq. (iii), } \frac{r_2}{r_1} = 2\right]$$

$$\text{so, } \frac{\Delta\omega}{\omega} = \frac{\omega_2 - \omega_1}{\omega_1} = \frac{\omega_2}{\omega_1} - 1 = \frac{1}{4} - 1 = -\frac{3}{4}$$

Exercise 4.3

1. (a) Equation of motion

$$F_1 + F_2 = ma$$

$$\text{or } 2F + F = ma \Rightarrow a = \frac{3F}{m} \quad \dots(i)$$

Torque equation about centre of mass,

$$2F \cdot \frac{l}{2} - F \cdot x = \left(\frac{ml^2}{12}\right) \alpha$$

$$\Rightarrow \alpha = \frac{12F(l-x)}{ml^2} \quad \dots(ii)$$

- (b) If point P does not acceleration

$$\vec{a}_P = \vec{a}_{P,C} + \vec{a}_C$$

$$\text{As } a_P = 0 = -\alpha x + a \Rightarrow \alpha x = a$$

$$\frac{12F(l-x) \cdot x}{ml^2} = \frac{3F}{m}$$

$$\text{which given } x = \frac{l}{2}$$

$$2. \text{ Force equation, } F - 2F = ma \Rightarrow a = -\frac{F}{m}$$

$$\text{Hence } \vec{a} = -\frac{F}{m} \hat{i}$$

Torque equation about centre of mass

$$F \cdot \frac{l}{2} + \left(\frac{2}{3}l - \frac{l}{2}\right) = I_C \alpha$$

$$F \cdot \frac{l}{2} + F \cdot \frac{l}{6} = \frac{ml^2}{12} \cdot \alpha$$

$$\frac{3}{2}Fl = \frac{ml^2}{12} \cdot \alpha \Rightarrow \alpha = \frac{8F}{ml}$$

$$\text{Hence } \vec{\alpha} = -\frac{8F}{ml} \hat{k}$$

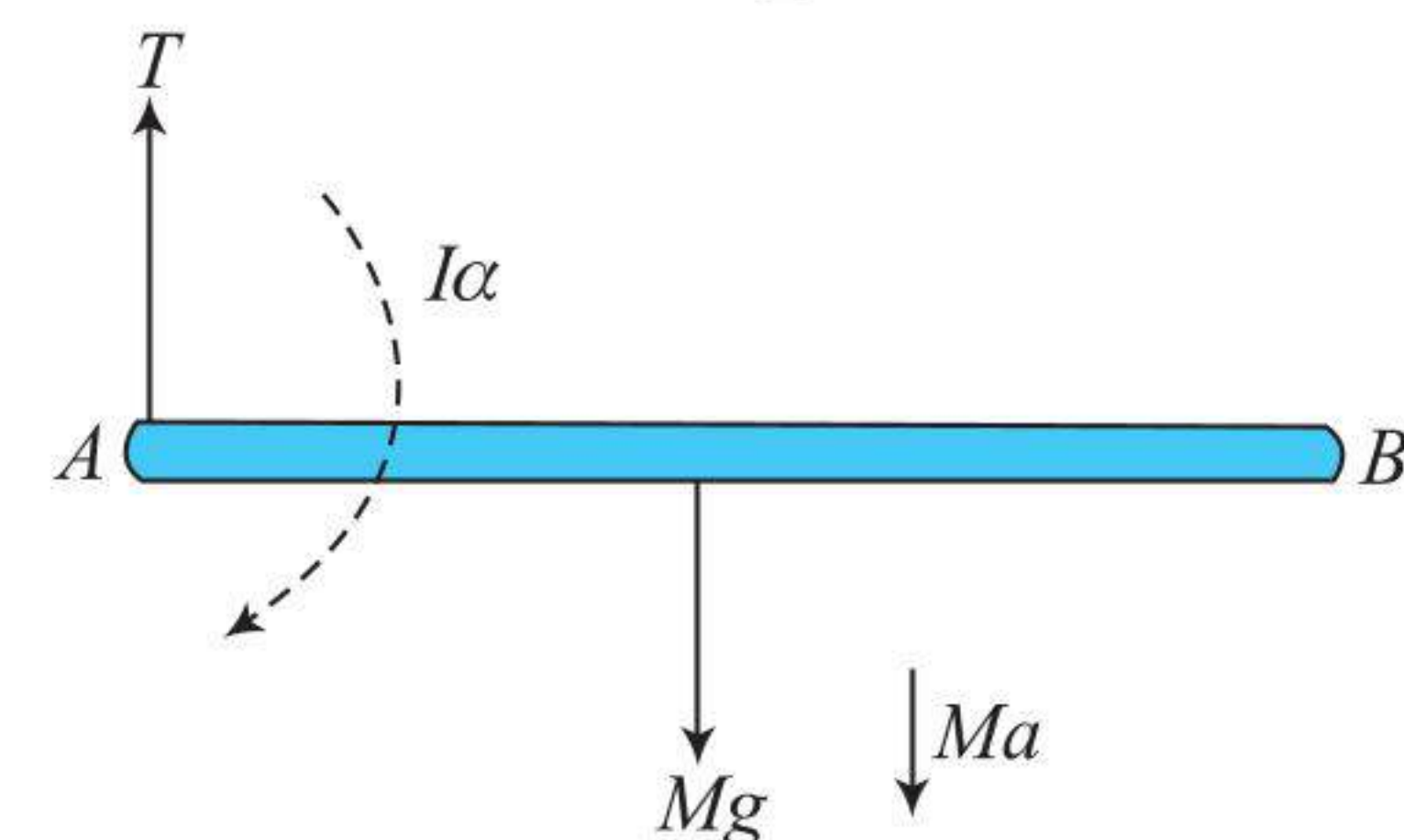
3. When right string snaps, only two forces act on the rod,

(i) Weight Mg of the rod, and

(ii) Tension T in left string

Since, both these two forces are vertical and non-collinear therefore they produce a moment and rod starts to rotate and has no horizontal acceleration. But string is inextensible, therefore left end of rod remains stationary. Hence, the rod rotates about its left end.

Let angular acceleration of rod be α clockwise. Then downward acceleration of its centre is $a = \frac{l}{2}\alpha$



Considering free body diagrams of the rod

For vertical forces, $Mg - T = Ma$

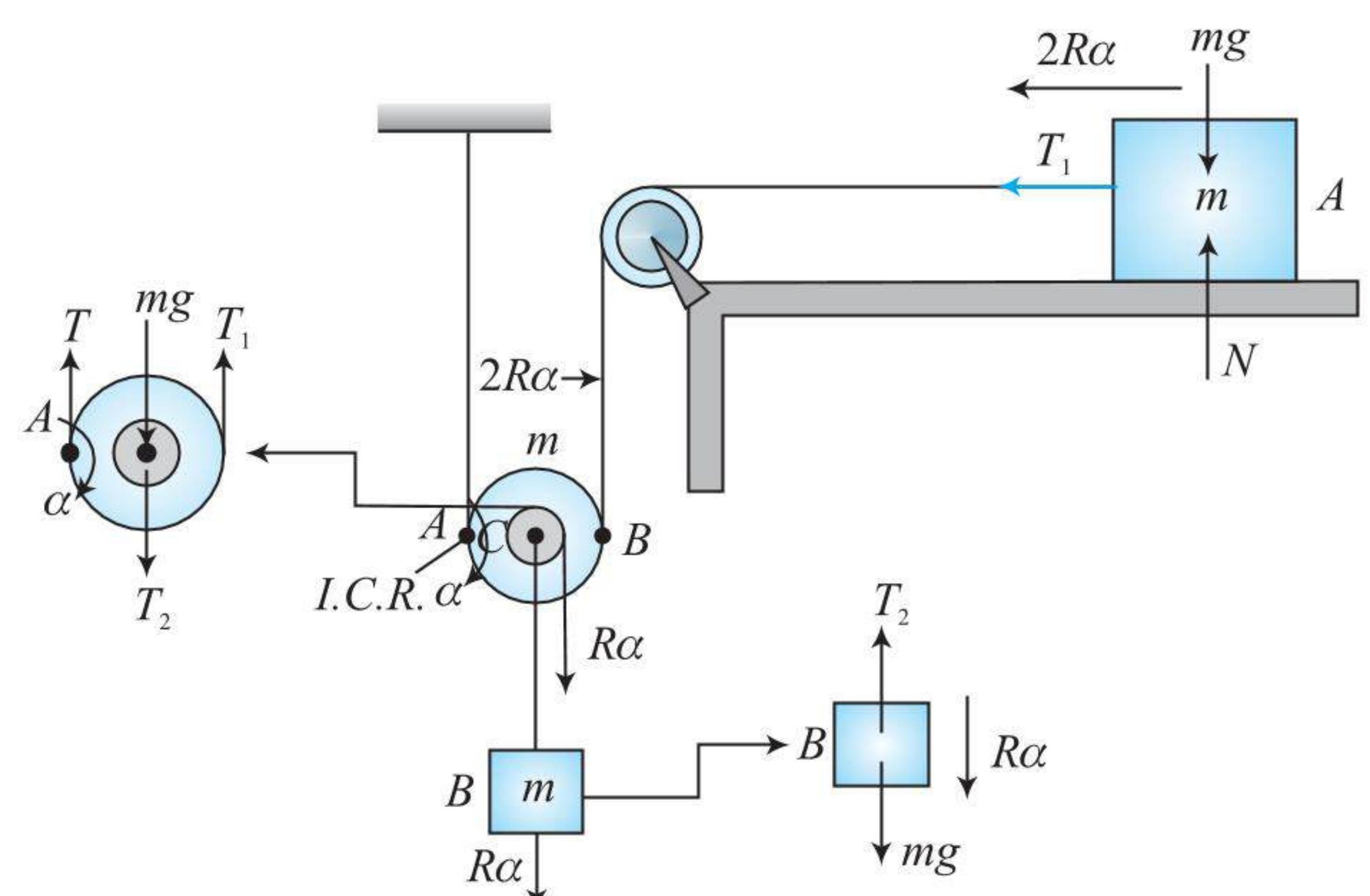
$$Mg - T = M \frac{la}{2} \quad \dots(i)$$

Point A is ICR. Applying torque equation about A . $Mg \frac{l}{2} = I_A \alpha$

$$Mg \frac{l}{2} = \frac{Ml^2}{3} \alpha \quad \dots(ii)$$

From Eqs. (i) and (ii), $T = \frac{1}{4} Mg$

4. The instantaneous axis of rotation will pass through the point A as shown in figure.



Let angular acceleration of disc be α (clockwise), then downward acceleration of its centre and block B will be equal to $a = R\alpha$ where R is the radius of the disc and acceleration of upper block A will be equal to the acceleration of point B , $b = 2R\alpha$ (leftward).

FBD of different parts will be as shown in figure.

$$\text{For horizontal forces on } A, T_1 = m(2R\alpha) \quad \dots(i)$$

$$\text{For vertical forces on } B, mg - T_2 = m(R\alpha)$$

$$\text{or } T_2 = mg - mR\alpha \quad \dots(ii)$$

Now taking moment of forces acting on the pulley about A ,

$$(mg + T_2) \cdot R - T_1 \cdot (2R) = I\alpha \quad \dots(iii)$$

$$\text{where, } I = \frac{3}{2}mR^2$$

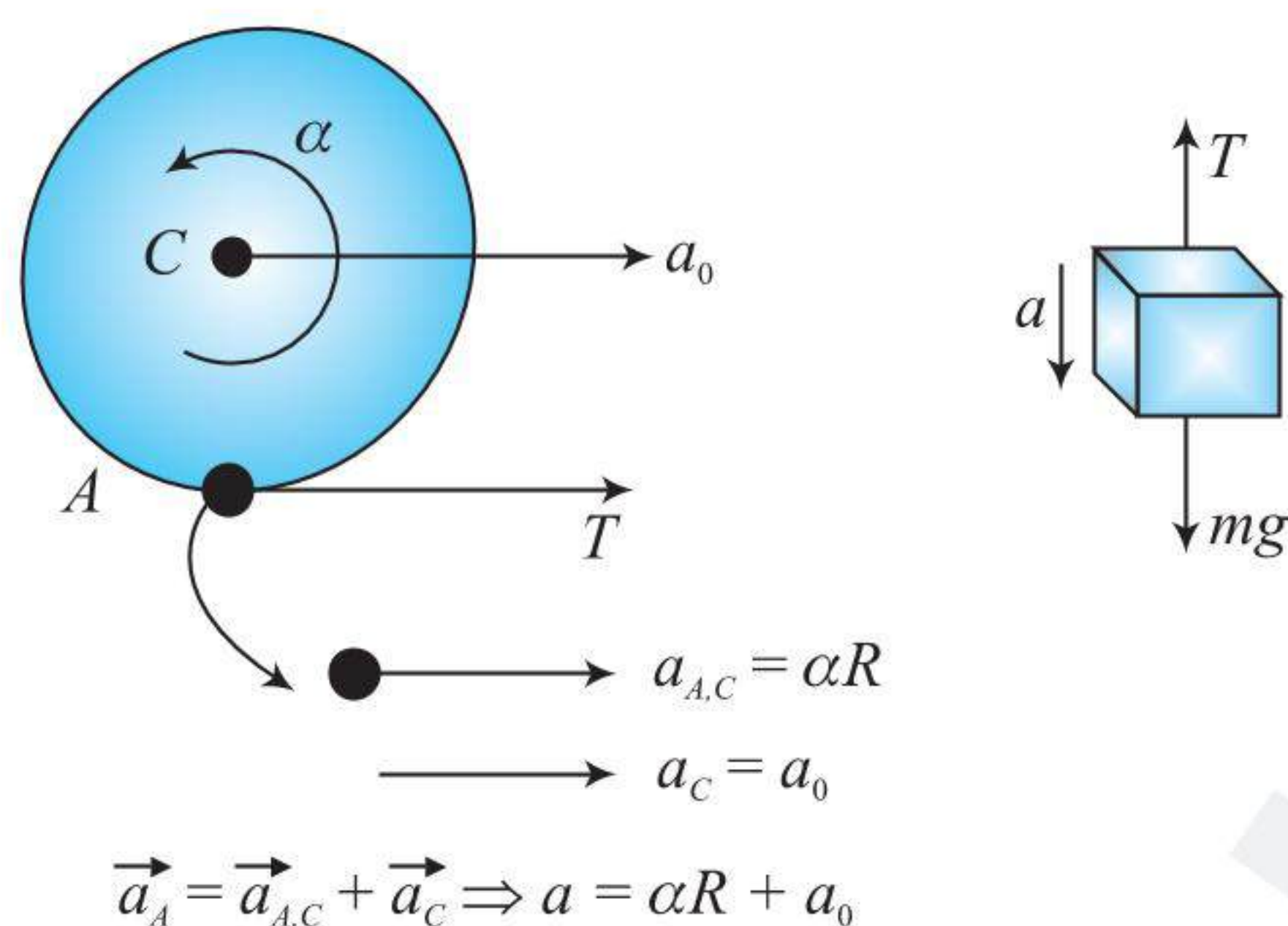
Substituting, $T_1 = 2mR\alpha$ and $T_2 = (mg - mR\alpha)$, we get

$$R\alpha = \frac{4g}{13}$$

Hence, acceleration of block B is equal to $\frac{4g}{13}$

Acceleration of block A is equal to, $b = 2R\alpha$ or $b = \frac{8g}{13}$

5. Let a = acceleration of mass m , a_0 = acceleration of COM of the disc, α = angular acceleration of the disc about its COM



Acceleration of point A is: $a_A = a_0 + R\alpha$

But a_A must be equal to a

$$\therefore a = a_0 + R\alpha \quad \dots(i)$$

$$\text{For motion of 'm': } mg - T = ma \quad \dots(ii)$$

$$\text{For translation of disc: } T = Ma_0 \quad \dots(iii)$$

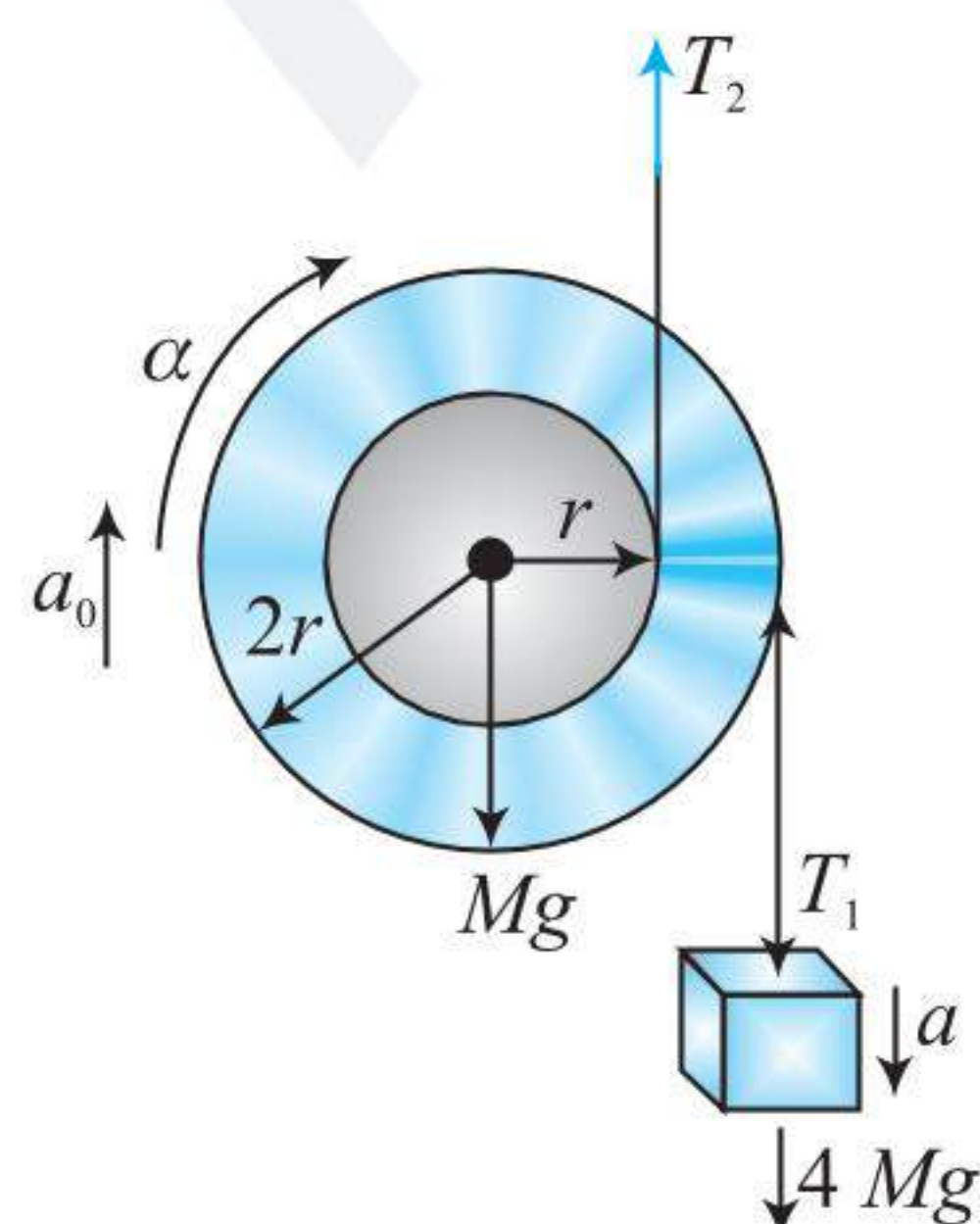
$$\text{For rotation of disc: } TR = \frac{1}{2}MR^2\alpha$$

$$\text{or } T = \frac{1}{2}MR\alpha \quad \dots(iv)$$

From above equations we get

$$a = \frac{3mg}{3m + M} \quad \text{and} \quad a_0 = \frac{mg}{3m + M}$$

6. Let α = angular acceleration of the pulley
 a_0 = upward acceleration of the centre of the pulley
 a = Acceleration of the block in downward direction.



The constraint relations are $a_0 = r\alpha$

$$\text{and } a = R\alpha - a_0 = 2r\alpha - r\alpha = r\alpha$$

For motion of the block:

$$4Mg - T_1 = 4M(r\alpha) \quad \dots(i)$$

For translation of pulley:

$$T_2 - T_1 - Mg = Ma_0 \quad \dots(ii)$$

For rotation of the pulley:

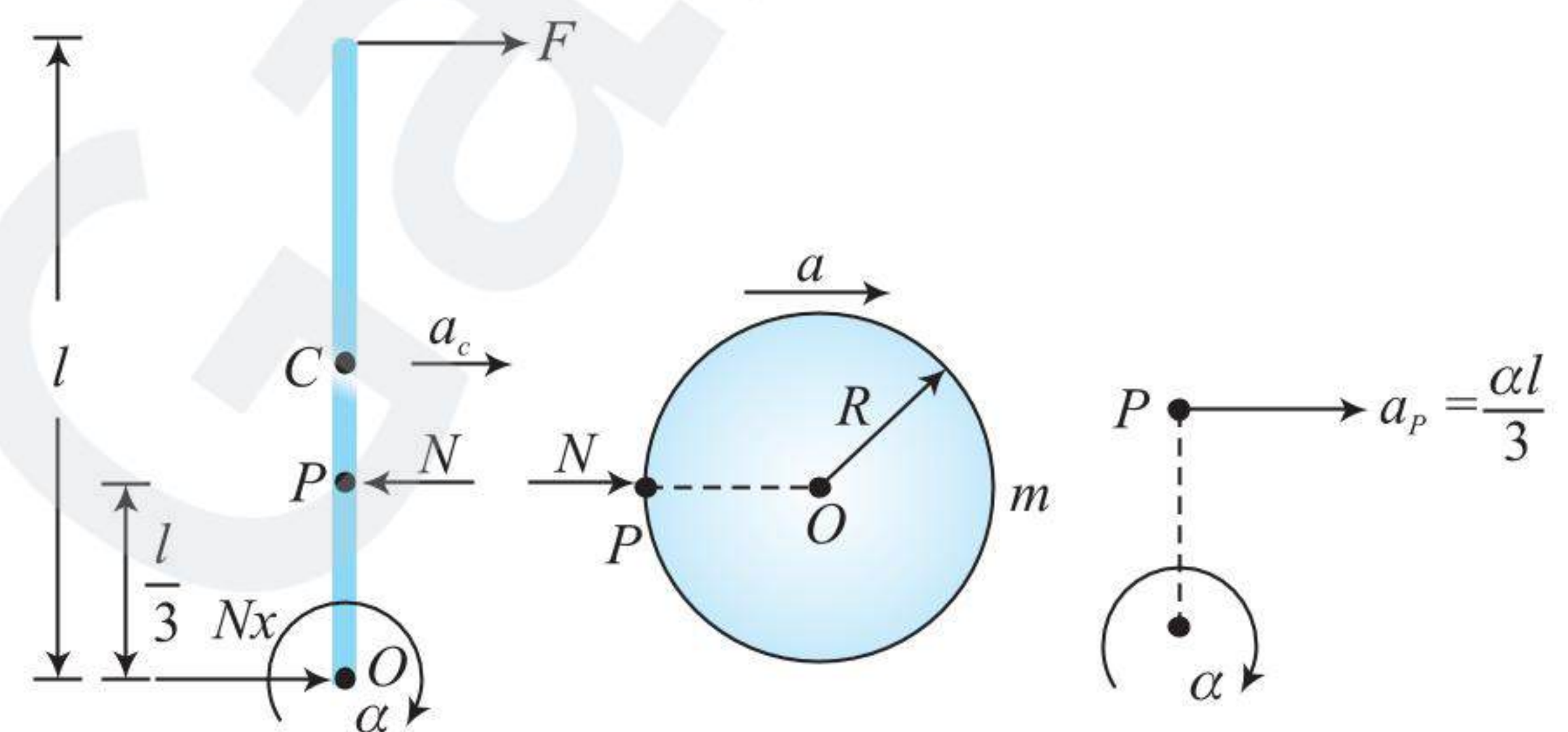
$$T_1(2r) - T_2(r) = I\alpha \quad \dots(iii)$$

Adding Eqs. (i), (ii) and (iii)

$$3Mg = \left(4 + 1 + \frac{1}{2}\right)Ma_0 \quad [\because r\alpha = a_0]$$

$$\therefore a_0 = \frac{6}{11}g$$

7. The following figure demonstrates the given condition.



Torque equation for rod about O,

$$I_0\alpha = Fl - N\frac{l}{3}$$

$$\therefore \frac{ml^2}{3} \cdot \alpha = Fl - \frac{Nl}{3} \quad \text{or} \quad m\frac{l\alpha}{3} = F - \frac{N}{3} \quad \dots(i)$$

The motion of the sphere is translational only

acceleration of the sphere (a) = acceleration of point P

$$\frac{l\alpha}{3} = a \quad \dots(ii)$$

$$\text{From (i) and (ii) } ma = F - \frac{N}{3} \quad \dots(iii)$$

$$\text{For sphere, } N = ma \quad \dots(iv)$$

Solving (iii) and (iv) we get, $a = \frac{3F}{4m}$ and $N = \frac{3F}{4}$

Acceleration of the COM of the rod is

$$a_{cm} = \alpha \frac{l}{2} = \frac{3a}{2} = \frac{9F}{8m}$$

Let horizontal component of hinge reaction is N_x

Force equation for rod,

$$F - \frac{3F}{4} + N_x = ma_{cm} \Rightarrow F - \frac{3F}{4} + N_x = m\left(\frac{9F}{8m}\right)$$

$$N_x = \frac{9F}{8} - \frac{F}{4} = \frac{7F}{8}$$

Exercise 4.4

1. Zero. Since lowest point of the sphere is stationary with respect to the plank, therefore friction force acting on the plank is zero. Thus, no force is required to keep the plank stationary.
2. The cylinder will roll without slipping so long as the friction force is sufficiently large and it never reaches the limiting value. The forces acting on the cylinder are: (a) its weight Mg , (b) normal reaction N , (c) tension T to the right, (d) f_f to the right.

Let a be the acceleration of the falling mass m .

Then $a_{\text{CM}} = \frac{a}{2}$

For the load,

$$mg - T = ma$$

$$\Rightarrow T = mg - ma$$

For the cylinder

$$T + f_{fr} = \frac{Ma}{2}$$

and $\tau = TR - f_{fr} R = \left(\frac{1}{2}MR^2\right) \alpha$

$$T - f_{fr} = \frac{1}{2}MR\alpha = \frac{1}{2}MR\left(\frac{a_{\text{CM}}}{R}\right) = \frac{1}{2}Ma_{\text{CM}} = \frac{1}{4}Ma$$

Solving $f_{fr} = \frac{1}{8}Ma$

$$T = \frac{3}{8}Ma \text{ and } a = \frac{mg}{m + \frac{3}{8}M}$$

There will be no slip if

$$f_{fr} \leq \mu mg \Rightarrow \mu \geq \frac{Ma}{8mg}$$

or $\mu \geq \frac{M}{8m + 3M}$

Rolling + slipping: Now $a_{\text{CM}} \neq \alpha R$, rather acceleration of the highest point of the cylinder $= a_{\text{CM}} + \alpha R =$ acceleration of m .

and $\tau = TR - f_{\text{lim}} R = \left(\frac{1}{2}MR^2\right) \alpha$

$$T - \mu Mg = \frac{1}{2}MR\alpha$$

For load: $mg - T = ma$

Solving for α ,

$$a = \frac{3mg - \mu Mg}{M + 3m}$$

3. Since the cylinder does not slip, the displacement, velocity and acceleration of the hanging mass are, respectively, equal to the displacement, velocity and acceleration of the upper point B of the cylinder as the string leaves the upper point tangentially.

Now v (velocity of the upper point) $= v_m + \omega r = 2v_{\text{CM}}$ ($\therefore \omega r = v_m$)

From conservation of energy, when m lowers through h

$$mgh = \frac{1}{2}mv^2 + \left(\frac{1}{2}mv_{\text{CM}}^2 + \frac{1}{2}I\omega^2\right)$$

($\therefore$ loss in PE = gain in KE)

$$= \frac{1}{2}mv^2 + \frac{1}{2} \frac{Mv^2}{4} + \frac{1}{2} \left(\frac{1}{2}Mr^2\right) \frac{v_{\text{CM}}^2}{r^2} \quad \left(\because I = \frac{1}{2}Mr^2\right)$$

$$= \frac{1}{2}mv^2 + \frac{1}{8}Mv^2 + \frac{1}{4}M \frac{v^2}{4}$$

$$= \frac{1}{2}mv^2 + \frac{3}{16}Mv^2$$

$$16mgh = 8mv^2 + 3Mv^2$$

$$\Rightarrow v^2 = \frac{16mgh}{8m + 3M}$$

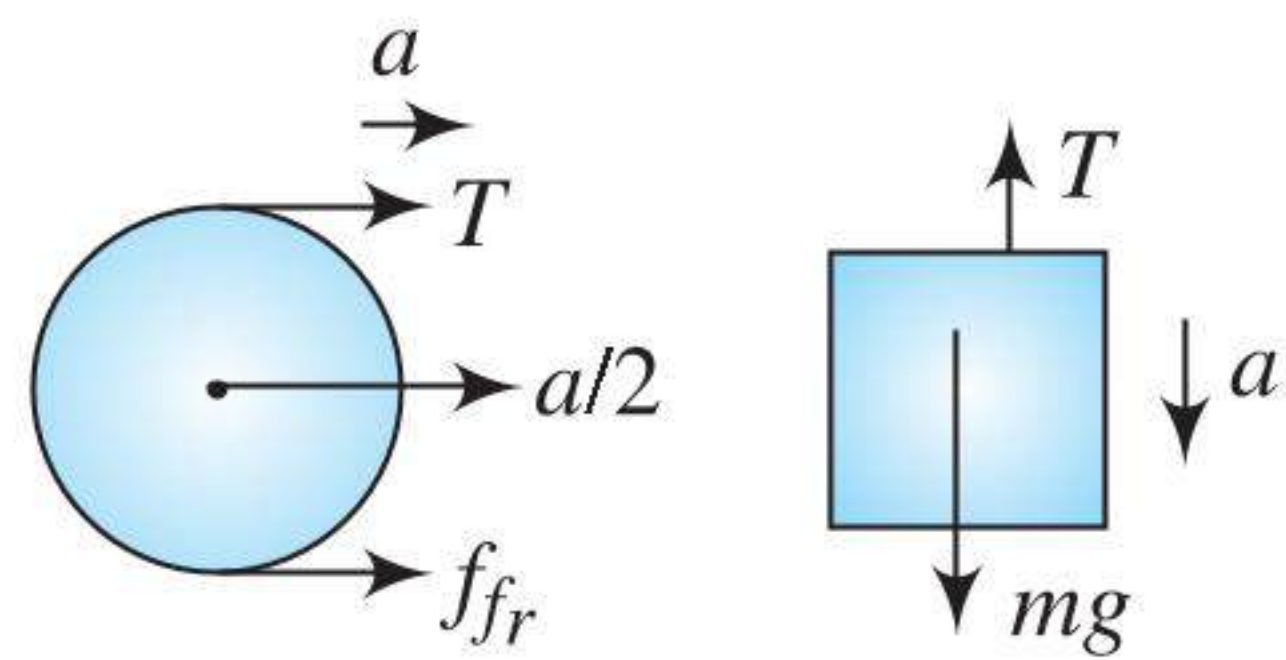
From $v^2 = 2ah$

$$\Rightarrow a = \frac{8mg}{8m + 3M} = \frac{g}{1 + 3M/8m} = 4 \text{ m/s}^2$$

Considering downward motion of m ,

$$mg - T = ma$$

or $T = mg - ma = 2(10 - 4) = 12 \text{ N}$



4. Let sphere rolls down with acceleration a' , writing force and torque equations for sphere

$$N = \frac{mg}{2} \quad \dots(i)$$

$$mg - f = ma' \quad \dots(ii)$$

$$fR = \left(\frac{2}{5}mR^2\right) \cdot \alpha$$

$$f = \frac{2}{5}mR\alpha \quad \dots(iii)$$

$$\alpha R = a' \quad \dots(iv)$$

$$\Rightarrow f = \frac{2}{5}ma' \text{ or } a' = \frac{5}{2} \frac{f}{m} \quad \dots(v)$$

From (ii) and (v),

$$mg - f = \frac{5}{2}f \Rightarrow mg = \frac{7}{2}f \text{ or } f = \frac{2}{7}mg$$

But, $f \leq \mu N$

$$\frac{2}{7}mg \leq \frac{\mu mg}{2} \text{ or } \mu \geq \frac{4}{7}$$

5. Since there is no slipping, the velocity of the point of contact of the string with the pulley is the vector sum of the velocity of the point of contact due to rotation plus the velocity of the centre of mass of the spool. That is

$$v_B = v_{\text{CM}} - \omega r, \omega \text{ is in clockwise direction}$$

But $v_{\text{CM}} = \omega R$

$$v_B = v_{\text{CM}} - v_{\text{CM}} \frac{r}{R} = v_{\text{CM}} \left(\frac{R-r}{R}\right)$$

This is also the velocity (v) of the rolling mass.

$$v_B = v_{\text{CM}} \left(\frac{R-r}{R}\right) = v \left(\frac{R-r}{R}\right)$$

Considering the conservation of energy when m lowers by h , we get

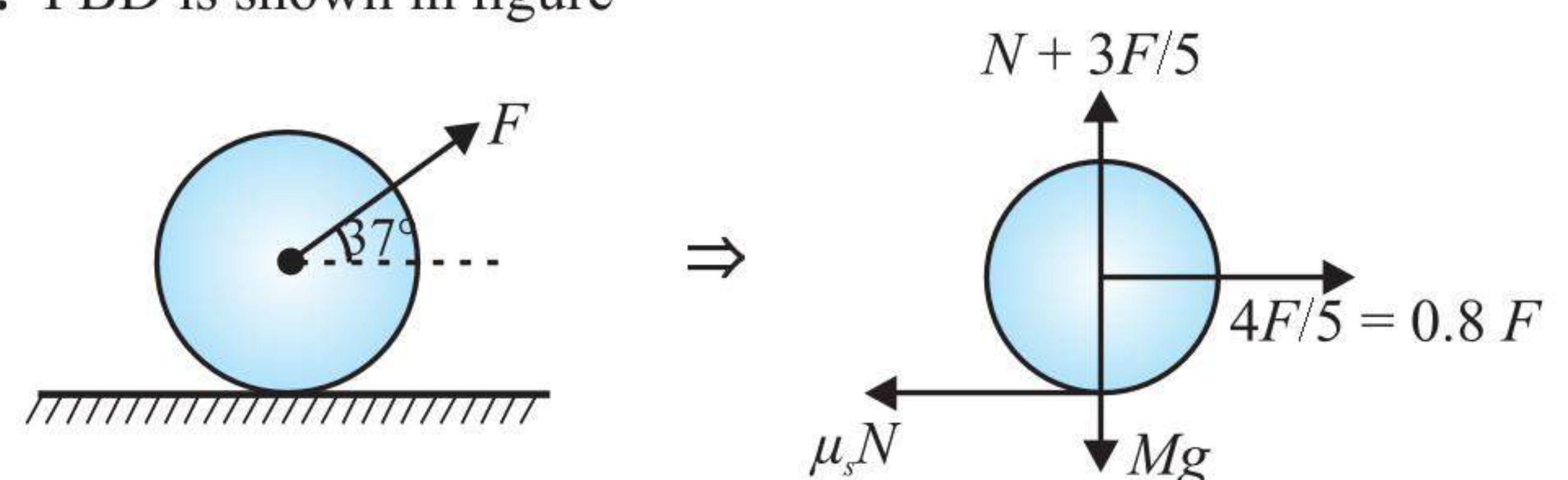
$$\begin{aligned} mgh &= \frac{1}{2}mv^2 + \left(\frac{1}{2}Mv_{\text{CM}}^2 + \frac{1}{2}I\omega^2\right) \\ &= \frac{1}{2}mv^2 + \left(\frac{1}{2}Mv_{\text{CM}}^2 + \frac{1}{2} \times \frac{1}{2}MR^2 \frac{v_{\text{CM}}^2}{R^2}\right) \\ &= \frac{1}{2}mv^2 + \frac{3}{4}Mv_{\text{CM}}^2 \\ &= \frac{1}{2}mv^2 + \frac{3}{4}Mv^2 \left(\frac{R}{R-r}\right)^2 \end{aligned}$$

$$v^2 = \frac{4mgh}{2m + 3M[R/(R-r)]^2}$$

comparing with $v^2 = 2ah$

$$a = \frac{v^2}{2h} = \frac{2mg}{2m + 3M[R/(R-r)]^2}$$

6. FBD is shown in figure



$$N = Mg - \frac{3F}{5} = Mg - 0.6F$$

For pure rolling, $a = R\alpha$

$$\text{or } \frac{0.8F - \mu_s N}{M} = \frac{(R)(\mu_s N)(R)}{MR^2/2} = \frac{2\mu_s N}{M}$$

$$\therefore 0.8 F = 3 (0.4) (Mg - 0.6 F)$$

$$\text{or } F = 0.79 Mg$$

Therefore, maximum value of F is $0.79 Mg$.

7. The force acting on the cylinder are (a) mg , downwards, (b) F , applied force downwards, (c) N , reaction of each plank upwards, (d) f_r , frictional force (not limiting) between each plank and the cylinder to the right.

Considering vertical motion

$$F + mg = 2N \quad \dots(i)$$

Considering horizontal motion

$$2f_r = ma \quad \dots(ii)$$

where a is the horizontal acceleration of the centre of mass of the cylinder to the right. Considering rotational dynamics of the cylinder about its axis

$$\tau = FR - 2f_r R = \left(\frac{1}{2}mR^2\right) \times \alpha$$

$$(F - 2f_r)R = \frac{1}{2}mR^2 \times \frac{a}{R} \quad (\because \alpha = a/R)$$

$$F - 2f_r = \frac{1}{2}ma$$

$$F = \frac{1}{2}ma + 2f_r = \frac{3}{2}ma \quad \dots(iii)$$

$$[(\because 2f_r = ma \text{ by Eq. (ii)}]$$

$$F = 3f_r \quad \dots(iv)$$

Now $f_r \leq f_{\text{lim}}$ or $f_r \leq \mu N$ ($\because f_{\text{lim}} = \mu N$)

$$\frac{1}{3}F \leq \mu \frac{F + mg}{2}$$

$$2F \leq 3\mu F + 3\mu mg$$

$$F \leq \frac{3\mu mg}{2 - 3\mu}$$

$$F_{\text{max}} = \frac{3\mu mg}{2 - 3\mu}$$

When the force is maximum, acceleration is automatically maximum.

$$a_{\text{max}} = \frac{2F_{\text{max}}}{3m} = \frac{2\mu g}{2 - 3\mu}$$

8. As the force F is applied, the cylinder tends to rotate along anticlockwise direction with the point of contact 'C' tending to move to the right. Hence, friction acts along the left (as shown in figure). If α is the angular acceleration of the cylinder, then from $\tau = I\alpha$, we have

$$r(F - f) = (mr^2/2)\alpha \quad \dots(i)$$

If a is the linear acceleration of the cylinder's CM, then from $F = ma$, we have

$$f = ma \quad \dots(ii)$$

For pure rolling,

$$a = \alpha r \quad \dots(iii)$$

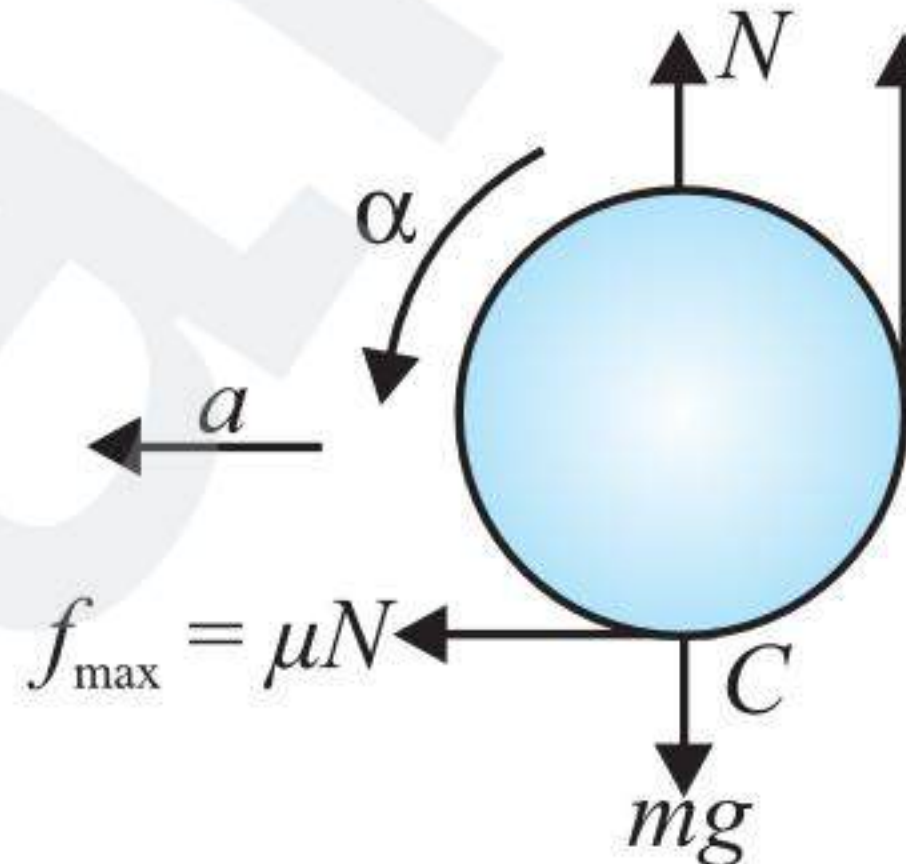
Dividing Eq. (i) by Eq. (ii), we get

$$\frac{(F - f)}{f} = \frac{r\alpha}{2a} = \frac{1}{2}$$

$$\Rightarrow F = (3f/2) \Rightarrow F_{\text{max}} = (3/2)f_{\text{max}}$$

$$[f_{\text{max}} = \mu N = \mu(mg - F)]$$

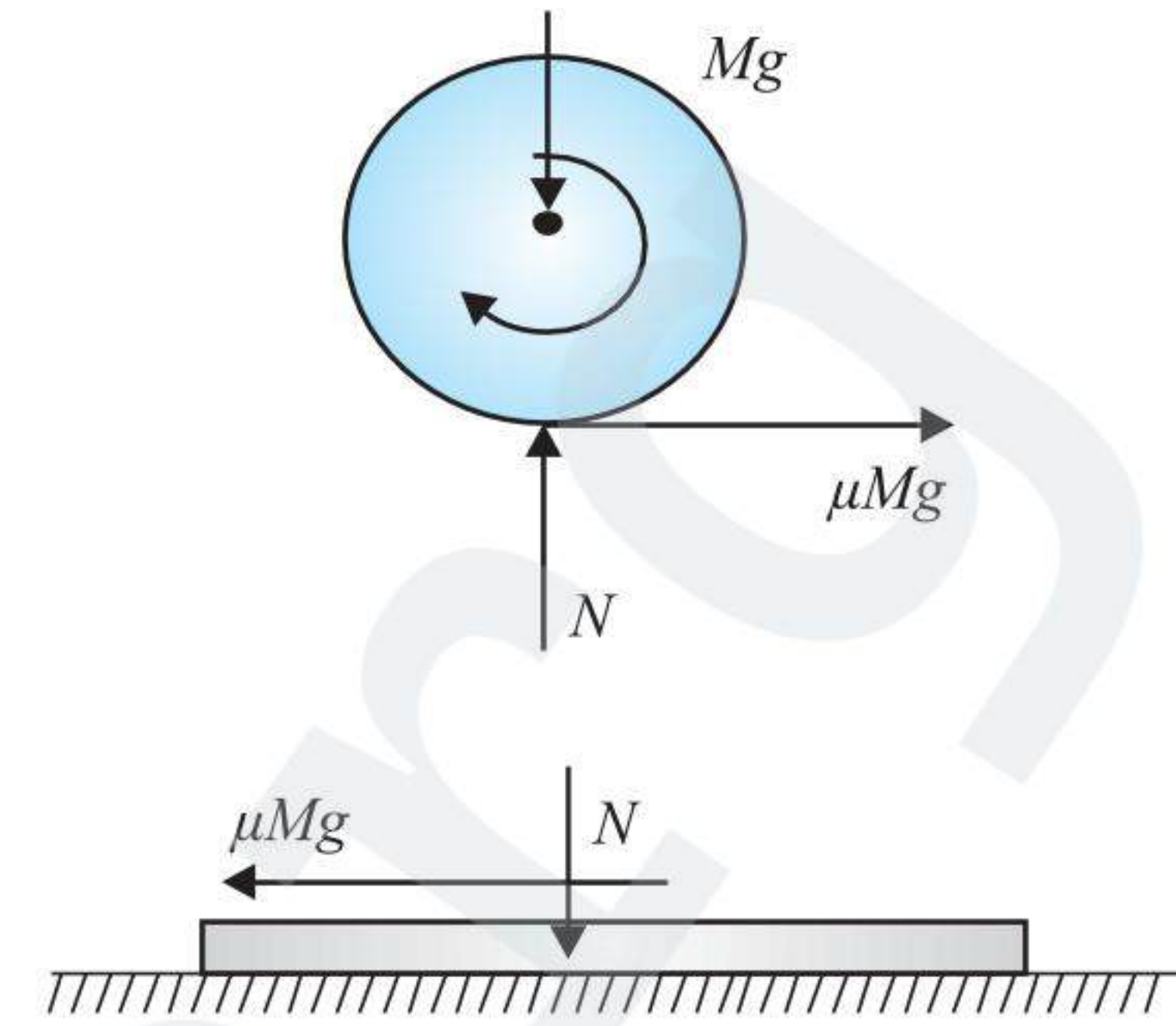
$$\Rightarrow F_{\text{max}} = \frac{3\mu}{2} [mg - F_{\text{max}}] = \frac{3\mu mg}{2 + 3\mu}$$



$$\text{From Eq. (ii), } a = \frac{f}{m}$$

$$\Rightarrow a_{\text{max}} = \frac{f_{\text{max}}}{m} = \frac{2}{3m} F_{\text{max}} = \frac{2}{3m} \left(\frac{3\mu mg}{2 + 3\mu} \right) = \frac{2\mu g}{2 + 3\mu}$$

9. Step 1: Free-body diagram,



Step 2: Equation of motion,

$$\text{Plank: } \mu Mg = MA \Rightarrow A = \mu g \quad (\text{backward})$$

(A will be positive)

$$\text{Sphere: } \mu Mg = Ma \Rightarrow a = \mu g \quad (\text{forward})$$

(a will be positive)

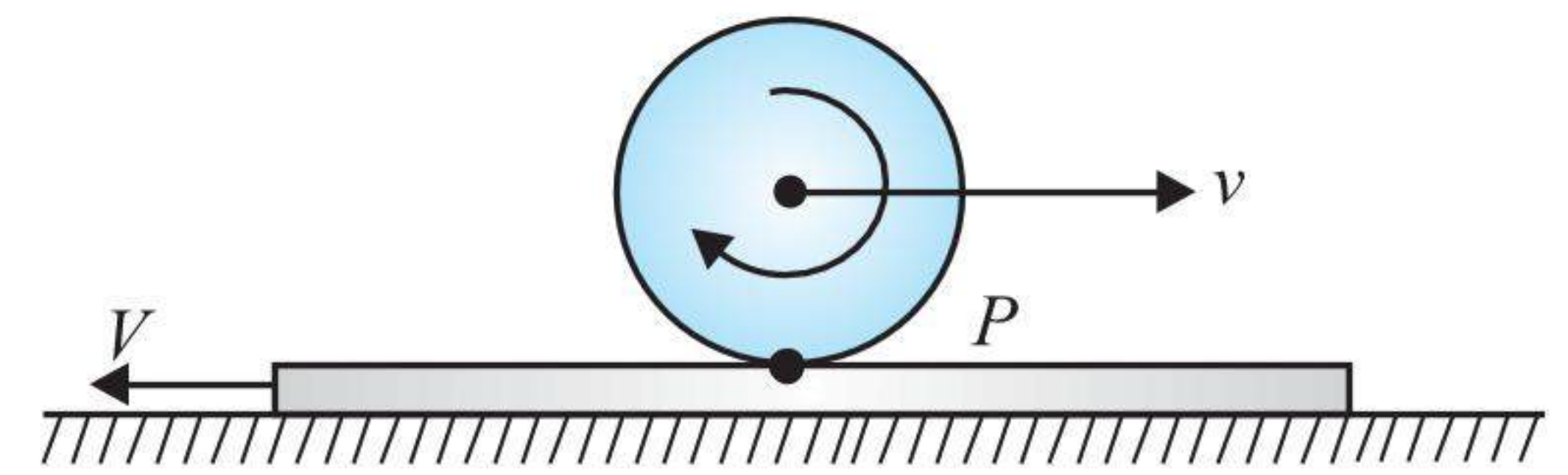
Step 3: Torque equation,

$$\mu Mg R = \left(\frac{2}{5}MR^2\right) \alpha$$

$$\alpha = \frac{5}{2} \frac{\mu g}{R} \quad (\text{anticlockwise})$$

(it will be negative)

Step 4: Constraint relation



$$V_p = V = \omega R - v$$

$$(0 + \mu g t) = \left(\omega_0 - \frac{5}{2} \frac{\mu g}{R} t\right) R - (0 + \mu g t)$$

$$\left(2\mu g t + \frac{5}{2}\mu g t\right) = \omega_0 R \Rightarrow \frac{9\mu g t}{2} = \omega_0 R$$

$$\Rightarrow t = \frac{2\omega_0 R}{9\mu g}$$

Velocity of the plank when rolling starts,

$$V = 0 + \mu g t$$

$$V_{\text{plank}} = \mu g \times \frac{2\omega_0 R}{9\mu g} = \frac{2}{9} \omega_0 R$$

Velocity of the sphere when rolling starts,

$$V_{\text{sphere}} = 0 + \mu g t = \frac{2}{9} \omega_0 R$$

Exercise 4.5

1. (a) Kinetic energy of rotation, $K_{\text{rotational}} = \frac{1}{2} I_C \omega^2$

$$= \frac{1}{2} \left(\frac{ml^2}{12} \right) \left(\frac{\sqrt{g}}{l} \right)^2 = \frac{1}{24} mgl$$

- (b) Kinetic energy of translation, $K_{\text{translational}} = \frac{1}{2} m v_c^2$

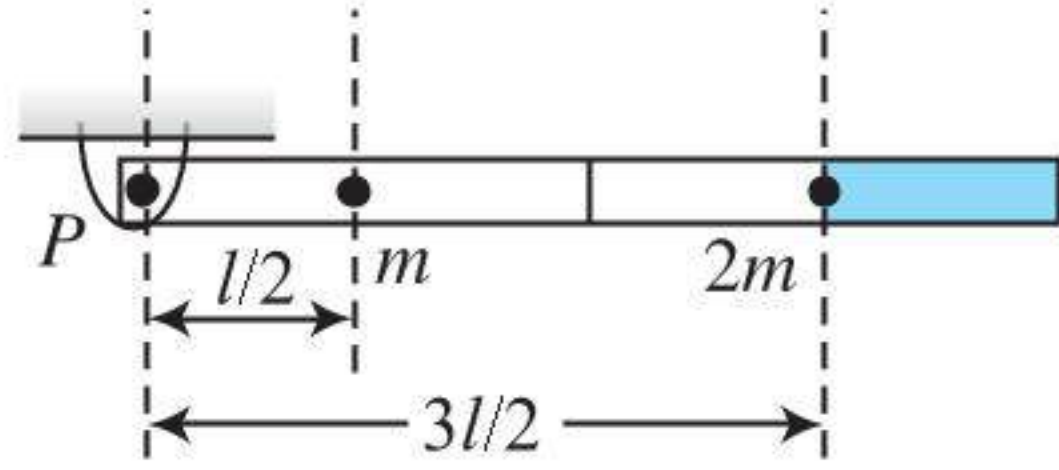
$$= \frac{1}{2} m \left[\left(\omega \frac{l}{2} \right)^2 \right] = \frac{1}{2} m \left[\sqrt{\frac{g}{l}} \times \frac{l}{2} \right]^2 = \frac{1}{8} mgl$$

(c) Total kinetic energy,

$$K_{\text{total}} = K_{\text{translational}} + K_{\text{rotational}}$$

$$= \frac{1}{6}mgl = \frac{1}{2}I_P\omega^2 = \frac{1}{2}\left[\frac{ml^2}{3}\right]\left[\sqrt{\frac{g}{l}}\right]^2 = \frac{1}{6}mgl$$

2. Moment of inertia of composite rod about P



$$I_P = \frac{ml^2}{3} + \frac{(2m)l^2}{12} + 2m\left(\frac{3}{2}l\right)^2 = 5ml^2$$

$$\text{Total kinetic energy, } K_{\text{total}} = \frac{1}{2}I_P\omega^2$$

$$= \frac{1}{2}(5ml^2)\omega^2 = \frac{5}{2}ml^2\omega^2 = 250 \text{ J}$$

3. $\Delta K + \Delta U = 0$

$$\left[\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 - 0\right] + \left[-mg\frac{l}{2}\right] = 0 \Rightarrow \omega = \sqrt{\frac{3g}{l}}$$

$$\text{Velocity of point } P, V_P = \omega l = \sqrt{3gl}$$

4. Using principle of conservation of mechanical energy, we have

$$\frac{mgl}{2} + mgl = \frac{1}{2}I\omega^2$$

where ω is the angular speed of the rod when it becomes vertical, and $I = (ml^2/3) + ml^2$, assuming mass of each rod is m .

Substituting value of I in the above equation, we get

$$\omega = \sqrt{\frac{9g}{4l}}$$

5. Using conservation of energy principle, we have fall in PE of the block = Gain in KE of the block + Rotational KE of the pulley + Rotational KE of the shell.

$$\text{or } mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}\left(\frac{2MR^2}{3}\right)\omega^2$$

$$\text{where } \omega = \frac{v}{r} \text{ and } \omega' = \frac{v}{R}$$

After substituting these values in the above equation and solving, we get

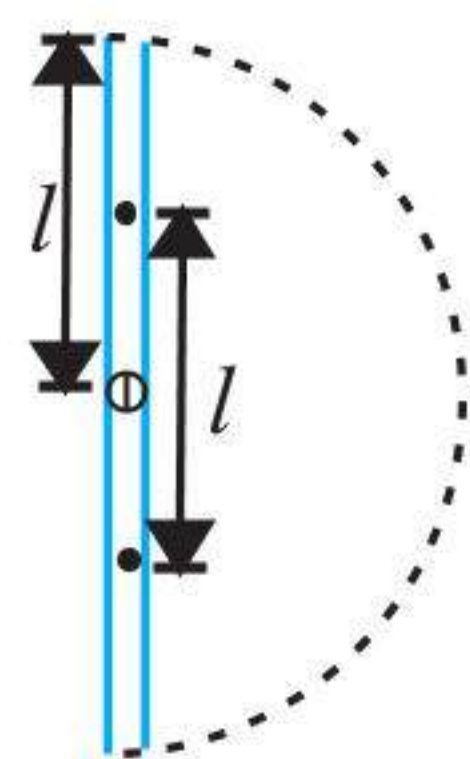
$$v^2 = \frac{mgh}{\left(\frac{m}{2} + \frac{I}{2r^2} + \frac{M}{3}\right)}$$

6. As the rod reaches its lowest position, the centre of mass is lowered by a distance l . Its gravitational potential energy is decreased by mg . As no energy is lost against friction, this should be equal to the increase in the kinetic energy. As the rotation occurs about the horizontal axis through the clamped end, the moment of inertia is $I = ml^2/3$. Thus,

$$\frac{1}{2}I\omega^2 = mg\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 = mg$$

$$\text{or } \omega = \sqrt{\frac{6g}{l}}$$

The linear speed of the free end is $v = l\omega = \sqrt{6gl}$



7. $\Delta K + \Delta U = 0$

$$\left(\frac{1}{2}I_0\omega^2 - 0\right) + \left[-mg\frac{l}{4}\right] = 0 \quad \dots(i)$$

$$I_0 = \frac{ml^2}{12} + m\left(\frac{l}{4}\right)^2 = \frac{7}{48}ml^2 \quad \dots(ii)$$

$$\text{From (i) and (ii), } \omega = 2\sqrt{\frac{6g}{7l}}$$

8. The conservation of energy yields

$$\Delta KE = \Delta PE$$

$$\frac{1}{2}I\omega^2 = mg(2r) + mgr$$

$$\Rightarrow \frac{1}{2}m(2r)^2\omega^2 + \frac{1}{2}\left\{\left(\frac{mr^2}{2}\right) + mr^2\right\}\omega^2 = 3mgr$$

$$\Rightarrow \frac{11}{4}mr^2\omega^2 = 3mgr \Rightarrow \omega = \sqrt{\frac{12g}{11r}}$$

$$9. \text{KE} = \frac{1}{2}\left(\frac{2}{5}\right)mv^2 \Rightarrow \text{KE} = \frac{2}{10}mv^2$$

If ω is to be doubled, v is also doubled.

$$\text{Therefore, } \Delta KE = KE_2 - KE_1 = \frac{2}{10}m[v_2^2 - v_1^2]$$

$$\Rightarrow \Delta KE = \frac{2}{10}m[r^2\omega_2^2 - r^2\omega_1^2]$$

$$= \frac{2}{10} \times 1 \times \left[\left(\frac{1}{10}\right)^2\right][4^2 - 2^2] = 2.4 \times 10^{-2} \text{ J}$$

10. Rolling can be considered as pure rotation about point of contact

$$K_{\text{rolling}} = \frac{1}{2}I_P\omega^2$$

$$= \frac{1}{2}\left(\frac{mR^2}{2} + mR^2\right)\omega^2 = \frac{3}{4}mR^2\omega^2 \quad \dots(i)$$

The rod translates with the velocity v hence velocity of centre of disc will also be v

$$v = \omega R \quad \dots(ii)$$

$$\text{Form (i) and (ii), } K_{\text{rolling}} = \frac{3}{4}mv^2$$

$$\text{Kinetic energy of the rod, } K_{\text{rod}} = \frac{1}{2}\left(\frac{m}{2}\right)v^2 = \frac{mv^2}{4} \quad \dots(iii)$$

$$K_{\text{total}} = K_{\text{rolling}} + K_{\text{rod}} = mv^2$$

11. In the process, angular momentum remains constant

$$I_1\omega_1 = I_2\omega_2$$

$$\text{where } I_1 = 2m(0.15)^2, \omega_1 = 4 \text{ rad/s}$$

$$I_2 = 2m(0.05)^2$$

Substituting these values in above equation, we get

$$\omega_2 = \frac{I_1\omega_1}{I_2} = 36 \text{ rad/s}$$

$$\text{Work done on the collar, } W = \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}I_1\omega_1^2$$

$$\text{or } W = \frac{1}{2}(0.05)^2 \times 36^2 - \frac{1}{2}2m(0.15)^2 \times 4^2 = 1.44 \text{ J}$$

$$\text{where } m = \frac{500}{1000} = 0.05 \text{ kg}$$

Exercise 4.6

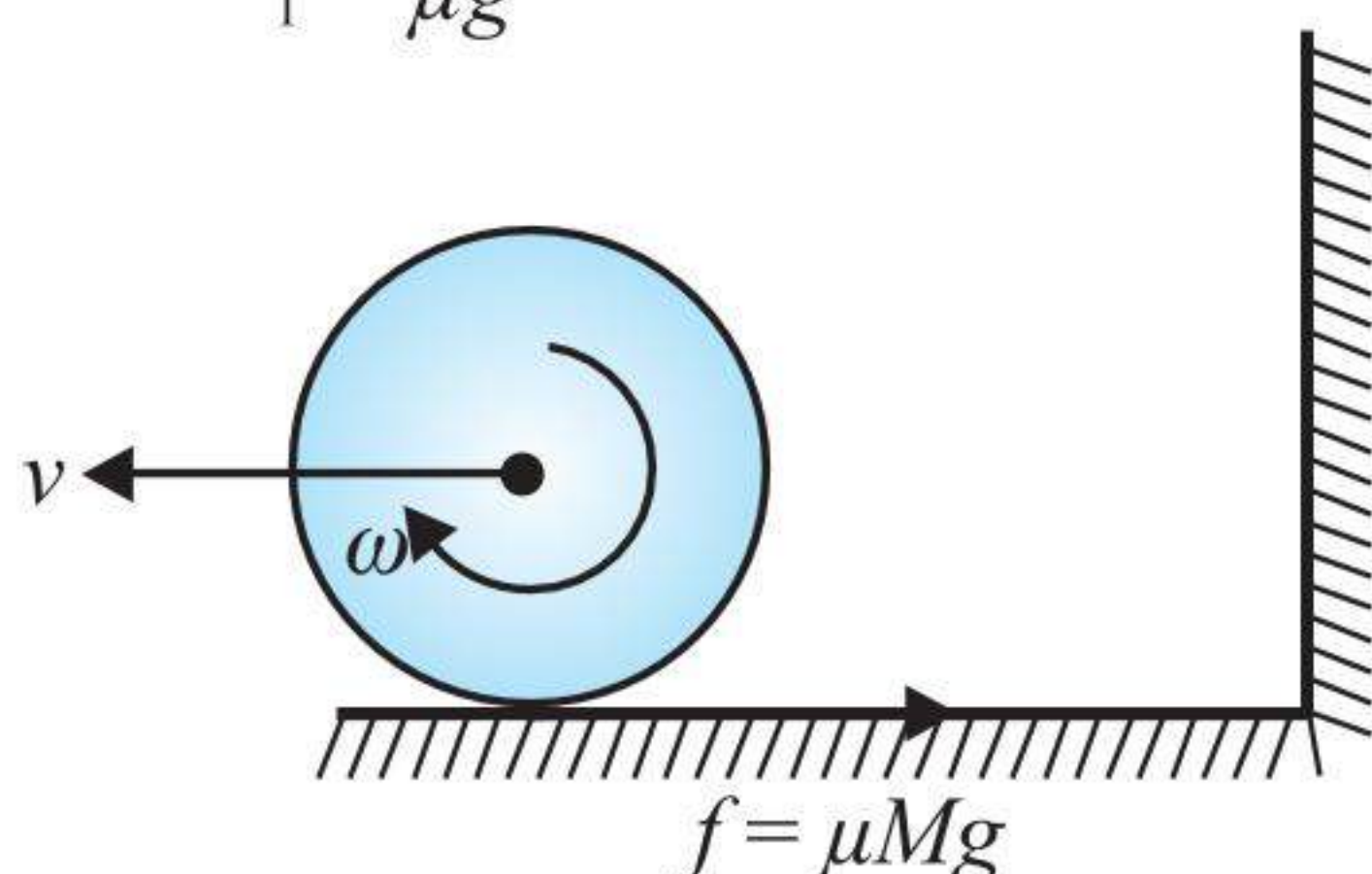
- (d) Since net force along the incline is zero, so cylinder will remain in position till it stops rotating. After that it will start moving downwards.
- (a) Since for $a = g \tan \theta$, no force acts along the incline, so it will continue its pure rolling.
- (a) If $v_0 > R\omega_0$, the sliding tendency of lowest point of the sphere will be in upward direction. Hence friction force will act downwards
If $v_0 < R\omega_0$, the sliding tendency of lowest point of the sphere will be in downward direction. Hence friction force will act upwards
If $v_0 = R\omega_0$, in this case the sphere is rolling up hence friction force will act upwards
- (d) Since energy is conserved for all the three bodies, so, final kinetic energy of all the three bodies will be same.
- (a) $v = \sqrt{\frac{2gh}{1+c}}$ where $c = \frac{k^2}{R^2}$
 c for ring = 1
 c for disc = $\frac{1}{2} = 0.5$
 c for sphere = $\frac{2}{5} = 0.4$
Hence, v (sphere) $>$ v (disc) $>$ v (ring)
- (b) We know $a = \frac{g \sin \theta}{1+c}$
Since a (sphere) $>$ a (disc) $>$ a (ring)
So, t (sphere) $<$ t (disc) $<$ t (ring)
Higher the acceleration smaller will be the time taken.

Exercise 4.7

- (a) True: Collision is elastic and the frictional impulse is not generated.
- (b) True: The net impulse passes through the point of contact with the wall. Therefore, angular impulse about the point of contact with the wall is zero and hence the angular momentum is conserved.
- (c) False: Angular momentum is not conserved because the net impulse exerts torque about a stationary point on the floor.
- (d) False: The point of contact with the horizontal surface moves away from the wall.
- (e) False: Friction force decreases both the speeds.
- (f) True: When the sphere starts rolling, it is moving away from the wall because the angular velocity becomes zero before the linear velocity. The linear acceleration of the CM of the sphere is $a = -\mu g$.

The angular acceleration about the CM is $\alpha = \frac{-5}{2} \frac{\mu g}{R}$.

The negative sign shows that both the accelerations have the retarding effect. The time after which the linear velocity will become zero is $t_1 = \frac{v}{\mu g}$.



and the time after which the angular velocity will become zero is

$$t_2 = \frac{2}{5} \frac{\omega R}{\mu g} = \frac{2}{5} \frac{v}{\mu g} \quad (\because v = \omega R)$$

Note that $t_2 < t_1$.

- (a) In the process of collision, the linear momentum remains constant
 $2mv - m(2v) + 0 = (2m + m + 8m) v_{CM}$
or $v_{CM} = 0$
- (b) Angular momentum of the system also remains constant. Therefore,
 $2mva + m(2v)(2a) + 0 = I\omega \quad \dots(i)$
where I is the moment of inertia of the system after collision.
 $I = MI$ of the rod + MI of the mass $2m$ + MI of the mass m
or $I = \frac{(8m)(6a)^2}{12} + 2ma^2 + m(2a)^2$
Substituting this value in Eq. (i), we get $\omega = \frac{v}{5a}$
- (c) Total energy after collision
 $E = \frac{1}{2} I \omega^2 = \frac{1}{2} (30m) a^2 \times \left(\frac{v}{5a}\right)^2$
or $E = \frac{3mv^2}{5}$

3. Conservation of linear momentum

$$mv_0 = M \frac{v_0}{8} - m \frac{v_0}{4}$$

$$8m = M - 2m \Rightarrow 10m = M$$

$$\Rightarrow M/m = 10$$

Conservation of angular momentum about the point of collision

$$0 = \frac{M(8l)^2}{12} \omega - M \frac{v_0}{8l}$$

$$\Rightarrow \omega = \frac{3v_0}{128l}$$

Using coefficient of restitution equation

$$\left[\left(\frac{v_0}{8} + \omega l \right) - \left(-\frac{v_0}{4} \right) \right] = e(v_0 - 0) \Rightarrow e = \frac{51}{128}$$

Velocity at end A

$$v_A = \frac{v_0}{8} + \omega \cdot 4l = \frac{28}{128} v_0$$

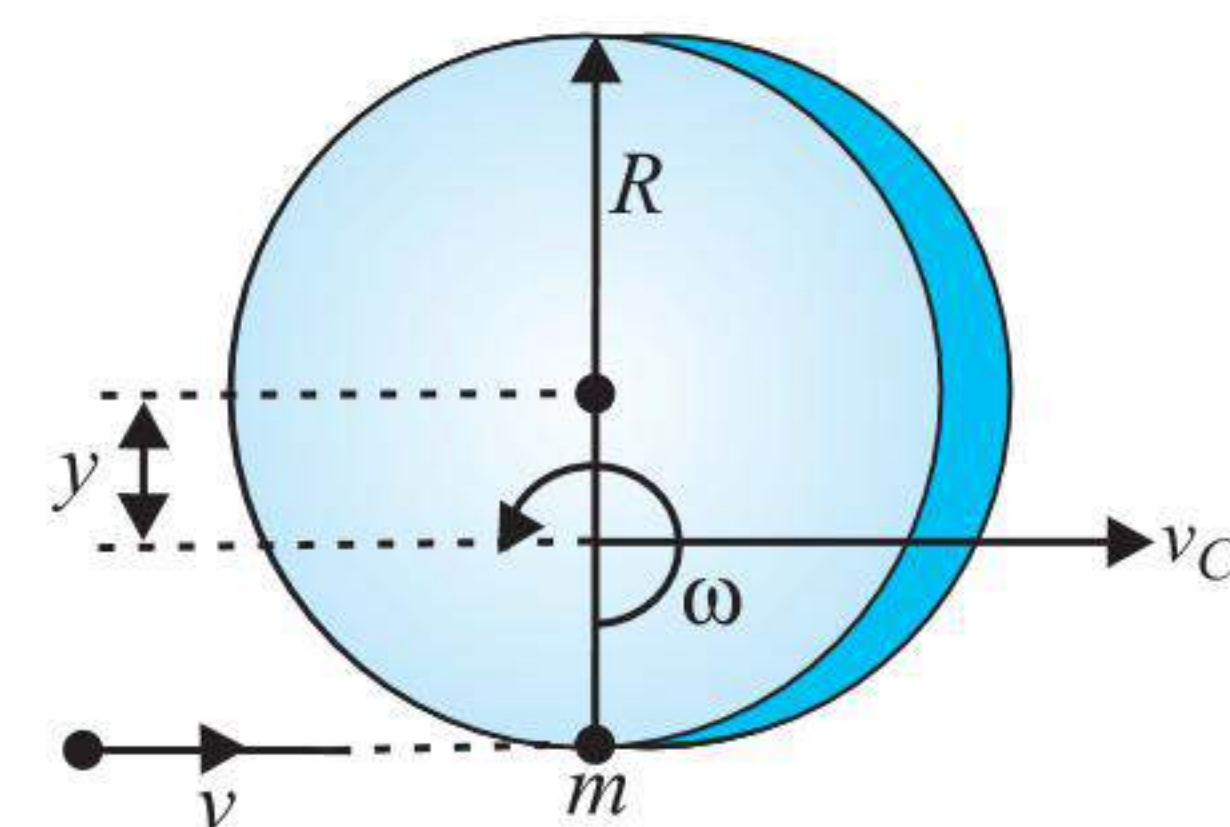
$$\Rightarrow v_A = \frac{7}{32} v_0$$

Velocity at end B

$$v_B = \frac{v_0}{8} + \omega \cdot 4l = \frac{4}{128} v_0$$

$$\Rightarrow v_B = \frac{1}{32} v_0$$

- Let the velocity of the CM of the system after the strike be v_c . By conservation of linear momentum,



$$mv = 2mv_c \Rightarrow v_c = \frac{v}{2}$$

$$\text{Position of CM, } y = \frac{m \cdot 0 + m \cdot R}{(m + m)} + m = \frac{R}{2}$$

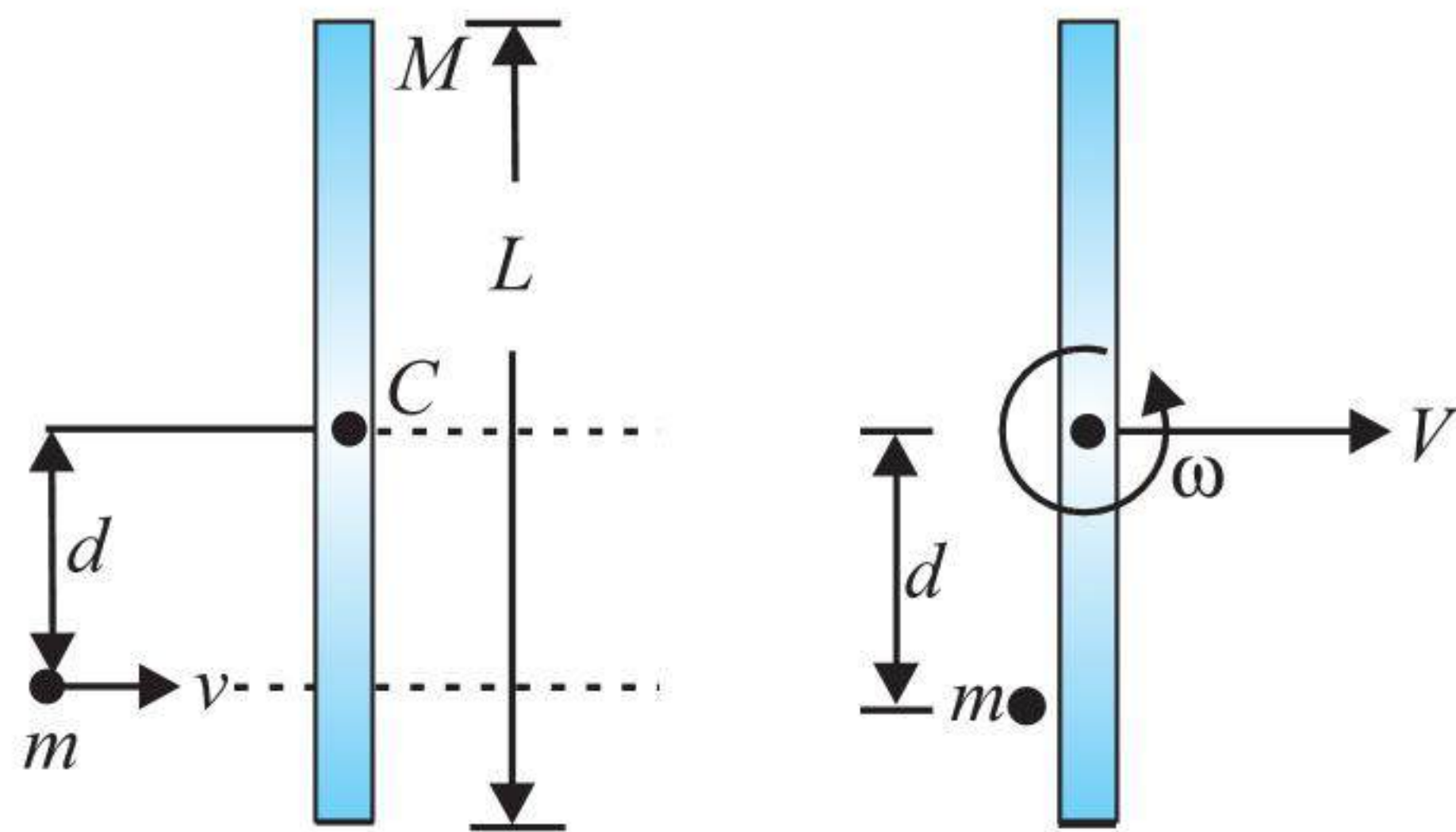
Using conservation of angular momentum about CM of the system (hoop + bullet),

$$mv \frac{R}{2} = I_c \omega = (I_{\text{bullet}} + I_{\text{hoop}})_c \omega$$

$$= \left[m \left(\frac{R}{2} \right)^2 + \left\{ mR^2 + m \left(\frac{R}{2} \right)^2 \right\} \omega \right]$$

After solving, we get $\omega = \frac{v}{R}$

5.



By conservation of linear momentum and angular momentum, we have

$$mv + 0 = m \cdot 0 + MV \Rightarrow V = \frac{mv}{M}$$

$$\text{and } mvd = I\omega \Rightarrow \omega = \frac{mvd}{I} = \frac{mvd}{\left(\frac{ML^2}{12} \right)}$$

Since collision is elastic, therefore

$$\frac{1}{2} mv^2 = \frac{1}{2} M \left(\frac{mv}{M} \right)^2 + \frac{1}{2} \left(\frac{ML^2}{12} \right) \left(\frac{mvd}{\frac{ML^2}{12}} \right)^2$$

$$\Rightarrow m = \frac{ML^2}{L^2 + 12d^2}$$

6. Let Δt be the duration of impact, then

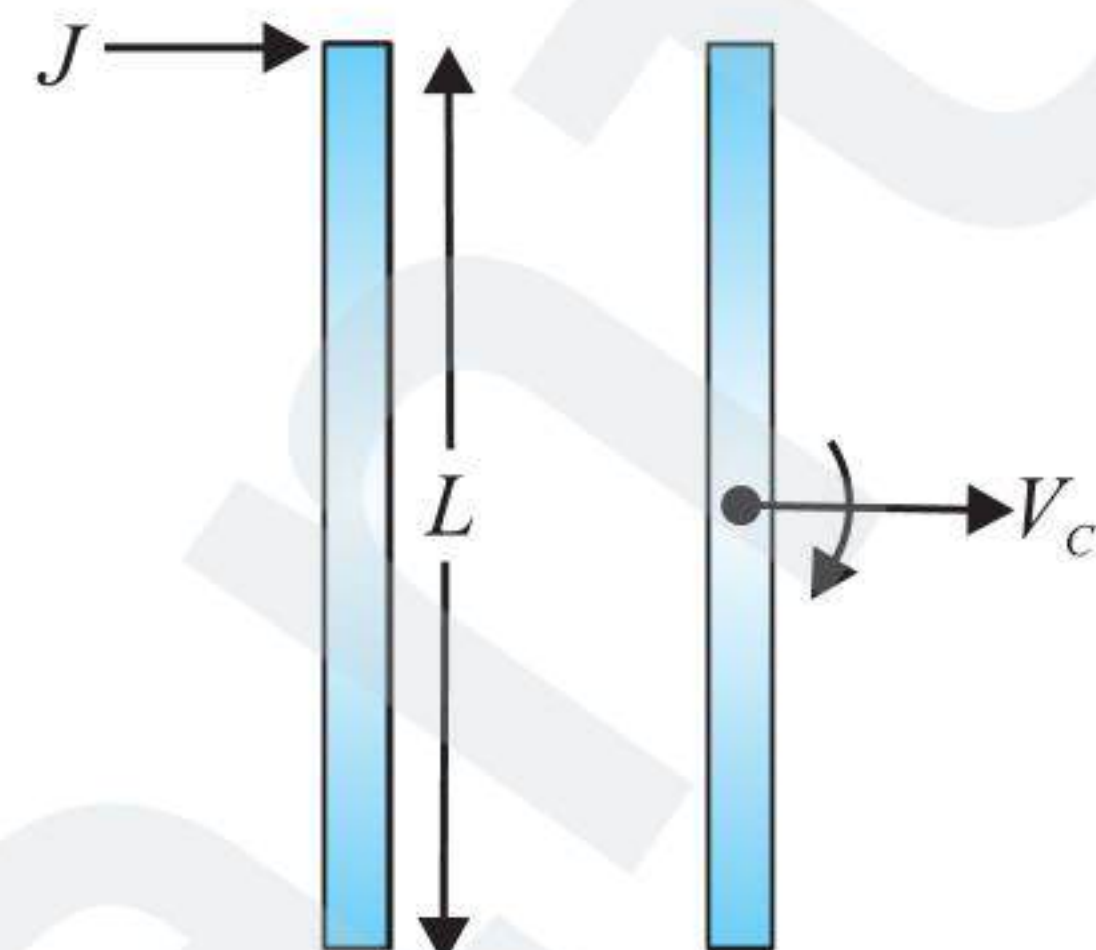
$$J = \Delta \vec{P}$$

$$\text{or } J = (Mv_c - 0) \quad \dots(i)$$

Also angular impulse is equal to change in angular momentum

$$\text{or } \frac{Jl}{2} = I\Delta\omega$$

$$= I(\omega - 0) = \frac{ml^2}{12} \omega \quad \dots(ii)$$



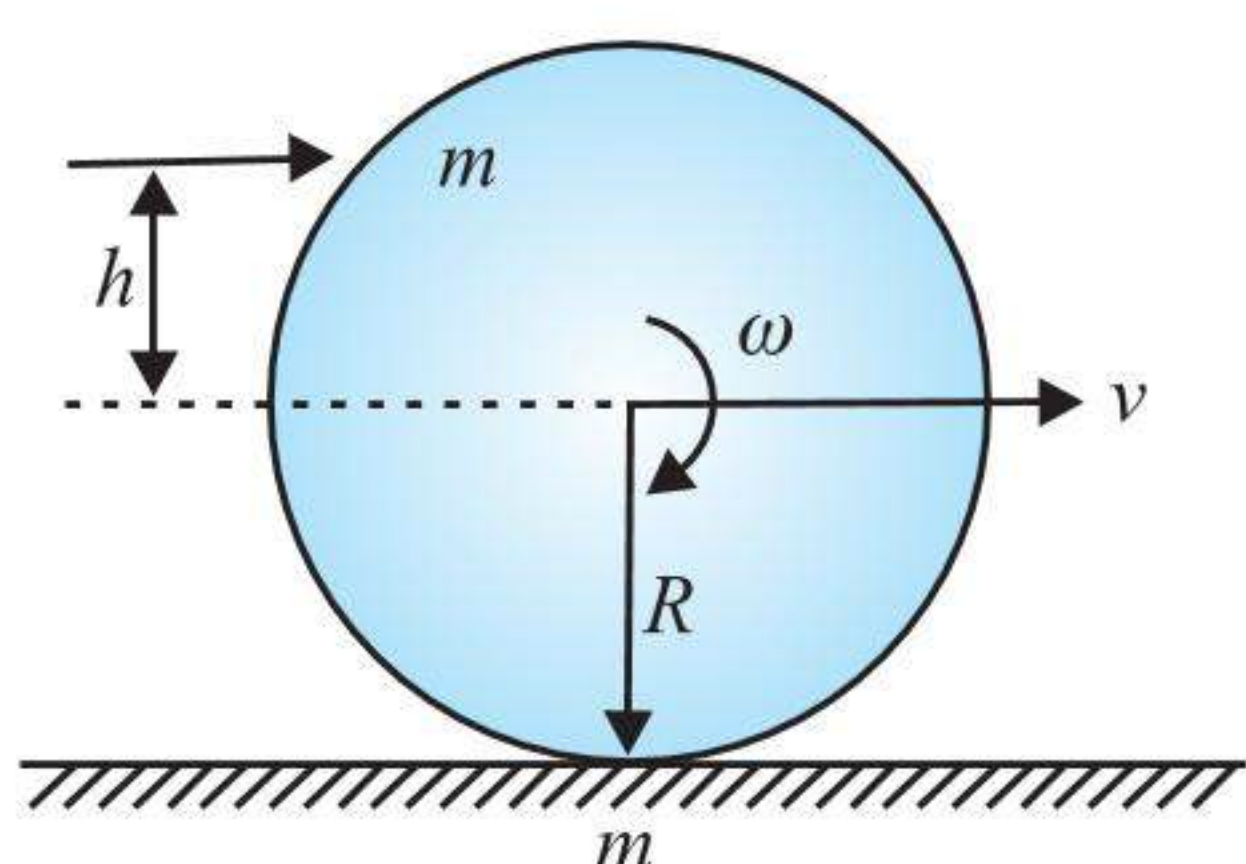
Dividing Eq. (i) by Eq. (ii), we get $\omega = \frac{6v_0}{l}$

Let t be the time in which the rod rotates at an angle 2π , then

$$\omega t = 2\pi \Rightarrow t = \frac{\pi L}{3v_0}$$

Hence, distance travelled by centre of mass $\Rightarrow x_c = \frac{l\pi}{3}$

7. Let v_z be the velocity attained by its centre of mass and ω be the angular velocity about the centre of mass by the impact of the cue.



Suppose J is the impulse provided by the cue

$$J = m(v - 0) \quad \dots(i)$$

$$\text{and } J.h = I(\omega - 0) \quad \dots(ii)$$

From Eqs. (i) and (ii), $Mv \times h = I\omega$

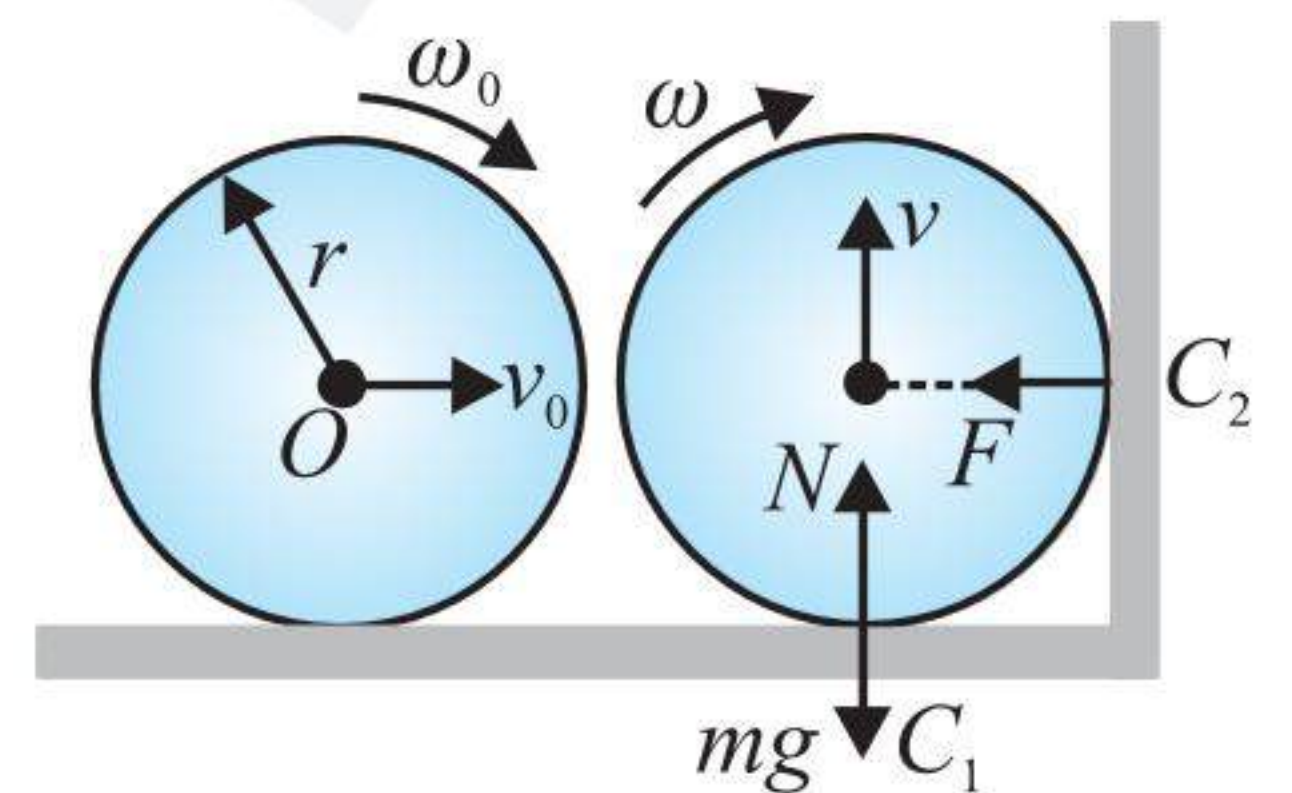
For pure rolling, $\omega = \frac{v}{R}$

$$\therefore mv \times h = \frac{2}{3} mR^2 \times \frac{v}{R}$$

$$\Rightarrow h = \frac{2R}{3}$$

8. Figure shows the two situations of a rolling wheel before and after its striking the wall.

If C_2 is the point on the wall where the wheel strikes, then the impulsive force F passes through it. Moreover, the weight of the wheel mg and the normal reaction offered by the horizontal surface on the wheel to balance each other. So, the angular momentum of the wheel remains conserved about point C_2 .



$\therefore$ Initial angular momentum = final angular momentum

$$\therefore I\omega_0 = I\omega + mvr$$

[where ω and v are the angular and linear speeds of the wheel, just after its striking]

Since slippage is prevented, so $v = \omega r$

$$\therefore I\omega_0 = I\omega + m\omega r^2$$

$$\Rightarrow \frac{mr^2\omega_0}{2} = \frac{mr^2\omega}{2} + m\omega r^2$$

$$\Rightarrow \omega = \frac{\omega_0}{3}$$

Exercises

Single Correct Answer Type

$$1. (2) E_1 = \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} (I_1 \omega_1) \omega_1 = \frac{1}{2} k \omega_1$$

Now $I_1 \omega_1 = I_2 \omega_2 = k$ (say)

$$E_2 = \frac{1}{2} I_2 \omega_2^2 = \frac{1}{2} (I_2 \omega_2) \omega_2 = \frac{1}{2} k \omega_2$$

$$I_1 > I_2 \text{ and } \omega_2 > \omega_1$$

so, $E_2 > E_1$

$$2. (4) \vec{L} = I_{\text{CM}} \vec{\omega} + m \vec{r} \times \vec{v}_{\text{CM}}$$

$$= \frac{MR^2}{2} \omega (-\hat{k}) + M \frac{3}{2} R (-\hat{j}) \times v_0 \hat{i}$$

$$= MRv_0 \hat{k}$$

3. (1) F will provide anticlockwise torque about the centre due to which the bottommost point will tend to move towards right, so friction will act towards left. So it will move towards left.

4. (2) As there is no friction, net force is F which is towards right. Torque due to this force will be in anticlockwise direction.

5. (2) Friction force will act towards right.

6. (1) For a body to roll without slipping on an incline of angle θ :

$$\mu \geq \frac{\tan \theta}{1 + \frac{mR^2}{I}}$$

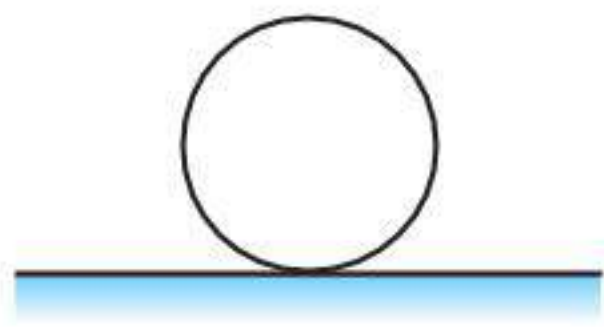
I is minimum for the sphere, hence μ required is least for the sphere. Here it is given that μ is just sufficient to roll the sphere purely. It means the ring and the cylinder will not roll purely, so kinetic friction will act on them. Ultimately, friction acting on each of the three will be $\mu mg \cos \theta$ and each of them will have the same acceleration. Hence, all of them reach at the same time.

7. (3) In first case, the impulse on the platform will act radially inward and it will not produce any angular impulse.

In second case, when man steps into platform, moment of inertia of system increases, so rotational speed decreases.

8. (4) Since $v_{\text{point of contact}} = 0$, no friction acts between ball and plank.

No impulse acts on plank or ball.



9. (2) Since all the forces pass through point of contact, so angular momentum remains conserved.

$$10. (4) \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} M v^2} = \frac{40}{60} = \frac{2}{3} \text{ or } \frac{\frac{I v^2}{R^2}}{M v^2} = \frac{2}{3} \text{ or } I = \frac{2}{3} M R^2$$

Clearly, the body is a hollow sphere.

$$11. (4) \frac{MgL}{2} \cos \theta = \frac{ML^2}{3} \alpha$$

$$\alpha = \frac{3}{2} \frac{g \cos \theta}{L}$$

$$a = \alpha L = \frac{3}{2} g \cos \theta$$

$$12. (4) \frac{mv^2}{2} + \frac{I\omega^2}{2} = mg \times \frac{3v^2}{4g} \Rightarrow I = \frac{mR^2}{2}$$

For pure rolling condition, $v = R\omega$.

13. (2) In both the cases, the loss of gravitational potential energy and the resulting gain 'total kinetic energy' is same.

14. (2) Equation of motion is

$$Mg - T = Ma \quad \dots(i)$$

Taking torque about the axis passing through the centre of the spool and perpendicular to it,

$$TR = I\alpha = \frac{1}{2} MR^2 \left(\frac{a}{R} \right)$$

$$T = \frac{1}{2} Ma \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$Ma = Mg - \frac{1}{2} Ma$$

$$a = \frac{2g}{3}$$

$$\therefore T = \frac{Mg}{3}$$

15. (1) Work done = increase in KE of the rotation

$$W = \frac{1}{2} I \omega^2 = \frac{1}{2} \times \frac{1}{2} MR^2 \omega^2 = \frac{1}{4} MR^2 \omega^2$$

16. (1) From conservation of angular momentum,

$$\frac{ML^2}{12} \omega_0 = \left[\frac{ML^2}{12} + 2m \left(\frac{L}{2} \right)^2 \right] \omega,$$

$$\omega = \left(\frac{M}{M+6m} \right) \omega_0$$

$$17. (2) mg \left(\frac{l}{4} \right) = \frac{1}{2} \left[\frac{ml^2}{12} + m \left(\frac{l}{4} \right)^2 \right] \omega^2$$

$$\omega = \sqrt{\frac{24g}{7l}} = 2\sqrt{\frac{6g}{7l}}$$

18. (4) As the inclined plane is smooth, the sphere can never roll, rather it will just slip down. Hence, the angular momentum remains conserved about any point on a line parallel to the inclined plane and passes through the centre of the ball.

19. (4) As $\sum \tau = 0$, angular momentum and linear momentum remain conserved. As the two balls will move radially out, I changes. In order to keep the angular momentum ($L = I\omega$) conserved, angular speed (ω) should change.

20. (3) If the track is smooth (case A), only translational kinetic energy changes to gravitational potential energy.

But, if the track is rough (case B), both translational and rotational kinetic energies change to potential energy.

Therefore, potential energy ($= mgh$) will be more in case B than in case A. Hence, $h_1 > h_2$.

21. (4) Since linear acceleration is same for all ($a = Mg \sin \theta - \mu Mg \cos \theta$) as they have same mass ' M ' and same ' μ '.

Hence, all will reach the bottom simultaneously.

22. (2) For all the bodies, torque is the same.

$$\text{Now, KE} = \frac{1}{2} mv^2 + \frac{L^2}{2I}$$

Linear velocity ' v ' is the same for all as the same force acts on them.

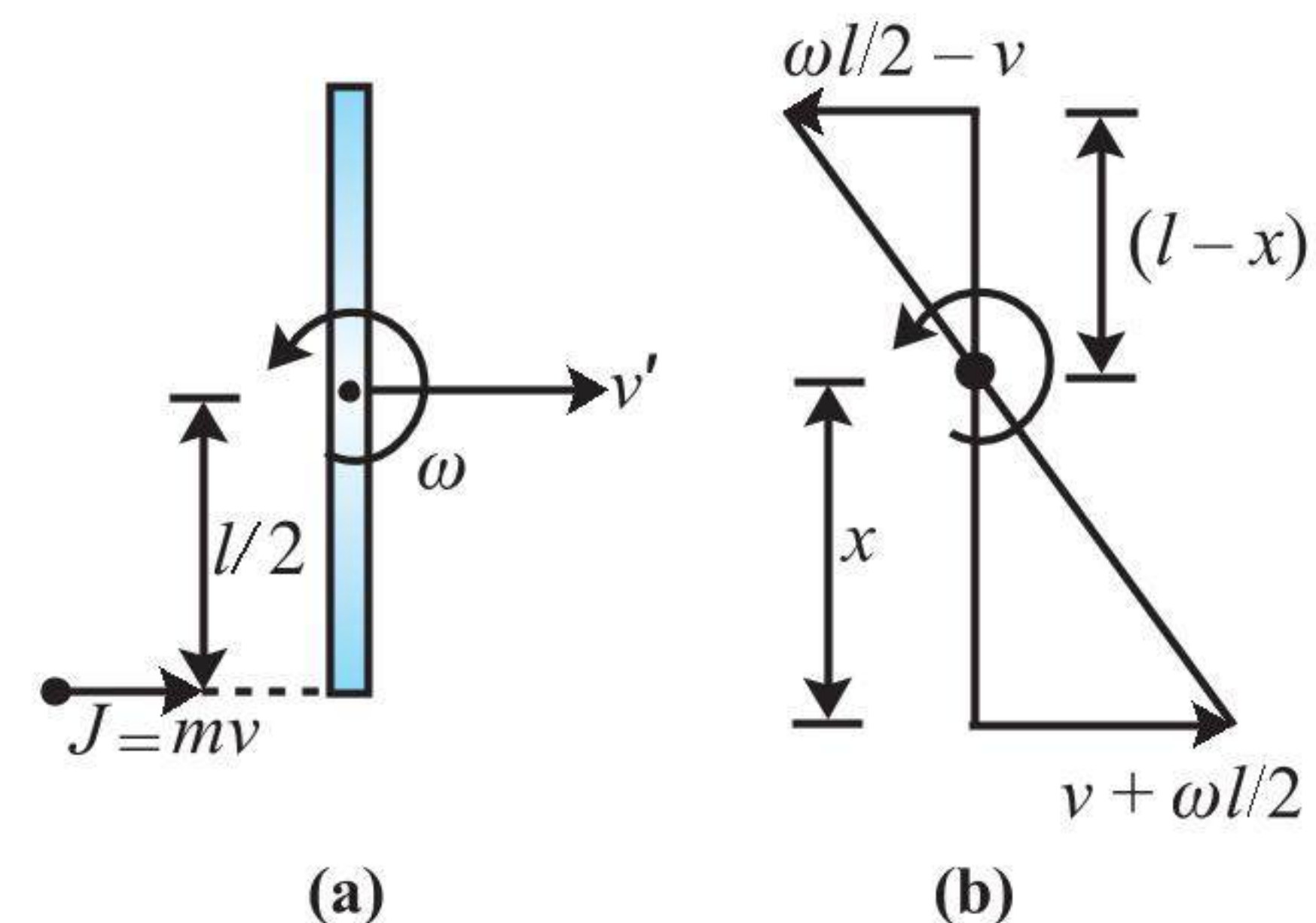
Therefore, the more value of moment of inertia implies the lesser kinetic energy.

Among all, the hollow sphere has the maximum moment of inertia, $I = \left(\frac{2}{3} MR^2 \right)$

23. (4) $J = mv = mv'$

$\Rightarrow$ Velocity of the CM of rod = v

Applying impulse momentum equation about the CM of rod



$$J \frac{l}{2} = I_{\text{CM}} \omega \Rightarrow \frac{mvl}{2} = \left(\frac{ml^2}{12} \right) \omega \Rightarrow \omega = \frac{6v}{l}$$

About instantaneous axis of rotation the rod is considered to have pure rotation.

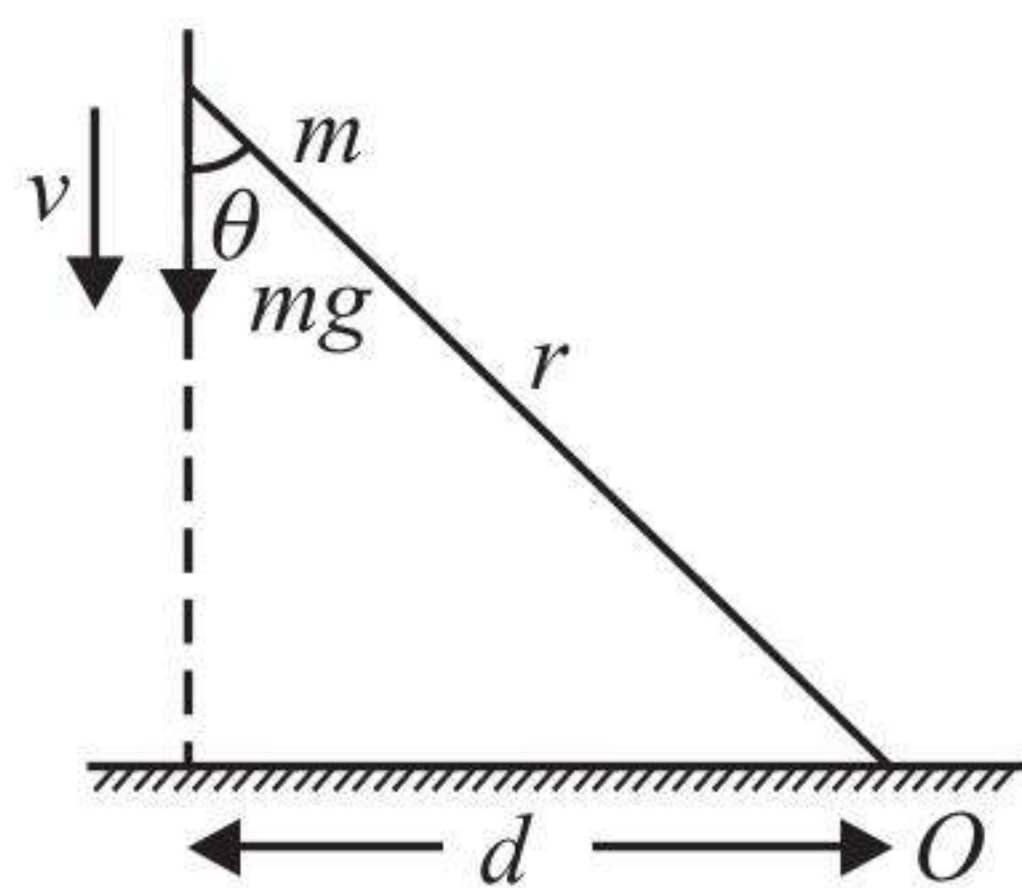
Let instantaneous axis of rotation be located at a distance x from the colliding end [Fig. (b)]

$$\frac{\omega l}{2} - v = \frac{v + \omega l}{2} \quad \dots(i)$$

Substituting the value of $\omega = 6v/l$ in Eq. (i), we get $x = \frac{2}{3} l$.

24. (3) The magnitude of angular momentum of the particle about $O = mvd$.

Since speed v of the particle increases, its angular momentum about O increases.



Magnitude of inertia of the particle about $O = mr^2$

Hence, MI of the particle about O decreases.

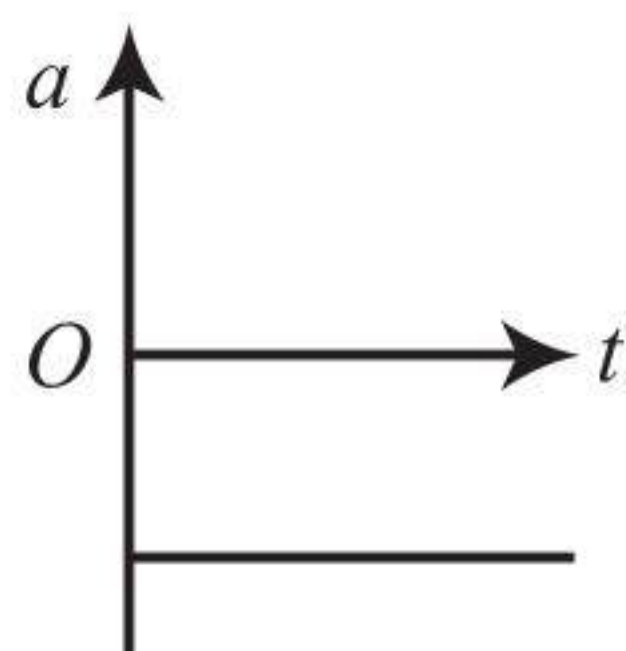
Angular velocity of the particle about $O = \frac{v \sin \theta}{r}$

Therefore, v and $\sin \theta$ increase and r decreases.

Therefore, angular velocity of the particle about O increases.

25. (2) As the normal force exerted by the horizontal surface passes through point B , the external torque on the ball is zero about point B . So the angular momentum of the ball is conserved about point B . ($\because \tau = \frac{dL}{dt}$)

26. (4) As the sphere rolls up its speed is decreasing and while rolling down its speed is increasing. Hence the acceleration of its centre of mass is down the incline and is thus always negative. Therefore the correct graph is shown in figure.



$$27. (3) a = \frac{g \sin \theta}{1 + \frac{I}{MR^2}}$$

a is minimum, if I is maximum.

$$I_{\max} = MR^2 \text{ [for ring]}$$

$$\Rightarrow a_{\min} = \frac{g \sin \theta}{1 + \frac{MR^2}{MR^2}} = \frac{g \sin \theta}{2}$$

28. (2) For A and B , energy will remain conserved but energy of C will dissipate due to action of frictional force.
29. (3) Both the sphere and the plank will slide down with same acceleration $g \sin \theta$

$$30. (1) \text{ As } \sqrt{\frac{2gh}{1 + \frac{I}{MR^2}}}$$

Hence velocity is independent of the inclination of the plane and depends only on height h through which body descends.

But because $t + \frac{1}{\sin \theta} \sqrt{\frac{2h}{g} \left(1 + \frac{I}{MR^2}\right)}$ depends on the

inclination also, hence greater the inclination lesser will be the time of descent. Hence, in present case, the speeds will be same (because h is same) but time in descent will be different (because of different inclinations)

31. (4) Let the angular velocity of disc after child jumps off. be ω'

$\therefore$ From conservation of angular momentum

$$(I + mR^2)\omega = mvR + I\omega'$$

$$\therefore \omega' = \frac{(I + mR^2)\omega - mvR}{I}$$

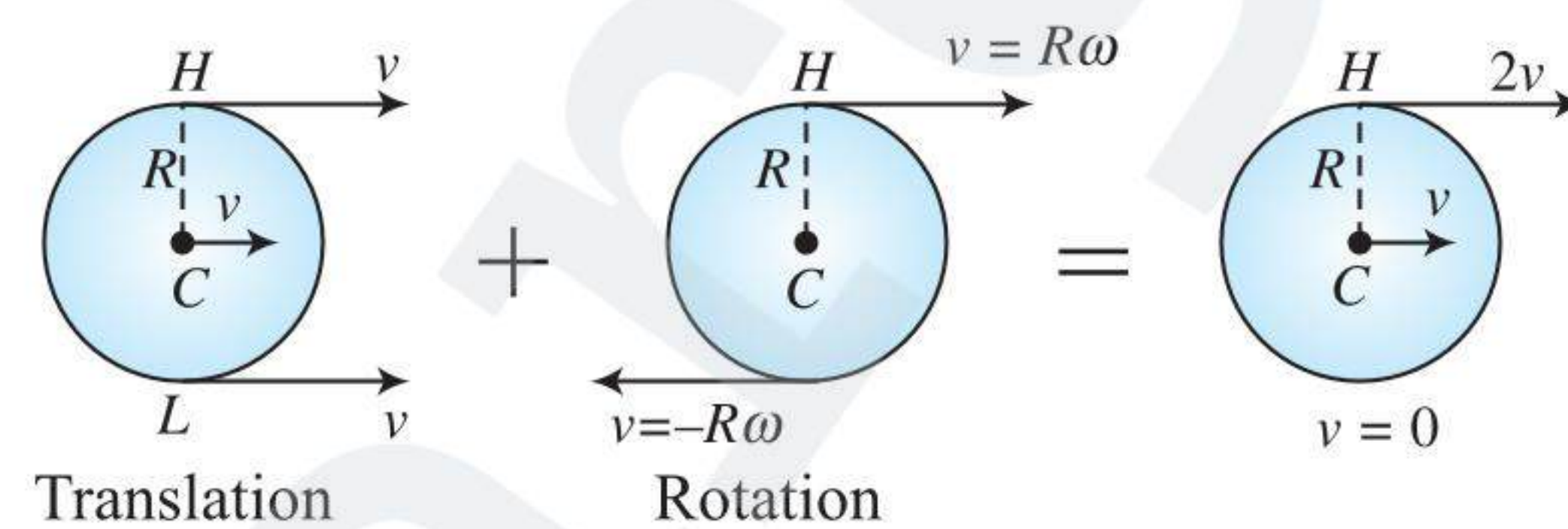
$$32. (3) \text{ Case-1 } mgh = \frac{1}{2}mv^2 + \frac{1}{2}mk^2\omega^2$$

$$\text{Case-2 } mgh = \frac{1}{2}m \times \left(\frac{5v}{4}\right)^2$$

$$\frac{1}{2}mk^2\frac{v^2}{R^2} + \frac{1}{2}mv^2 = \frac{1}{2}m \times \frac{25v^2}{16}$$

$$\frac{k^2}{R^2} + 1 = \frac{25}{16} \Rightarrow k = \frac{3R}{4}$$

33. (1) In case of rolling without slipping, all points of a rigid body have same angular speed but different linear speed. The linear speed is maximum ($= 2v$) at the highest point H , while minimum for the lowest point ($=$ zero) L . This all clearly follows from the given figure.



34. (3) Applying conservation of angular momentum,

$$(5 + 5)\omega' = 5 \times 10 + 5 \times 20 = 150$$

$$\text{or } \omega' = 15 \text{ rad s}^{-1}$$

$$\begin{aligned} \text{Initial kinetic energy} &= \frac{1}{2} \times 5 \times 10 \times 10 + \frac{1}{2} \times 5 \times 20 \times 20 \\ &= 250 + 1000 = 1250 \text{ J} \end{aligned}$$

$$\text{Final kinetic energy} = \frac{1}{2} \times 10 \times 15 \times 15 = 1125 \text{ J}$$

$$\text{Loss of kinetic energy} = (1250 - 1125) \text{ J} = 125 \text{ J}$$

35. (4) $F + f = ma$... (i)

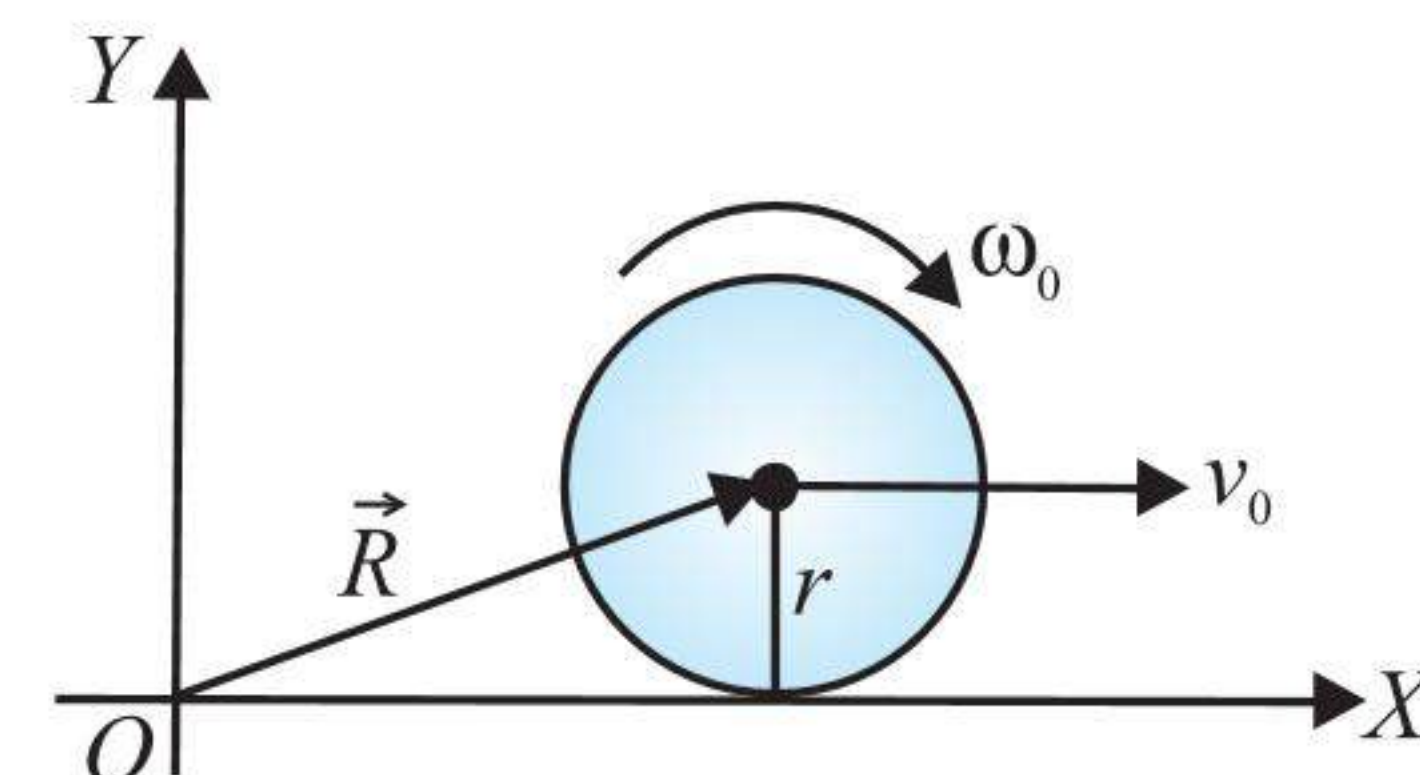
$$FR - fR = \frac{mR^2}{2} \frac{a}{R} \text{ ... (ii)}$$

$$F - f = \frac{ma}{2} \text{ ... (iii)}$$

$$2F = \frac{3ma}{2} \Rightarrow a = \frac{4F}{3m}$$

$$\begin{aligned} 36. (1) \vec{L}_O &= I_{\text{CM}} \vec{\omega} + m(\vec{r} \times \vec{v}) = \frac{1}{2} mR^2 \omega \hat{k} + m(4R\hat{i} + 3R\hat{j}) \times (\omega R\hat{i}) \\ &= \frac{1}{2} mR^2 \omega \hat{k} - 3mR^2 \omega \hat{k} = -\frac{5}{2} mR^2 \omega \hat{k} \end{aligned}$$

$$37. (4) \vec{L} = \vec{R} \times m\vec{v}_0 + I\vec{\omega}_0 = mv_0 + I\omega_0 \text{ (which is constant)}$$



38. (2) During the impact, the impact forces pass through point P . Therefore, the torque produced by it about P is equal to zero.

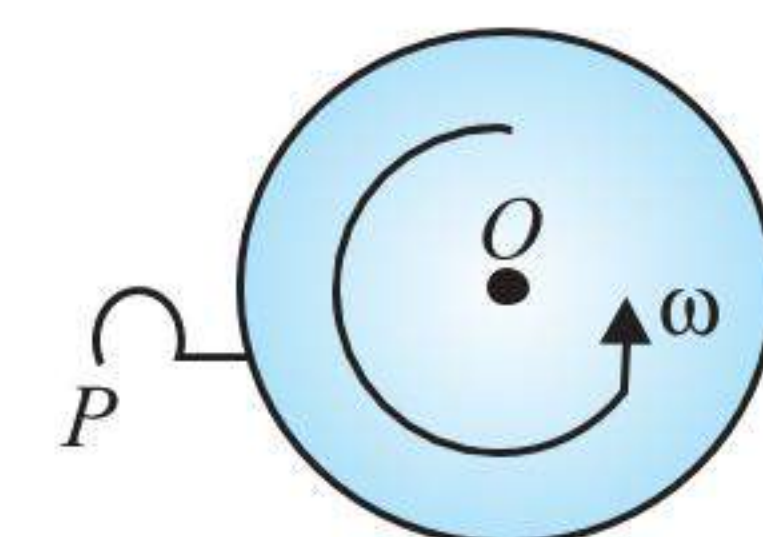
Consequently, the angular momentum of the disc about P , just before and after the impact, remains the same

$$\Rightarrow L_2 = L_1 \text{ ... (i)}$$

where L_1 = angular momentum of the disc about P just before the impact

$$I_0 \omega = \frac{1}{2} mr^2 \omega$$

L_2 = angular momentum of the disc about P just after the impact



$$I_0 \omega = \left(\frac{1}{2} mr^2 + mr^2 \right) \omega' = \frac{3}{2} mr^2 \omega'$$

Just before the impact, the disc rotates about O . But just after the impact, the disc rotates about P .

$$\Rightarrow \frac{1}{2} mr^2 \omega = \frac{3}{2} mr^2 \omega' \Rightarrow \omega' = \frac{1}{3} \omega$$

39. (3) Using conservation of angular momentum about O , we get

$$mvR = (mR^2 + mR^2)\omega; \omega = \frac{v}{2R}$$

40. (4) Apply conservation of angular momentum about the hinge, we get

$$mvR = \frac{m}{2} R^2 \omega + m(R\omega)R$$

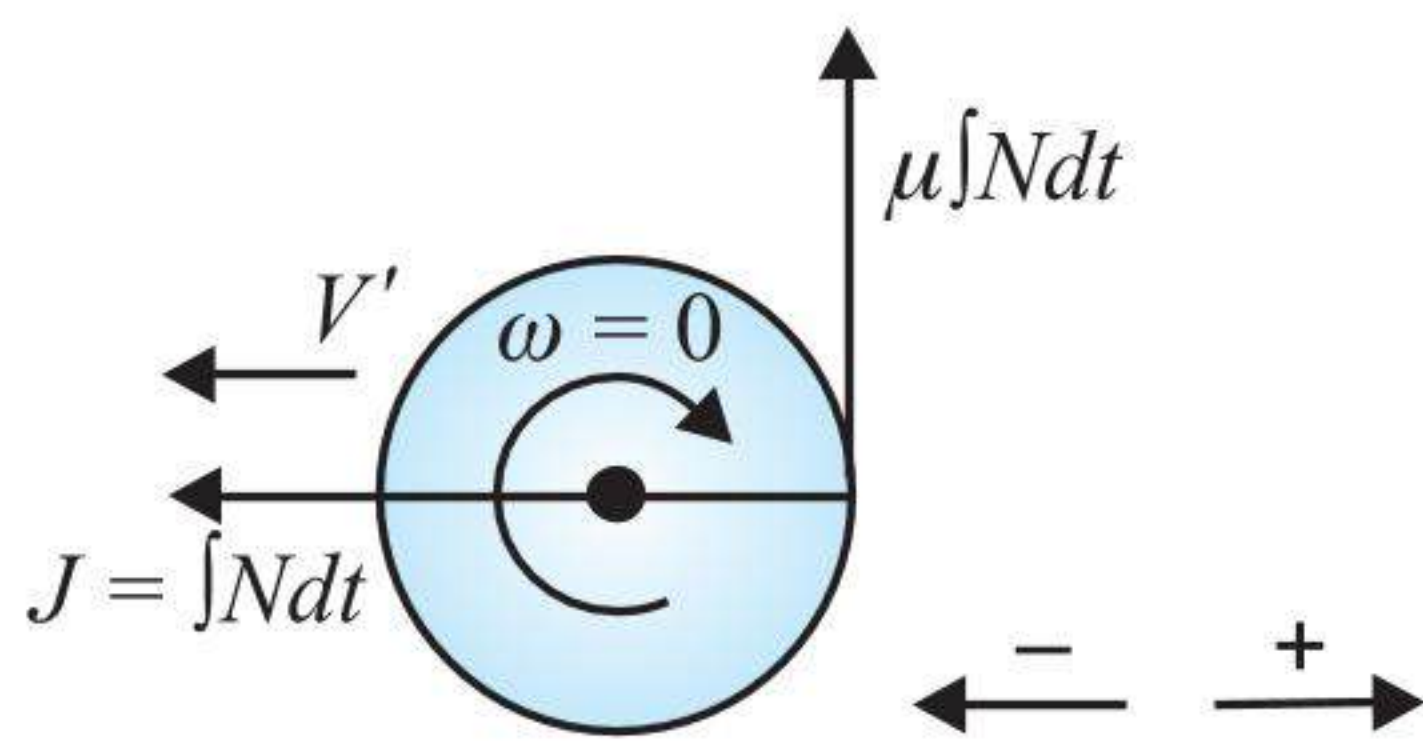
$$R\omega = \frac{2v}{3} = \frac{2 \times 5}{3} = \frac{10}{3} \text{ m/s}$$

41. (1) Impulse provided by the edge in the horizontal direction:

$$-\int N dt = -mV' - (mV) \quad \dots(i)$$

Friction impulse in the vertical direction:

$$\mu R \int N dt = \frac{2}{5} mR^2 \left(\frac{V}{R} \right) \quad \dots(ii)$$



From Eqs. (i) and (ii), we get

$$\int N dt = 2mV \text{ and } V' = V$$

42. (3) **Method 1:** The distance of $\vec{L}$ is perpendicular to the line joining the bob at point C . Since this line keeps changing its orientation in space, direction of $\vec{L}$ keeps changing, however, as ω is constant, magnitude of $\vec{L}$ remains constant.

Method 2: The torque about point is perpendicular to the angular momentum vector about point C . Hence it can only change the direction of L , and not its magnitude.

43. (4) Taking τ about CM

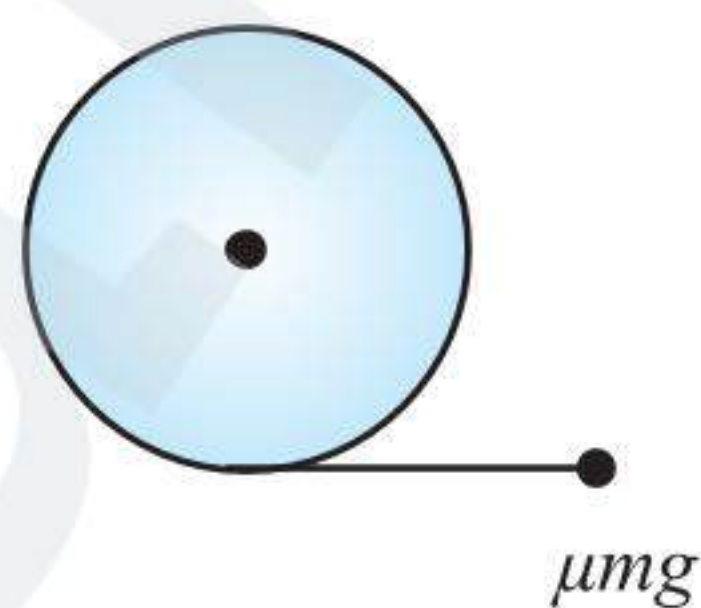
$$\mu mgR = MR^2 \alpha$$

$$\mu g = R\alpha$$

$$\alpha = \frac{\mu g}{R}$$

$$\omega = \omega_0 - \frac{\mu g}{R} f = \frac{\omega_0}{2}$$

$$\frac{\omega_0}{2} = \frac{\mu g}{R} f \Rightarrow f = \frac{\omega_0 R}{2\mu g}$$



44. (4) As sphere is rolling we can apply torque equation about point of contact (about P).

$$FR = I_P \alpha$$

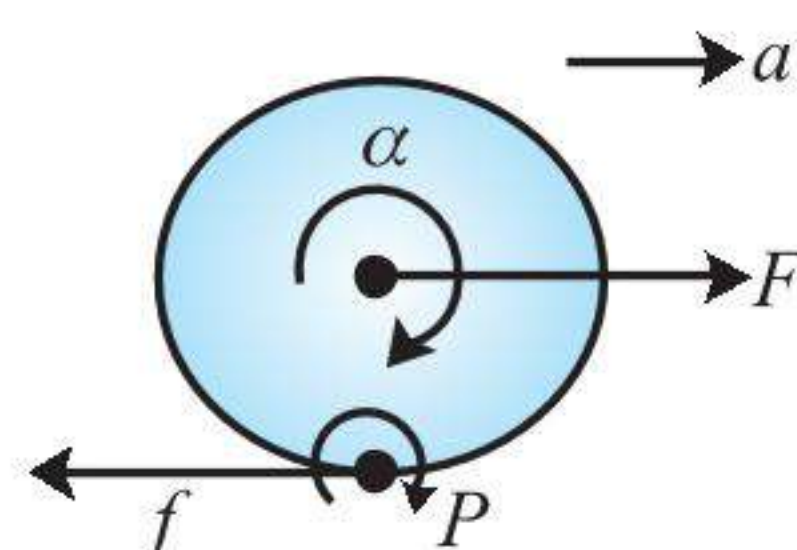
$$\Rightarrow FR = \left(\frac{7}{5} MR^2 \right) \alpha$$

$$\Rightarrow R\alpha = \frac{5F}{7M} = a \text{ (acceleration of CM)}$$

Equation of translatory motion

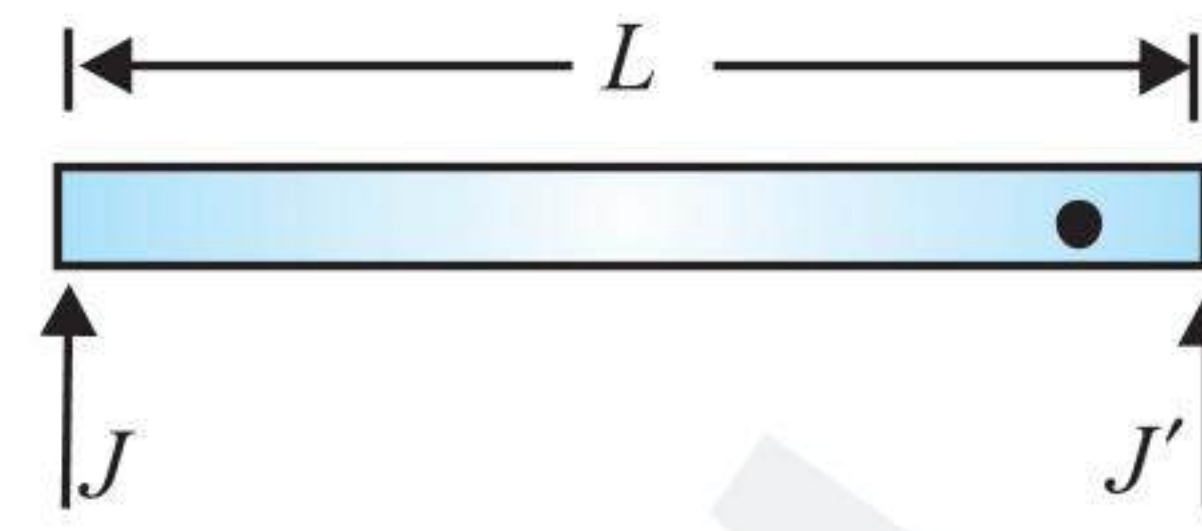
$$F - f = Ma \Rightarrow f = \frac{2}{7} F \quad \dots(i)$$

$$\text{For no sliding, } f \leq \mu mg \Rightarrow F = \frac{7}{2} \mu mg$$



45. (2) Let J' be the impulse exerted by the pivot on the rod.

Then from impulse momentum theorem, $J + J' = mv_0$, where v_0 is the velocity acquired by the centre of mass.



Now apply, angular impulse = change in angular momentum about the pivot.

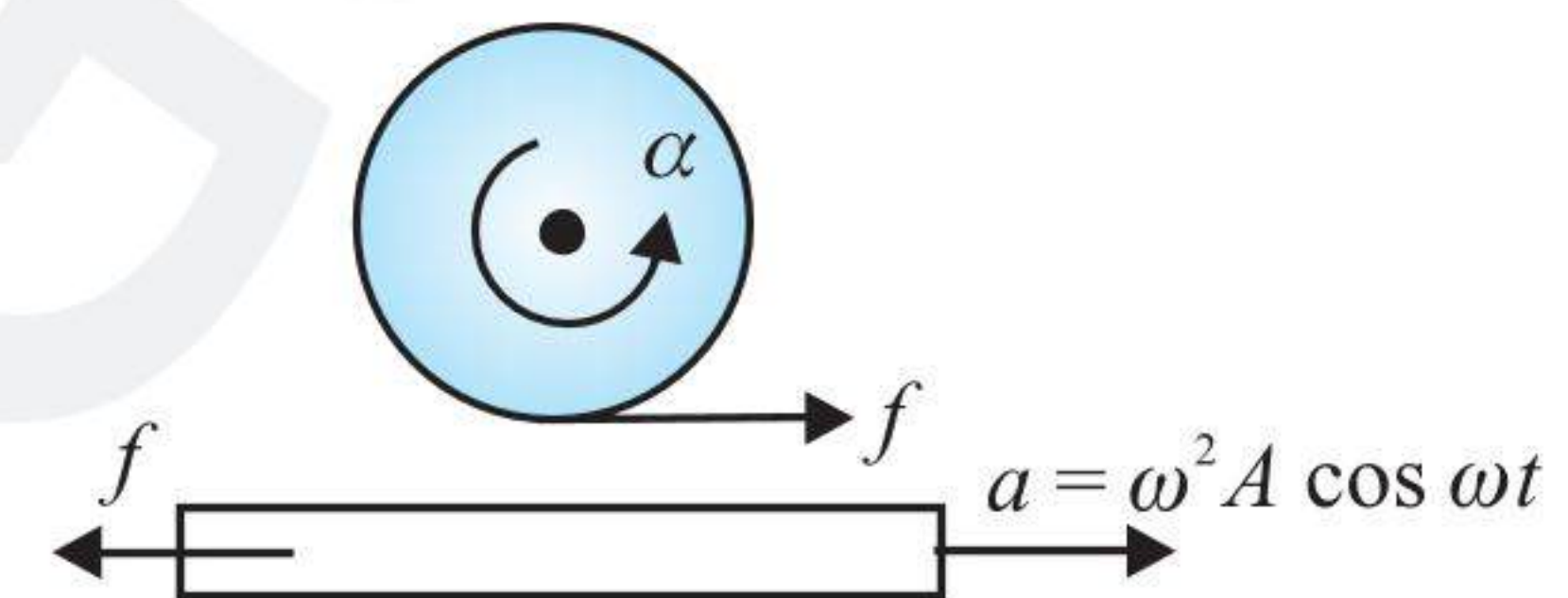
$$J \times L = \frac{ML^2}{3} \omega \text{ or } J = \frac{ML}{3} \omega,$$

where ω is the angular velocity of the rod.

$$v_0 = \frac{\omega L}{2} \Rightarrow Mv_0 = \frac{M\omega L}{2}$$

$$J + J' = \frac{3J}{2} \Rightarrow J' = \frac{J}{2}$$

46. (2) The cylinder is having a fixed axis, so it can perform rotational motion only.



$$fR = \frac{MR^2}{2} \alpha \Rightarrow f = \frac{MR}{2} \alpha$$

For no slipping,

$R\alpha$ = acceleration of the platform

$$\frac{2f}{M} = \omega^2 A \cos \omega t$$

For the maximum torque, f would be maximum and

$$f_{\max} = M\omega^2 A/2$$

So the maximum torque, $\tau_{\max} = f_{\max} \times R = \frac{M\omega^2 AR}{2}$

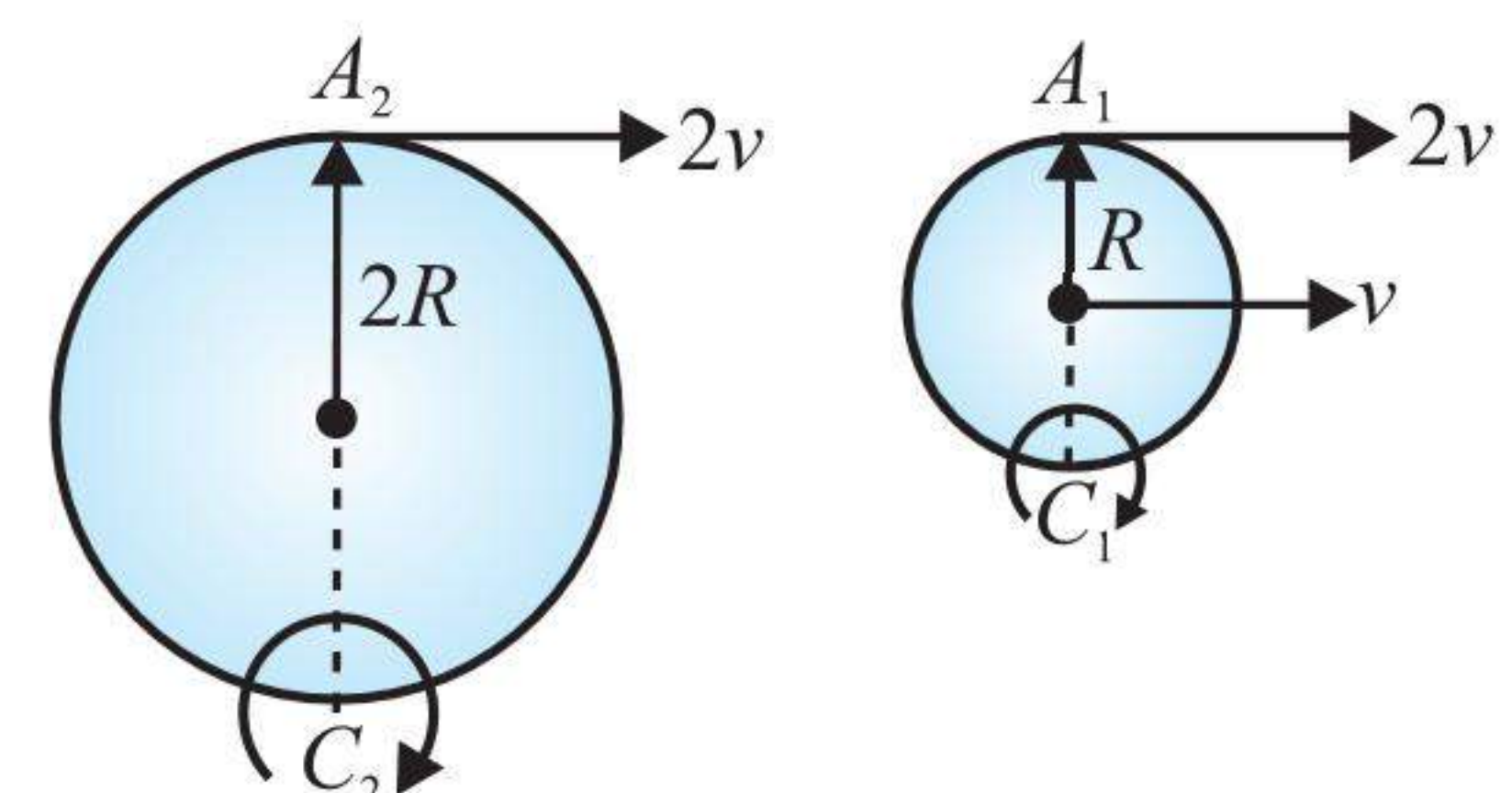
47. (2) Since the disc comes to rest, it stops rotating and translating simultaneously $v = 0$ and $\omega = 0$. That means, the angular momentum about the instantaneous point of contact just after the time of stopping is zero. We know that the angular momentum of the disc about P remains constant because frictional force f , N and mg pass through point P and thus produce no torque about this point.

$$\Rightarrow L_{\text{initial}} = L_{\text{final}} \Rightarrow mvr - I_0 \omega_0 = 0$$

$$\Rightarrow mvr = \frac{1}{2} mr^2 \omega_0 \Rightarrow 2v_0 = \omega_0 r$$

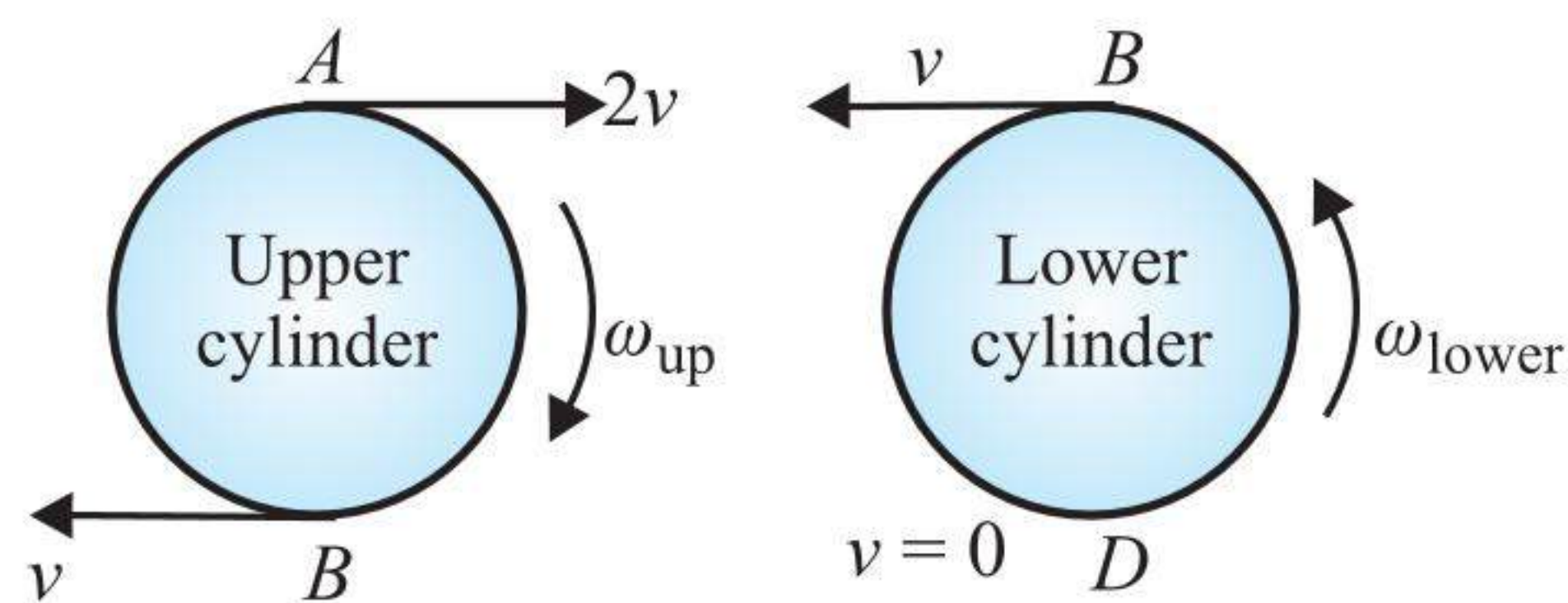
48. (2) In figure, C_1 and C_2 are ICR (instantaneous centre of rotation) of the two cylinders. The cylinder can be considered as rotating about C_1 and C_2 . In the absence of slipping between the plank and the cylinders, points A_1 and A_2 have the same velocity.

Angular velocity of the larger cylinder is $\frac{2v}{4R} = \frac{v}{2R}$



$$v_{\text{CM}} = (2R) \left(\frac{v}{2R} \right) = v$$

49. (2)



Angular velocity of the upper cylinder is

$$\omega_{\text{up}} = \frac{2v - (-v)}{2R} = \frac{3v}{2R}; \text{ for the lower cylinder}$$

$$\omega_{\text{lower}} = \frac{v - 0}{2R} = \frac{v}{2R}$$

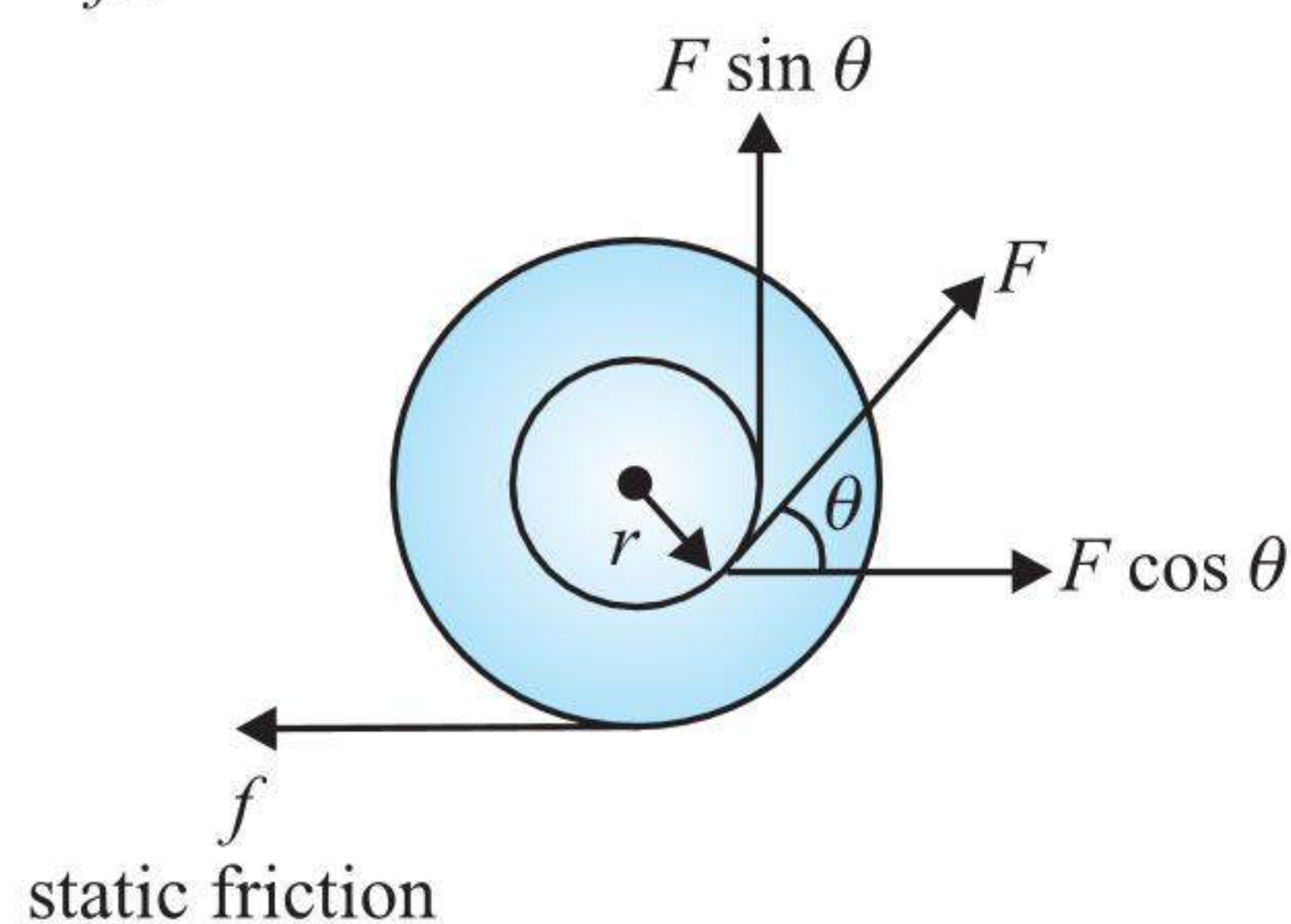
Hence, ratio, $\frac{\omega_{\text{up}}}{\omega_{\text{lower}}} = 3$

50. (1) If the spool is not to translate,

$$F \cos \theta = f \quad \dots(i)$$

If the spool is not to rotate,

$$Fr = fR \quad \dots(ii)$$



$$\text{or } \frac{FR}{r} \cos \theta = F$$

$$\text{or } \cos \theta = \frac{r}{R} \Rightarrow \theta = \cos^{-1} \left(\frac{r}{R} \right)$$

Also the line of action of F should pass through the bottom-most point.

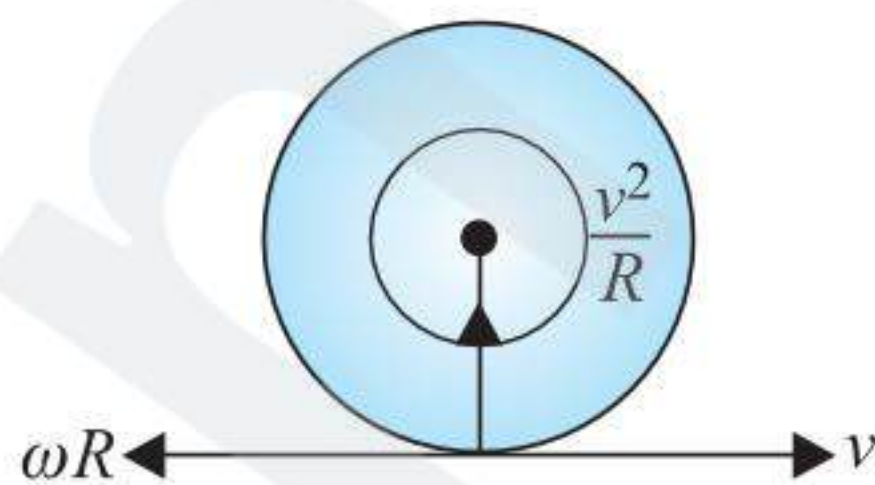
51. (4) As the disc is in combined rotation and translation, each point has a tangential velocity and a linear velocity in the forward direction.

From the figure,

$$v_{\text{net}} \text{ (for the lowest point)} = v - R\omega = v - v = 0$$

$$\text{and acceleration } \frac{v^2}{R} + 0 = \frac{v^2}{R}$$

(Since the linear speed is constant)

52. (2) $F = Ma$

$$FR = I\alpha$$

For pure rolling, $a = R\alpha$

Solving the above equations, we get

$$I = MR^2.$$

So the object may be a ring or a hollow cylinder.

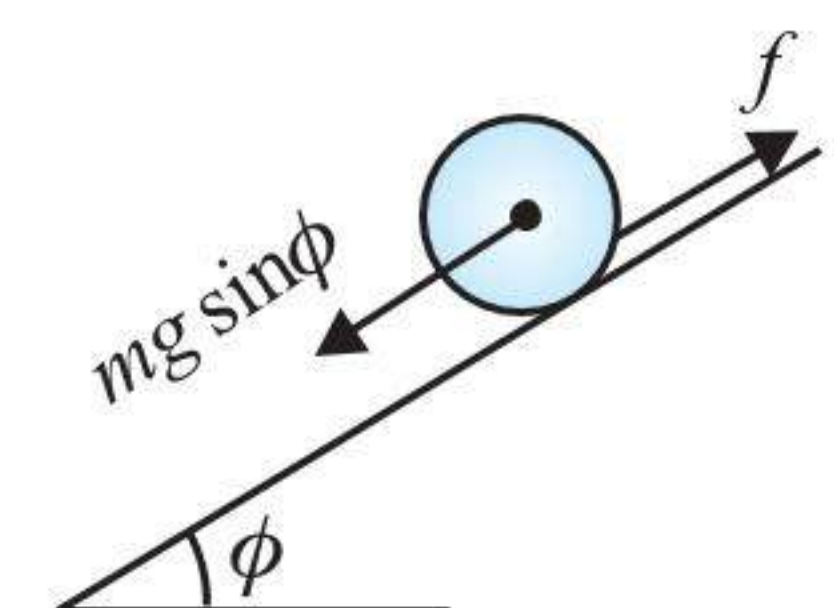
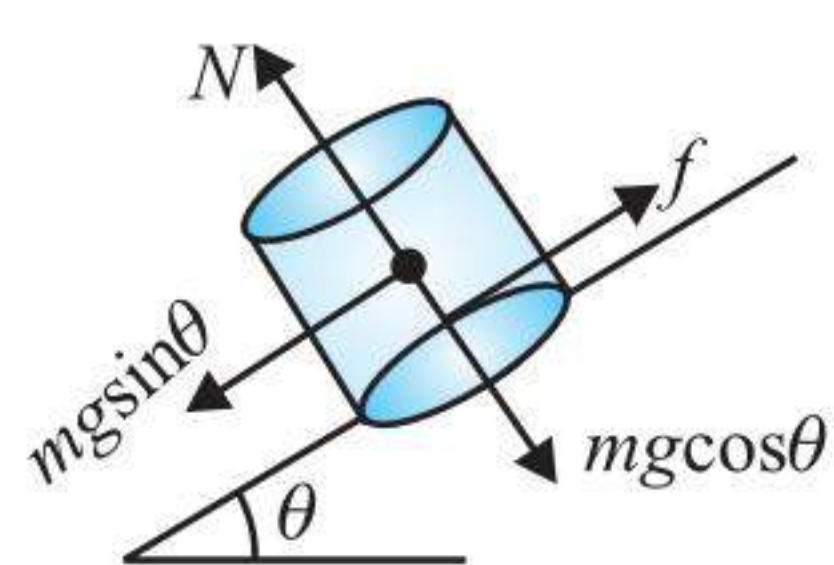
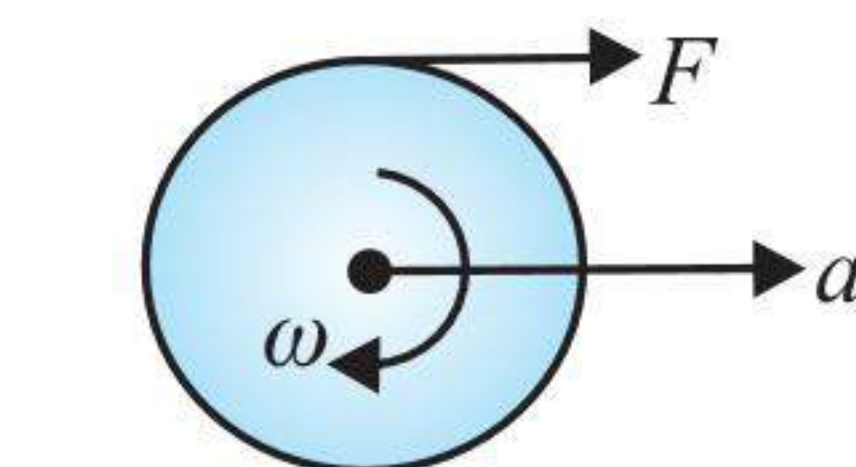
53. (1) $f = Mg \sin \theta$ For no sliding, $f < f_L$

$$mg \sin \theta < \mu_s mg \cos \theta$$

$$\tan \theta = \mu_s$$

For rolling friction, force

$$f = \frac{mg \sin \phi}{1 + \frac{R^2}{k^2}}$$

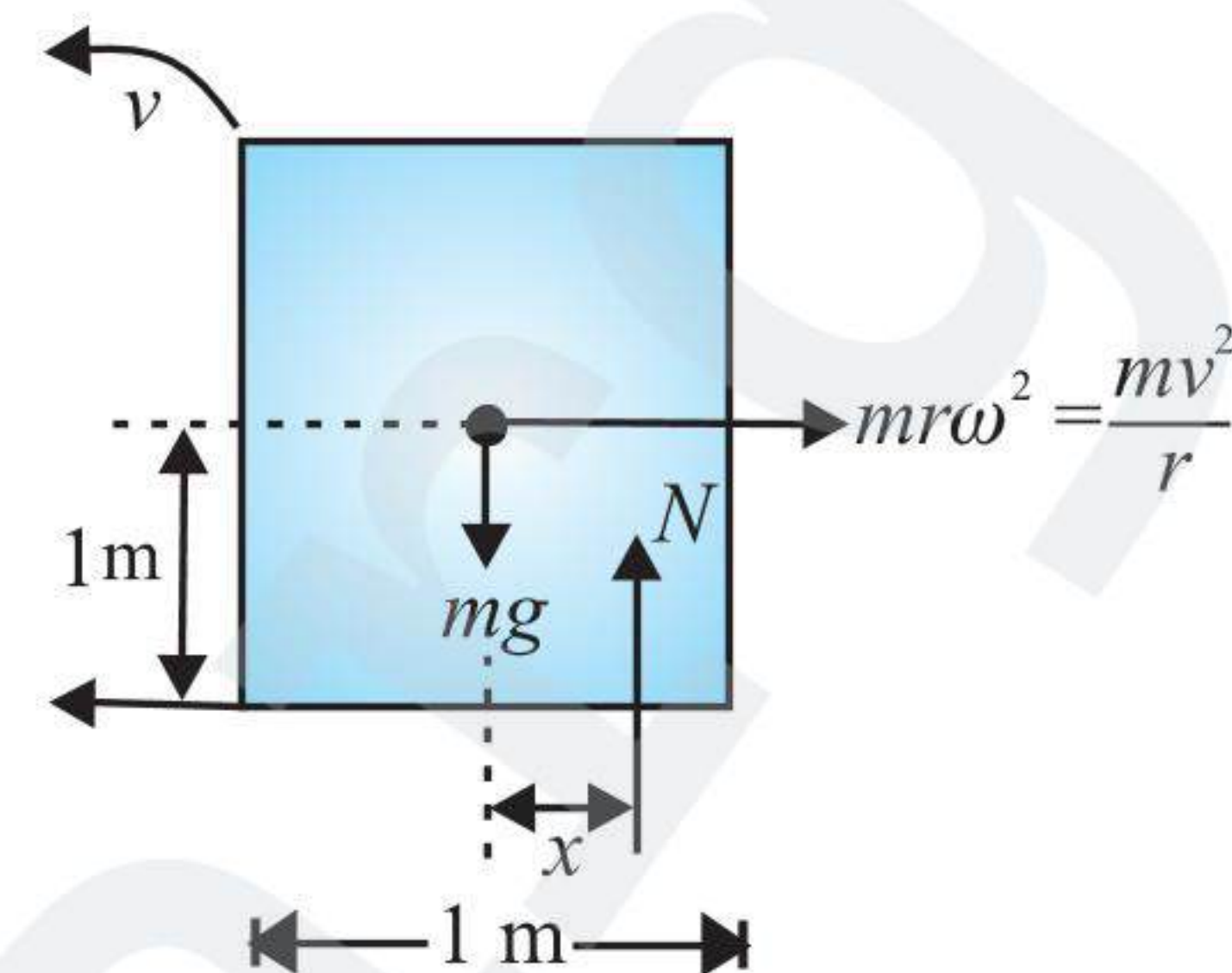


$$\Rightarrow f = \frac{mg \sin \phi}{3}$$

For pure rolling, $\frac{mg \sin \phi}{3} < \mu_s mg \cos \phi$

$$\tan \phi < 3\mu_s, \text{ so } \frac{\tan \phi}{\tan \theta} = 3$$

54. (2) The free-body diagram of the box w.r.t. the truck is as shown in figure.

For vertical equilibrium, $mg = N$ For horizontal equilibrium, $f = (mv^2)/r$ For rotational equilibrium, $f \times 1 = N \times x$ For no tipping to take place, $x < (1/2) \text{ m}$

$$\text{So, } mg \times x = \frac{mv^2}{r} \Rightarrow x = \frac{v^2}{rg} < \frac{1}{2} \Rightarrow v < \sqrt{\frac{rg}{2}} = 10 \text{ m/s}$$

55. (4) Since $v_0 > \omega_0 R$, so bottom-most point will have forward velocity initially. So friction on sphere will act in backward direction. Minimum velocity will occur when $v = \omega R$

$$\Rightarrow v_0 - \mu g t = \left(\omega_0 + \frac{5\mu g}{2R} t \right) R \Rightarrow t = \frac{2(v_0 - \omega_0 R)}{7\mu g}$$

56. (3) From conservation of angular momentum about point of contact.

$$I\omega_0 = I\omega + mR^2\omega$$

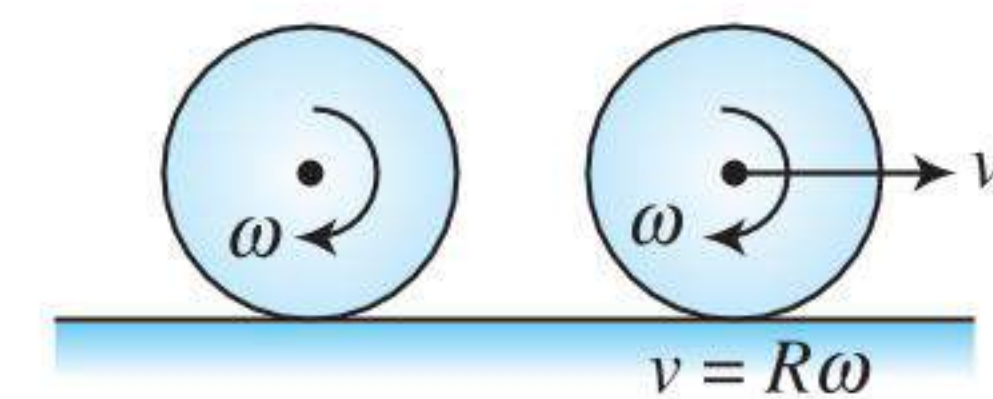
$$I\omega_0 = I \frac{v}{R} + mRv$$

$$v = \frac{I\omega_0}{\frac{I}{R} + mR} \Rightarrow v = \frac{\omega_0}{\frac{1}{R} + \frac{mR}{I}}$$

Now $I_{\text{solid sphere}} < I_{\text{hollow}}$

$$v_{\text{solid}} < v_{\text{hollow}}$$

$$v_1 < v_2$$

57. (2) Friction force μmg acts in forward direction till pure rolling is started. Hence linear acceleration

$$a = \frac{\mu mg}{m} = \mu g$$

$$v = at$$

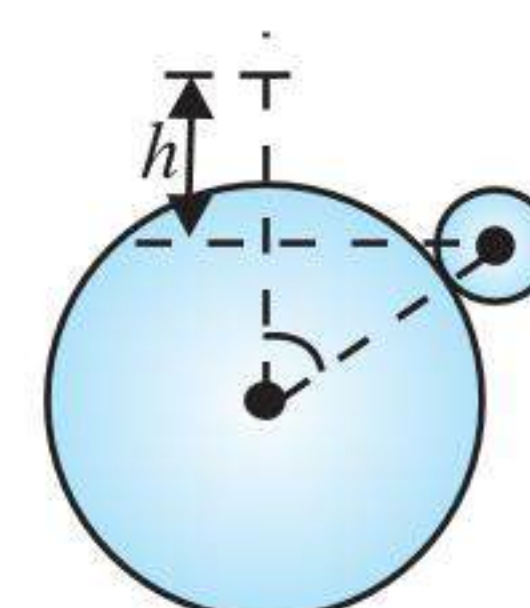
$$\text{or } t = \frac{v}{a} = \frac{\omega_0}{\mu g \left(\frac{1}{R} + \frac{mR}{I} \right)}$$

Again $I_{\text{solid}} < I_{\text{hollow}}$

$$\therefore t_{\text{solid}} < t_{\text{hollow}} \text{ or } t_1 < t_2$$

58. (2) $\frac{mv^2}{(R+r)} = mg \cos \theta$,

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 \quad \dots(i)$$



$$mg(R+r)(1-\cos\theta) = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

$$= \frac{7}{10}mv^2$$

$$mv^2 = \frac{10}{7}mg(R+r)(1-\cos\theta) \quad \dots(ii)$$

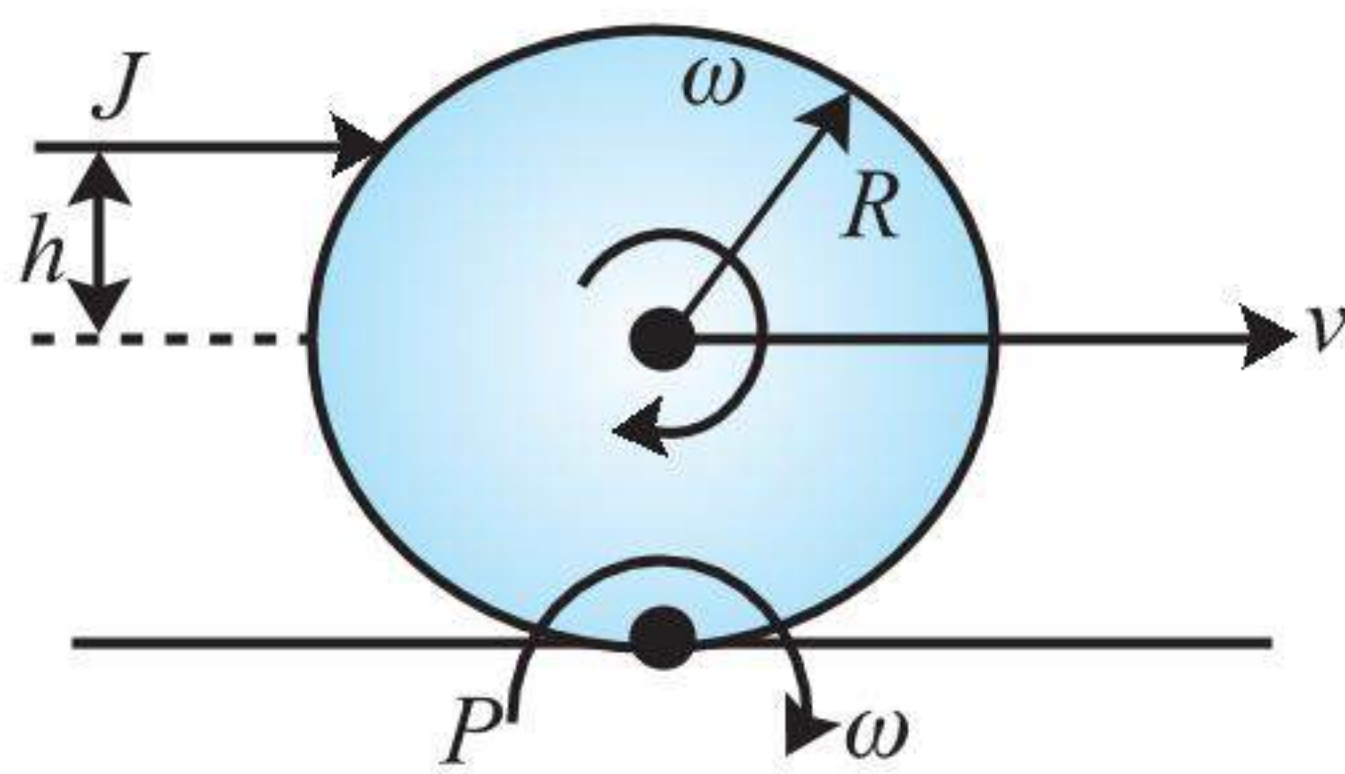
From (i) and (ii)

$$\frac{10}{7} = \frac{17}{7}\cos\theta \text{ or } \cos\theta = \frac{10}{17}$$

$$v = \sqrt{g(R+r)\cos\theta} = \sqrt{\frac{10}{17}g(R+r)}$$

$$\text{and } \omega = \frac{v}{r} = \sqrt{\frac{10g(R+r)}{17r^2}}$$

59. (2) Rolling is rotation about point of contact. Applying impulse momentum equation about P .



$$J(R+h) = I_P\omega \quad \dots(i)$$

$$\text{and } J = mv \quad \dots(ii)$$

$$\text{As sphere rolls } v = \omega R, \text{ and } I = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2$$

$$\text{After solving, we get } \frac{h}{R} = \frac{2}{5}$$

60. (3) From the figure in question, position vector of mass

$$\vec{r} = R\hat{i} - \sqrt{l^2 - R^2}\hat{k}$$

$$\text{Linear momentum } \vec{p} = Mv\hat{j}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

Substituting the values, we get result

$$Mvl \left[\frac{\sqrt{l^2 - R^2}}{l}\hat{i} + \frac{R}{l}\hat{k} \right]$$

61. (1) Let the impulse due to the particle be J , $J = mv_{\text{CM}}$

$$\Rightarrow v_{\text{CM}} = \frac{J}{m}$$

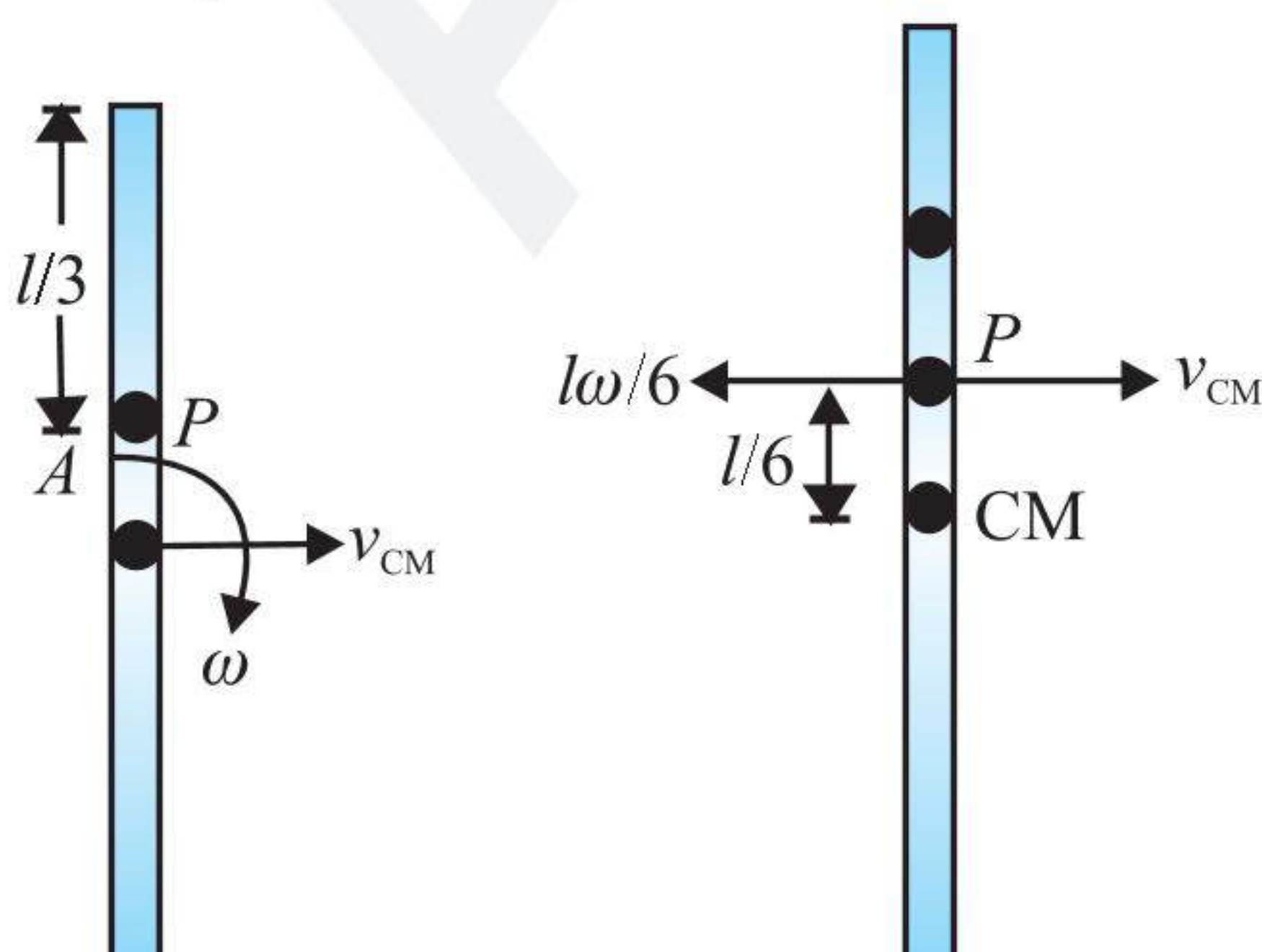
Angular impulse about centre of mass is

$$Jx = \frac{ml^2}{12}\omega \Rightarrow \omega = \frac{12Jx}{ml^2}$$

For point A to be at rest instantaneously,

$$\vec{v}_{\text{trans}} + \vec{v}_{\text{rot}} = 0$$

$$\text{or } v_{\text{CM}} = \frac{\omega l}{6}$$



$$\text{Thus } \frac{J}{m} = \frac{l}{6} \times \frac{12Jx}{ml^2}$$

$$\text{which gives, } x = \frac{l}{2}$$

62. (2) When the ring is at the maximum height, the wedge and the ring have the same horizontal component of velocity. As all the surfaces are smooth, in the absence of friction between the ring and the wedge surface, angular velocity of the ring remains constant.

For conservation of mechanical energy, we get

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv'^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv'^2 + mgh$$

where v' is final common velocity;

$$v' = \frac{v}{2} \text{ (from conservation of momentum) and } h = \frac{v^2}{4g}$$

63. (4) As torque = change in angular momentum

$$F\Delta t = mv \text{ (linear momentum)} \quad \dots(i)$$

$$\text{and } \left(F\frac{l}{2}\right)\Delta t = \frac{ml^2}{12}\omega \text{ (angular momentum)} \quad \dots(ii)$$

Dividing Eqs. (i) and (ii), we get

$$2 = \frac{12v}{\omega l} \Rightarrow \omega = \frac{6v}{l}$$

Using $S = ut$,

$$\text{Displacement of CM is } \frac{\pi}{2} = \omega t = \left(\frac{6v}{l}\right)t$$

and $x = vt$

Dividing, we get

$$\frac{2x}{\pi} = \frac{l}{6} \Rightarrow x = \frac{\pi l}{12}$$

$$\text{Coordinates of } A \text{ will be } \left[\frac{\pi l}{12} + \frac{l}{2}, 0\right].$$

64. (1) By conservation of angular momentum about pivot

$$L = I\omega$$

$$\frac{mvd}{2} = \left[\frac{Md^2}{12} + m\left(\frac{d}{2}\right)^2\right]\omega = \left(\frac{md^2}{2} + \frac{md^2}{4}\right)\omega$$

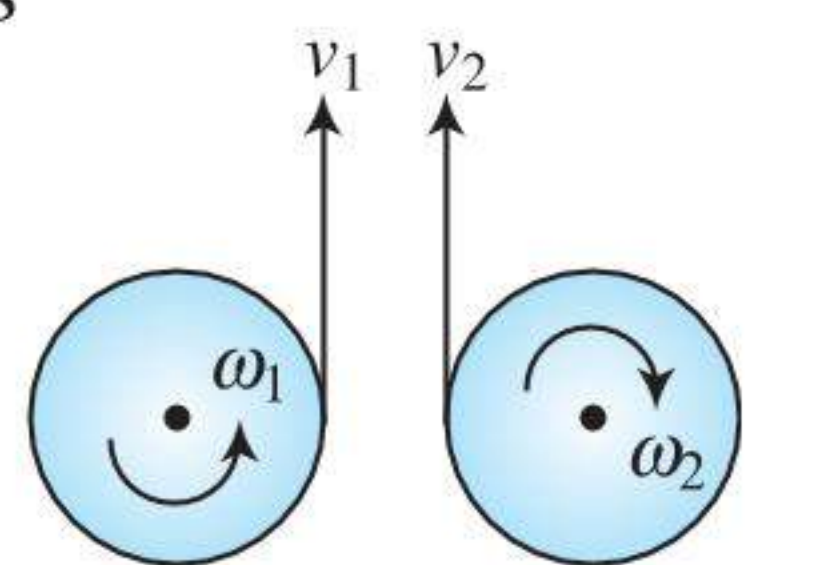
$$\frac{mvd}{2} = \frac{3}{4}md^2\omega \Rightarrow \frac{2}{3}\frac{v}{d} = \omega$$

65. (3) Angular momentum of system can not remain conserved as some external unbalanced torque is present due to forces at axels. Kinetic energy is not conserved, because slipping is there and work is done against friction.

66. (2) Let ω_1 and ω_2 be the final angular velocities when the slipping is ceased. Then

$$v_1 = v_2 \text{ or } \omega_1 R = \omega_2 (2R)$$

$$\text{or } \omega_2 = \frac{\omega_1}{2} \quad \dots(i)$$



Now applying angular impulse = change in angular momentum we have

$$J \cdot R = I\omega_1 \quad \dots(ii)$$

$$\text{and } J \cdot 2R = 4I(\omega_0 - \omega_2) \quad \dots(iii)$$

Here J = linear impulse due to friction which will be tangential and equal for both the cylinders.

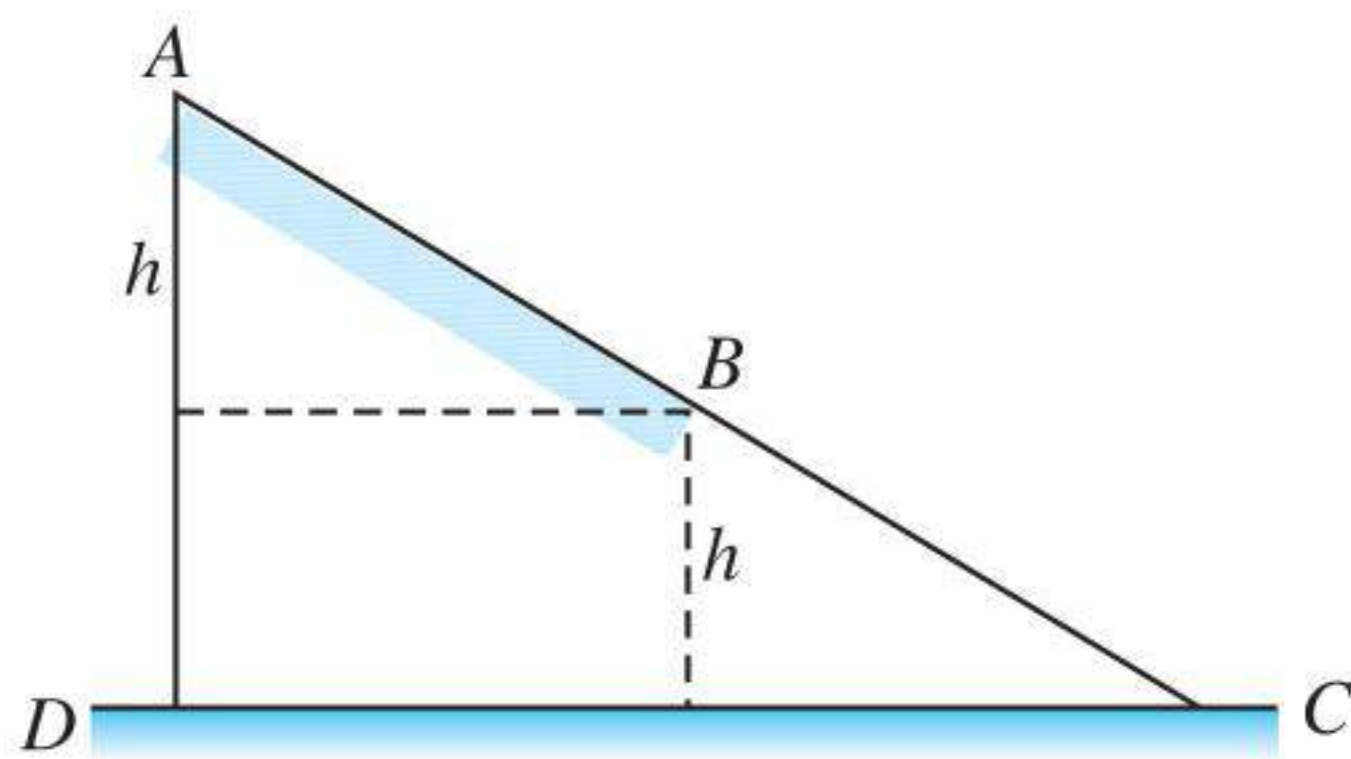
Solving Eqs. (i), (ii) and (iii) we get

$$\omega_1 = \omega_0 \text{ and } \omega_2 = \frac{\omega_0}{2}$$

67. (2) For a cylinder $\frac{K_T}{K_R} = 2$ in case of pure rolling.

Up to B:

$$K_T = \frac{2}{3} mgh \text{ and } K_R = \frac{1}{3} mgh$$



After B: rotational kinetic energy is constant; however, translational kinetic energy increases.

$$\text{At C: } K_T = \frac{2}{3} mgh + mgh = \frac{5}{3} mgh$$

$$\text{While } K_R = \frac{1}{3} mgh, \frac{K_T}{K_R} = 5$$

68. (3) Let v be the linear velocity of centre of mass of the spherical body and ω its angular velocity about centre of mass. Then

$$\omega = \frac{v}{2R}$$

$$\text{KE of spherical body, } K_1 = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

$$K_1 = \frac{1}{2} mv^2 + \frac{1}{2} (2mR)^2 \left(\frac{v^2}{4R^2} \right) = \frac{3}{4} mv^2 \quad \dots(i)$$

$$\text{Speed of the block, } v' = (\omega)(3R) = 3R\omega = (3R) \left(\frac{v}{2R} \right) = \frac{3}{2} v$$

$$\therefore \text{ KE of block, } K_2 = \frac{1}{2} mv'^2 = \frac{1}{2} m \left(\frac{3}{2} v \right)^2 = \frac{9}{8} mv^2 \quad \dots(ii)$$

$$\text{From Eqs. (i) and (ii), } \frac{K_1}{K_2} = \frac{2}{3}$$

69. (3) Since the cylinder rolls with a constant velocity, so there is no friction between the upper surface of the plank and the cylinder. The plank is at rest, under the influence of the weight of the cylinder. So, no external force is required to keep the plank in equilibrium.

70. (1) Given that I_0 is the moment of inertia of table and gun and m the mass of bullet.

Initial angular momentum of system about centre:

$$L_i = (I_0 + mr^2)\omega_0 \quad \dots(i)$$

Let ω be the angular velocity of table after the bullet is fired.

Final angular momentum

$$L_f = I_0\omega - m(v - r\omega)r \quad \dots(ii)$$

where $(v - r\omega)$ is absolute velocity of bullet to the right.

$$\text{Equating (i) and (ii): we get } (\omega - \omega_0) = \frac{mvr}{I_0 + mr^2}$$

This is also the increase in angular velocity.

71. (4) Net external torque is zero. Therefore angular momentum of system will remain conserved, i.e., $L_i = L_f$

Initial angular momentum $L_i = 0$

$\therefore$ Final angular momentum should also be zero.

or angular momentum of man = angular momentum of platform in opposite direction

$$\text{or } mv_0r = I\omega$$

$$\omega = \frac{mv_0r}{I} = \frac{(70)(1.0)(2)}{200}$$

$$\omega = 0.7 \text{ rad/s}$$

72. (2) Angular velocity of man relative to platform is

$$\omega_r = \omega + \frac{v_0}{r} = 0.7 + \frac{1}{2} = 1.2 \text{ rad/s}$$

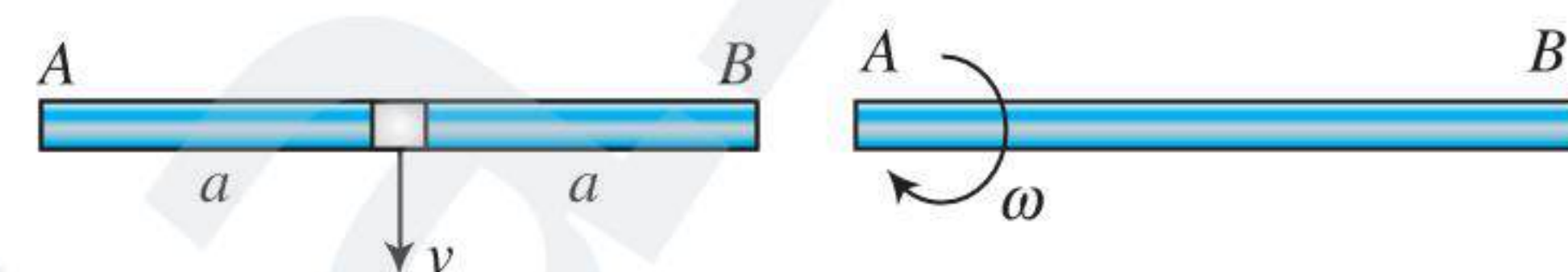
$\therefore$ Time taken to complete one round

$$t = \frac{2\pi}{\omega_r} \text{ or } t = \frac{2\pi}{1.2} \text{ s}$$

Angle rotated with respect ground in this time:

$$\theta = \left(\frac{v_0}{r} \right) \cdot t = \left(\frac{1}{2} \right) \left(\frac{2\pi}{1.2} \right) = \frac{5}{6} \pi$$

73. (4)



Angular momentum about A will be conserved i.e.,

$$L_i = L_f$$

$$\text{or } mva = I\omega$$

$$\text{or } mva = \frac{m(2a)^2}{3} \cdot \omega \therefore \omega = \frac{3v}{4a}$$

74. (2) KE of ball in position B = $mg(R - r)$

Here m = mass of ball.

Since it rolls without slipping the ratio of rotational to translational kinetic energy will be 2/5.

$$\frac{K_R}{K_T} = \frac{2}{5}$$

$$K_T = \frac{2}{7} mg(R - r); \frac{1}{2} mv^2 = \frac{2}{7} mg(R - r)$$

$$v = \frac{2}{\sqrt{7}} \sqrt{g(R - r)}; \omega = \frac{v}{R - r} = 2\sqrt{\frac{g}{7(R - r)}}$$

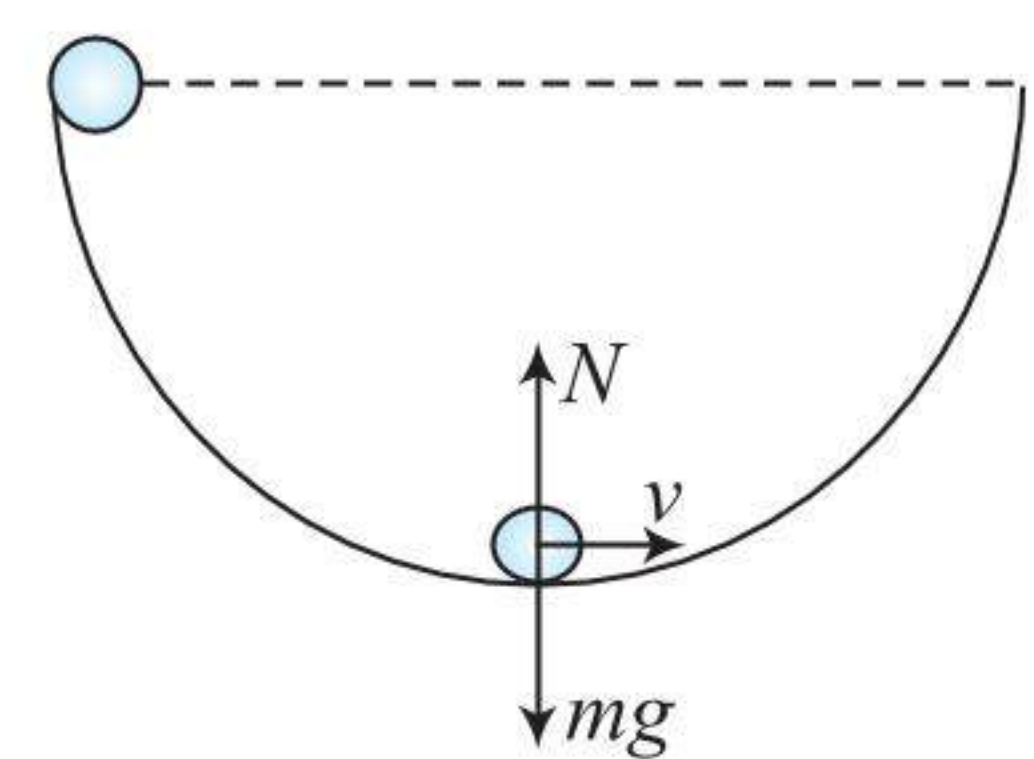
75. (3) From conservation of mechanical energy

$$mg(R - r) = \frac{7}{10} mv^2 \quad \dots(i)$$

$$\text{and } N - mg = \frac{mv^2}{R - r} \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get

$$N = \frac{17}{7} mg$$



Note: $(7/10) mv^2$ is the translational + rotational kinetic energy of sphere at bottom.

76. (1) Equilibrium of m gives: $T = mg$ (T = tension in string)

Net torque about point of contact of spool should be zero.

$$\text{Hence, } (2R)(Mg \sin \alpha) = TR$$

$$\text{or } 2Mg \sin \alpha = mg$$

$$\text{or } m = 2M \sin \alpha$$

77. (3) KE of $P = \frac{1}{2} m(2v)^2 = 2mv^2$

$$\text{KE of } S = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{3}{4}mv^2 \Rightarrow \frac{\text{KE of } P}{\text{KE of } S} = \frac{8}{3}$$

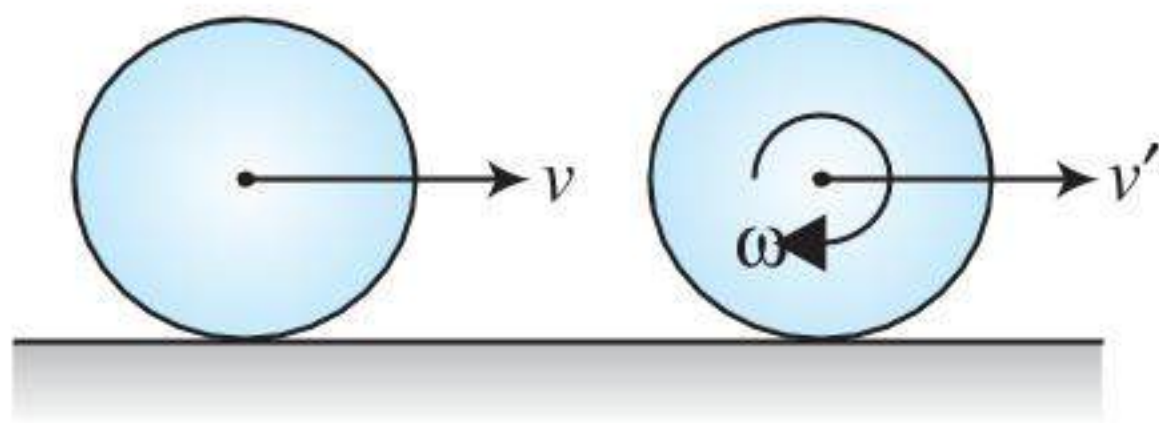
78. (3) Double torque, double angular acceleration, double linear acceleration, double distance.

79. (1) The kinetic energy is same in both the cases.

80. (3) Apply conservation of angular momentum:

$$\begin{aligned} I_i \omega_i &= I_f \omega_f \\ \Rightarrow \frac{1}{2}mR^2 \times 2.4 &= \left(\frac{1}{2}mR^2 + \frac{m(2R)^2}{12} \right) \omega_f \\ \Rightarrow \omega_f &= 1.44 \text{ rev/s} \end{aligned}$$

81. (2) Let v be the velocity of COM of ring just after the impulse is applied and v' its velocity when pure rolling starts. Angular velocity ω of the ring at this instant will be $\omega = v'/r$.



From impulse = change in linear momentum, we have

$$\begin{aligned} J &= mv \\ \text{or } v &= J/m \end{aligned} \quad \dots(i)$$

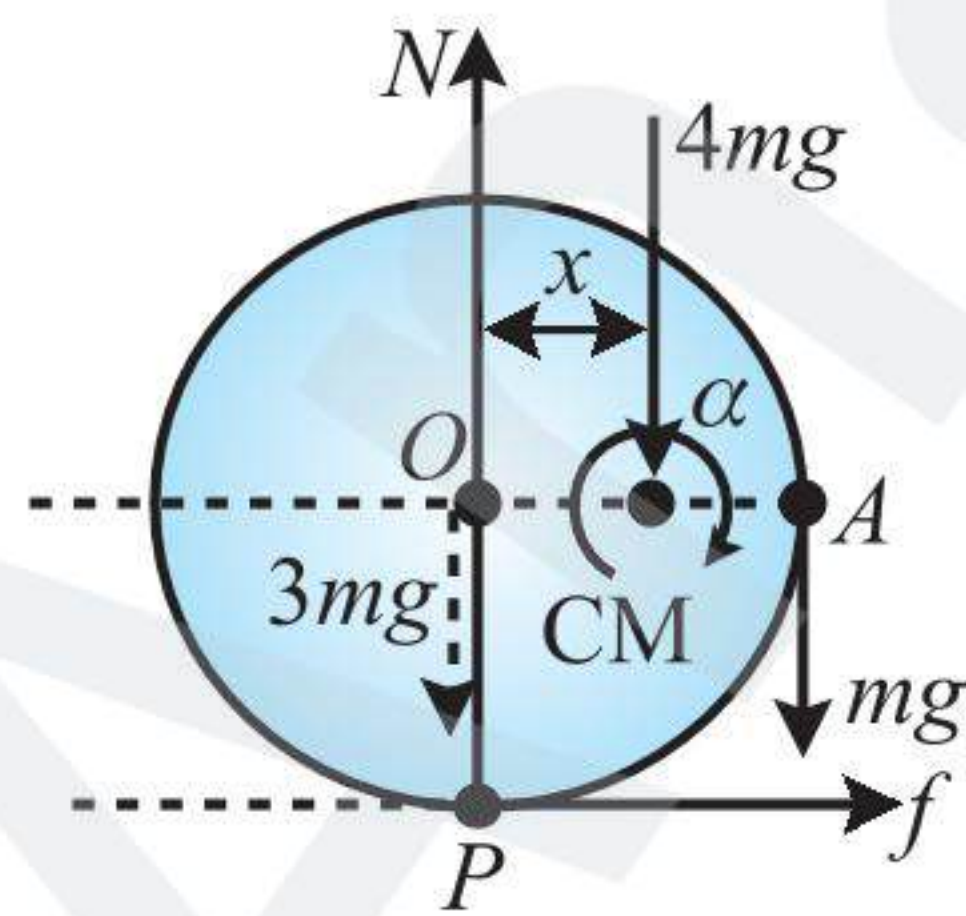
Between the two positions shown in figure, force of friction on the ring acts backwards. So, angular momentum of the ring about bottommost point will remain conserved

$$\begin{aligned} \therefore L_i &= L_f \quad \text{or} \quad mvr = mv'r + I\omega \\ mv'r + (mr^2)(v'/r) &= 2mv'r \\ v' &= \frac{v}{2} = \frac{J}{2m} \quad (\text{from equation}) \end{aligned}$$

82. (2) The distance of CM from the ring centre O

$$x = \frac{3m(0) + m(r)}{3m + m} = \frac{r}{4}$$

We can apply torque equation about point of contact as the ring is rolling.



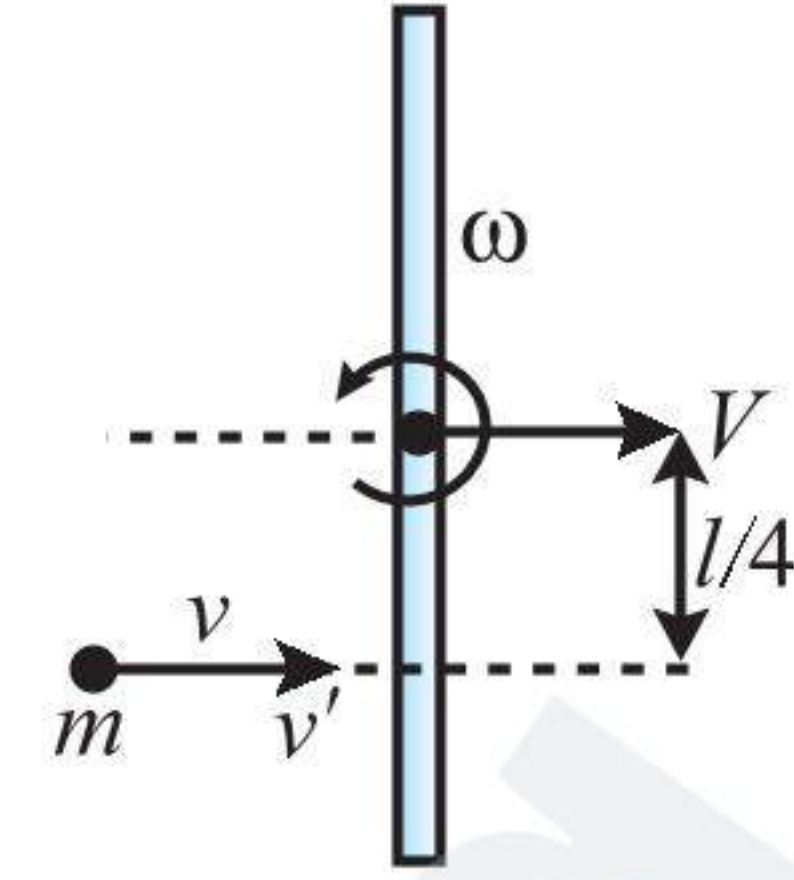
$$\tau_P = I_P \alpha$$

$$4mg \left(\frac{r}{4} \right) = \left[(3mr^2 + mr^2) + m(AP)^2 \right] \alpha$$

$$\Rightarrow mgr = \left[4mr^2 + m(\sqrt{2}r)^2 \right] \alpha$$

$$\Rightarrow mgr = 6mr^2 \alpha \Rightarrow \alpha = \frac{g}{6r}$$

83. (1) Applying conservation of linear momentum,



$$mv = mv' + mV \Rightarrow v = v' + V \quad \dots(i)$$

Applying conservation of angular momentum about point of collision.

$$0 = \left(\frac{ml^2}{12} \right) \omega - mV \left(\frac{l}{4} \right) \Rightarrow l\omega = 3V \quad \dots(ii)$$

Applying restitution equation,

$$(u_1 - u_2)_n = (v_2 - v_1)_n \Rightarrow (v - 0) = (V - v') \quad \dots(iii)$$

Solving Eqs. (i), (ii), and (iii) we get $V = v$ and $\omega = \frac{3v}{l}$

Time taken to complete three revolutions ($\theta = 6\pi$)

$$t = \frac{\theta}{\omega} = \frac{6\pi}{\omega} = \frac{6\pi l}{3v} = \frac{2\pi l}{v}$$

Hence, distance travelled by the centre of the rod is

$$s = vt = v \left(\frac{2\pi l}{v} \right) = 2\pi l$$

84. (1) Just before collision between two balls

Potential energy lost by ball A = kinetic energy gained by ball A .

$$\begin{aligned} mgh \frac{h}{2} &= \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2 \\ &= \frac{1}{2} \times \frac{2}{5}mR^2 \times \left(\frac{v_{cm}}{R} \right)^2 + \frac{1}{2}mv_{cm}^2 = \frac{1}{5}mv_{cm}^2 + \frac{1}{2}mv_{cm}^2 \end{aligned}$$

$$\Rightarrow \frac{5}{7}mgh = mv_{cm}^2 \Rightarrow \frac{mgh}{7} = \frac{1}{5}mv_{cm}^2$$

After collision, only translational kinetic energy is transferred to ball B .

So just after collision, rotational kinetic energy of ball A

$$= \frac{1}{5}mv_{cm}^2 = \frac{mgh}{7}$$

85. (4) Let after collision velocity of rod be v and angular velocity be ω , then

$$mv_0 = mv + MV \Rightarrow mv_0 x = mvx + \frac{Ml^2}{12} \omega$$

$$mv_0 = mv + \frac{M\omega \eta^2 x}{12}$$

$$\text{From above equations, } \frac{M\omega \eta^2 x}{12} = MV \Rightarrow \omega \eta^2 x = 12V$$

$$\text{Velocity of farther end of the rod} = V - \frac{l\omega}{2}$$

$$= \frac{\omega \eta^2 x}{12} - \frac{\omega \eta x}{2} = \frac{\omega \eta x}{2} \left(\frac{\eta}{6} - 1 \right)$$

If this velocity is opposite to v_0 , then,

$$\frac{\eta}{6} - 1 < 0 \Rightarrow \eta < 6$$

86. (1) $\vec{\tau}_{av} \cdot \Delta t = \Delta \vec{L}$

$$|\Delta \vec{L}| = |\vec{L}_f - \vec{L}_i| \text{ above point of projection.}$$

$$= (mu \sin \theta) (\text{Range})$$

$$= \frac{(mu \sin \theta)(u^2 \sin 2\theta)}{g} = \frac{mu^3 \sin \theta \sin 2\theta}{g}$$

$$|\vec{\tau}_{av}| = \frac{\Delta \vec{L}}{\Delta t} = \frac{mu^3 \sin \theta \sin 2\theta}{g} \frac{g}{2u \sin \theta}$$

$$= \frac{mu^2 \sin 2\theta}{2}$$

87. (1) Since the disc was rolling, the velocity of its top point at the instant of leaving the track was zero. It means $v_H = \omega r$.

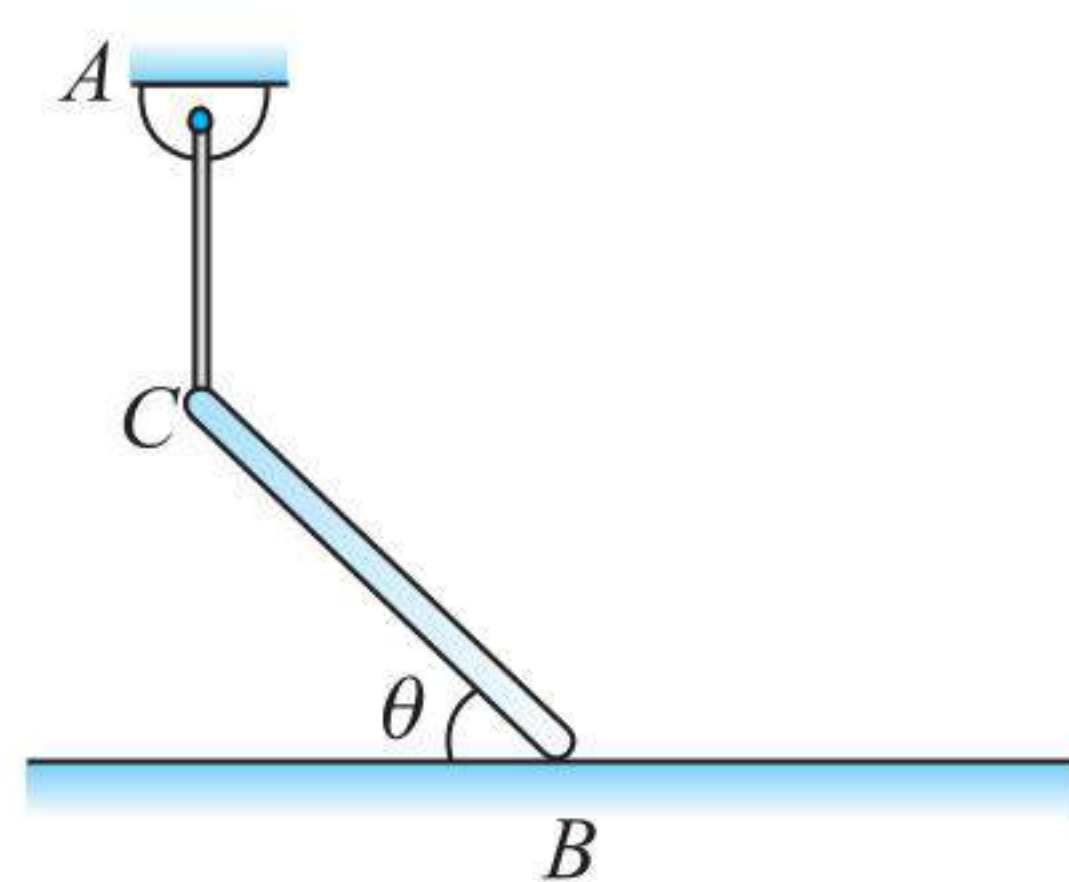
When the disc is in air v_H and ω both do not change. Hence the horizontal component of the velocity of the top point P of the disc at every instant is zero and the vertical component of the velocity of the point P is equal to the vertical component of velocity of the CM of disc.

$$\Rightarrow \sqrt{2gh} = \sqrt{2 \cdot g \cdot 2(R-0.1)} = 6$$

$$\Rightarrow 4 \times 10(R-0.1) = 36$$

$$R = 1 \text{ m}$$

88. (2) When the string gets vertical, the centre of the rod has horizontal velocity and the angular speed of the rod will be zero. At this point angle θ has become minimum (i.e. $\frac{d\theta}{dt} = 0$). Velocity of C is horizontal. Velocity of B is also horizontal.



$\therefore$ Rod has a horizontal velocity and no angular velocity.

Energy conservation gives:

$$\frac{1}{2} Mv^2 = Mg \frac{b}{4} - Mg \left(\frac{b-a}{2} \right)$$

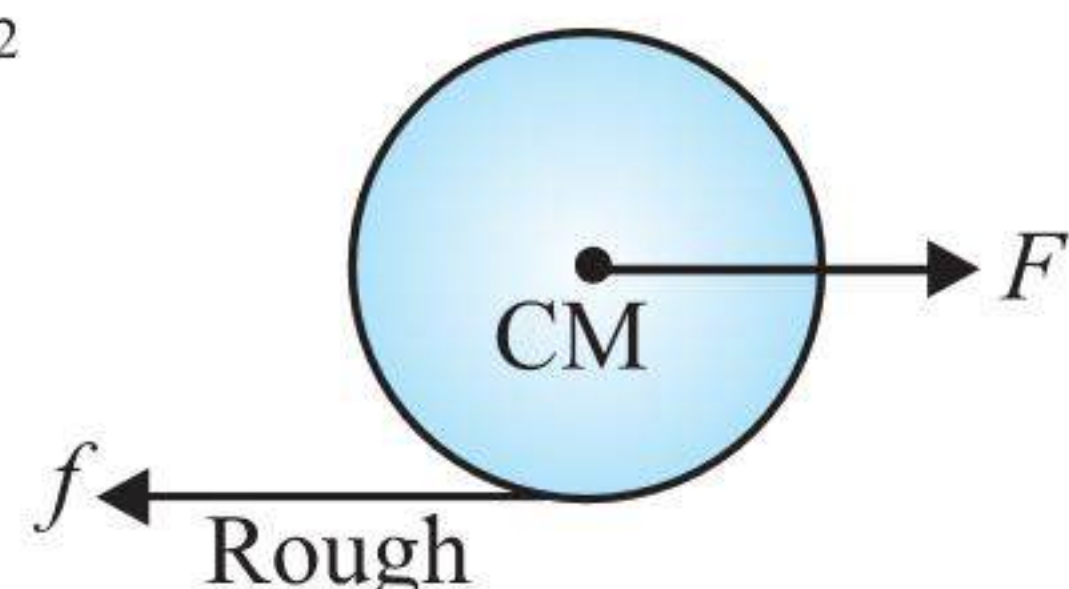
$$\therefore v = \sqrt{g \left(a - \frac{b}{2} \right)}$$

Multiple Correct Answers Type

1. (1), (2), (3), (4)

Acceleration of centre of mass,

$$a_c = \frac{F}{M} \frac{1}{1 + \frac{K^2}{R^2}} = \frac{5F}{7M} \quad \left(\because K = \sqrt{\frac{2}{5}} R \right)$$



$$\text{Frictional force, } f = F \left[\frac{1}{1 + \frac{R^2}{K^2}} \right] = \frac{2F}{7}$$

Since $f = +ve$, friction is directed backward.

Options (1), (2), (3) and (4) are incorrect.

2. (3), (4)

The velocity of the disc when rolling begins can be obtained using the conservation of angular momentum principle about the point through which the friction force acts. So, the coefficient of friction has no bearing on the final velocity. The work done by the force of friction will simply be changed to kinetic energy.

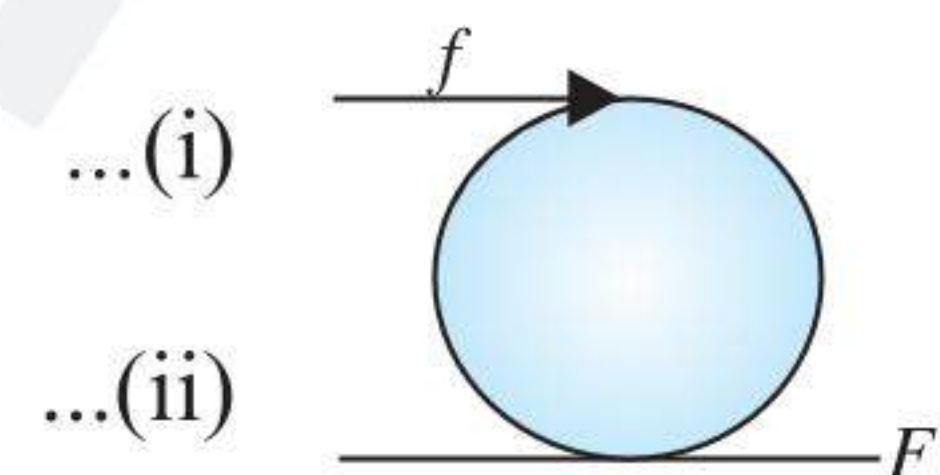
3. (3), (4)

$mgh = \frac{1}{2} mv^2 \left[1 + \left(\frac{k^2}{R^2} \right) \right]$, k = radius of gyration. For all solid spheres, $k^2/R^2 = 2/5$ = independent of m and R .

4. (2), (4)

$$F + f = mg \quad \dots(i)$$

$$FR - fR = \frac{2}{5} mR^2 \frac{a}{R} \quad \dots(ii)$$



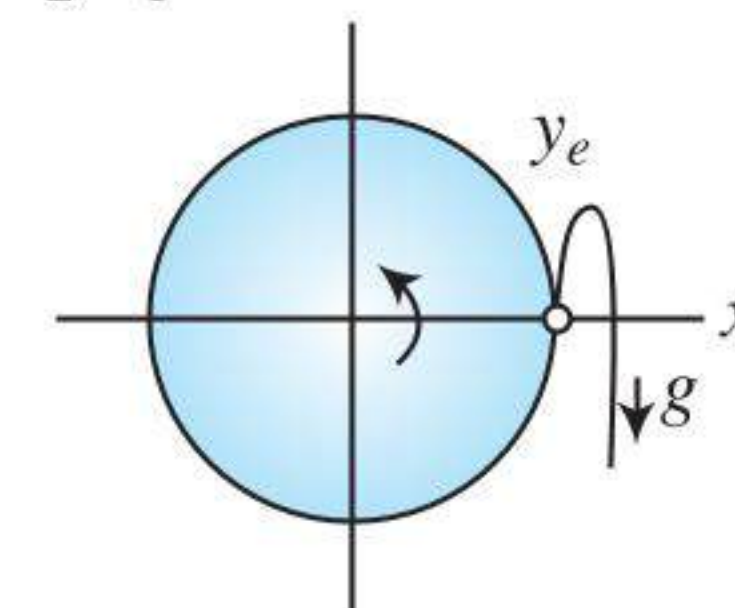
After solving these two equations we get the frictional force is positive. This implies that friction supports the motion.

5. (1), (2)

Since D is the instantaneous centre of rotation and as we go from D to B velocity increases. So upper half of the ring will have greater kinetic energy than lower half.

6. (1), (4)

$$I_1 \omega_T = I_T \omega_T + I_2 \omega_T \quad \dots(i)$$



Velocity of broken away piece decreases initially then increases.

7. (1), (4)

If $k^2 > Rx$, friction will act in backward direction, otherwise, in forward direction.

$$\text{For given spool } K^2 = \frac{R^2}{4}$$

$$\text{Fig. (a): } Rx = R \cdot \frac{R}{3} = \frac{R^2}{3} \text{ friction forward}$$

$$\text{Fig. (b): } Rx = R \cdot \frac{R}{5} = \frac{R^2}{5} \text{ friction backward}$$

In Fig. (c) as force is below the diametric plane, hence friction acts backward.

8. (1), (2), (4)

Let ω_1 = the initial angular velocity of the disc ω_2 = the final common angular velocity of the disc and the ring.

$$\text{For the disc, } I_1 = \frac{1}{2} mr^2$$

$$\text{For the ring, } I_2 = mr^2$$

By conservation of angular momentum,

$$L = I_1 \omega_1 = (I_1 + I_2) \omega_2$$

$$\omega_2 = \frac{I_1 \omega_1}{I_1 + I_2} = \frac{\omega_1}{3}$$

$$\text{Initial kinetic energy} = E_1 = \frac{1}{2} I_1 \omega_1^2$$

$$\text{Final kinetic energy} = E_2 = \frac{1}{2} (I_1 + I_2) \omega_2^2$$

Heat produced = loss in KE

$$\text{Ratio of heat produced to initial kinetic energy} = (E_1 - E_2)/E_1 = 2/3$$

9. (2), (4)

The force of friction between the two surfaces in contact disappears when there is no relative (linear) motion between them. Angular momentum will not be conserved as the discs will have the final angular velocities in the opposite directions.

10. (1), (2), (3)

Let L = angular momentum

$$\tau = \frac{dL}{dt} \text{ or } dL = \tau dt$$

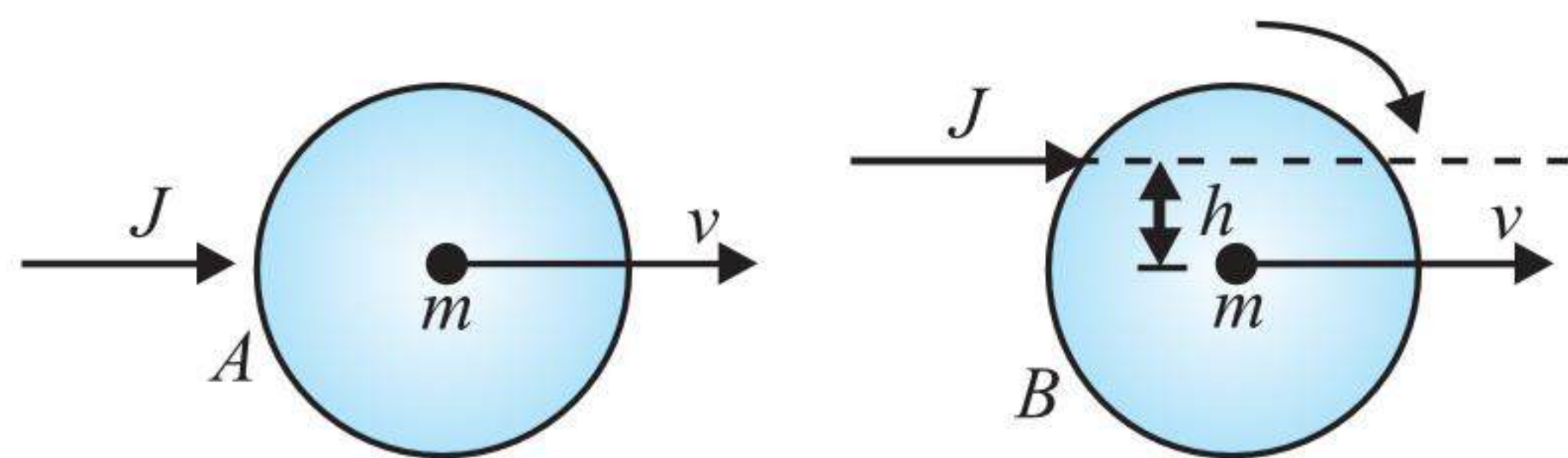
For constant torque, $\Delta L = \tau \Delta t = I \Delta \omega$

or $I\omega$ (if $\omega_i = 0$)

Rotational kinetic energy $= I\omega^2/2 = (\Delta L)^2/2I = (\tau \Delta t)^2/2I$

11. (1), (2), (4)

$J = mv$ for both. Sphere A has no angular motion. For sphere B , angular momentum imparted by $J = L$.



12. (2), (3), (4)

Let P = external force, F = force of friction between A and B , a_1 = acceleration between A and B , a_2 = acceleration beyond B

$$P - F = ma_1 \text{ and } P = ma_2$$

$$a_2 > a_1$$

Let α = angular acceleration between A and B

For one rotation, $\theta = 2\pi = \frac{1}{2} \alpha T^2$

$$T = \sqrt{\frac{4\pi}{\alpha}} = \text{time of travel from } A \text{ to } B$$

Angular velocity at $B = \omega_B = \alpha T$

For one rotation to the right of B , $\theta = 2\pi = \omega_B t$

$$t = \frac{2\pi}{\alpha T} = \frac{\frac{1}{2} \alpha T^2}{\alpha T} = \frac{T}{2}$$

13. (3), (4)

In rolling without slipping, no work is done against friction. Hence, loss in gravitational potential energy of a body is equal to its total kinetic energy, i.e., linear plus rotational kinetic energies.

Also, $v = \omega r$

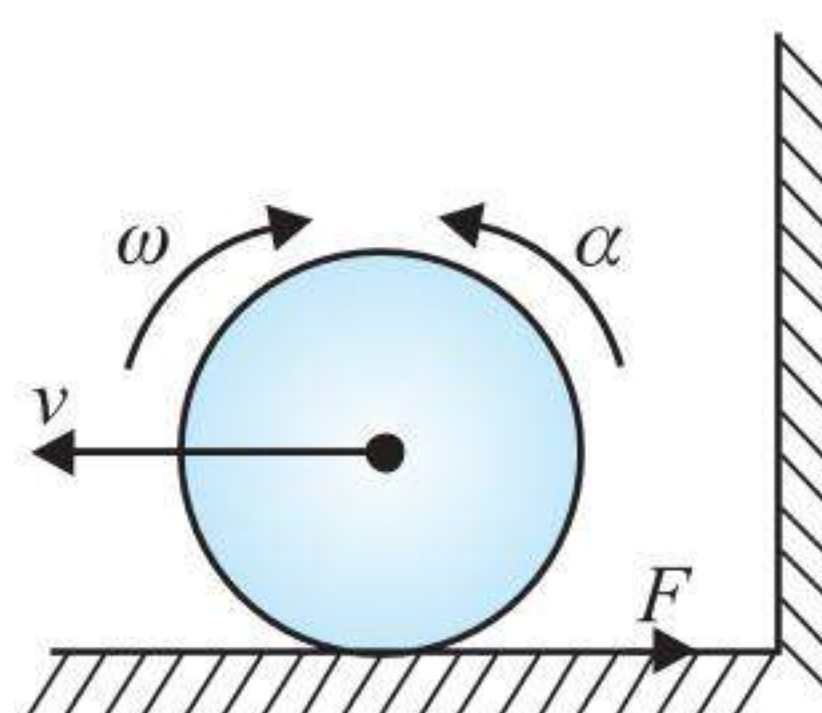
$$\begin{aligned} \text{Total kinetic energy} &= \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 \\ &= \frac{1}{2} mv^2 + \frac{1}{2} (mK^2) \left(\frac{v^2}{r^2} \right) = \frac{1}{2} mv^2 \left[1 + \frac{K^2}{r^2} \right] \end{aligned}$$

where K = radius of gyration

As all the bodies have the same final total kinetic energy, their final velocities will depend upon the ratio K/r . Bodies with the smaller value of K/r will have the greater v and hence will reach the bottom earlier.

14. (1), (3), (4)

After impact, the force of friction will act in a direction opposite to that of the motion. The body will retain its initial angular motion (clockwise). The force of friction will cause linear retardation, reducing v . It will also cause an anticlockwise angular acceleration, which will reduce ω to zero and then introduce anticlockwise ω till rolling without slipping begins. F will then disappear.



15. (1), (2), (3), (4)

Angular momentum = linear momentum $\times$ perpendicular distance from the point of rotation

$$\text{or } L = Jl$$

$$\text{Also, } I = \frac{ml^2}{3}$$

$$\omega = \frac{L}{I} = \frac{Jl}{ml^2/3} = \frac{3J}{ml}$$

$$\text{Kinetic energy} = \frac{L^2}{2I} = \frac{J^2 l^2}{2 \cdot \frac{ml^2}{3}} = \frac{3J^2}{2m}$$

$$v_c = \omega \frac{l}{2} = \frac{3J}{2m}$$

16. (1), (3), (4)

Since, the spool rolls over the horizontal surface, instantaneous axis of rotation passes through the point of contact of spool with the horizontal surface.

About the instantaneous axis of rotation, moment produced by F is clockwise. Therefore, the spool rotates clockwise. In that case, acceleration will be rightward and thread will wind.

If rotational motion of the spool is considered about its own, then the resultant moment on it must be clockwise. But moment produced by the force is anticlockwise and its magnitude is equal to F_1/r . Hence, moment produced by the friction (about its own axis) must be clockwise and its magnitude must be greater than Fr . It is possible only when friction acts leftwards. Therefore, option (2) is correct.

17. (1), (2), (3)

Weight of the particle is balanced by normal reaction of the floor. Hence, the resultant force acting on the particle is provided by the tension in the thread.

The line of action of this tension passes through 'B' at the initial moment only. After that it does not pass through 'B'. Hence, the tension produces a moment about B . It means, angular momentum about B does not remain conserved. The tension always produces a moment about O . Magnitude of this moment is equal to TR in an anticlockwise direction. Hence, options (1), (2) and (3) are incorrect. Since neither external work is done on the particle nor energy is lost against friction, therefore KE of the ball remains constant. Hence, option (4) is correct.

18. (1), (2), (3), (4)

When the particle strikes at one end of the rod then an impulse is exerted by the particle on the rod along the direction of its original motion. An equal but opposite impulsive reaction is exerted by the rod on the particle. If system of rod and the particle is considered, then there is no external impulse on the system, hence momentum of the system remains conserved. Hence, option (1) is correct.

Since, both the impulsive reactions act at the same point, one acts on the rod and other on the particle, hence no angular impulse is produced by these impulsive reactions about any point. It means, angular momentum of the system remains conserved about every point. Hence, options (2) and (3) are correct.

19. (1), (2), (4)

There is no slipping, therefore $\tau = I\alpha$

$$T[R_2 - R_1] = [MK^2 + MR_2^2]\alpha$$

$$\alpha = \frac{200(0.6 - 0.3)}{50(0.3^2 + 0.6^2)} = 2.67 \text{ rad/s}^2$$

Using Newton's IInd law

$$T - f = MR_2\alpha$$

$$f = (T - MR_2 \alpha) = \left[200 - 5 \times 0.6 \times \frac{8}{3} \right] = 120 \text{ N}$$

$$\text{Now } F_{\max} = \mu_s mg = 0.1 \times 50 \times 10 = 50 \text{ N}$$

$$f_k = \mu_s Mg = 0.08 \times 50 \times 10 = 40 \text{ N}$$

$$Ma = -T + 40 \Rightarrow a = -3.2 \text{ m/s}^2$$

20. (2), (3), (4)

Due to force F , the plank has a tendency to slide to the right below the sphere. Hence, friction acts on the plank to the left and that on the sphere acts to the right. The friction not only acts on accelerating force on the sphere but also produces an anticlockwise moment. Therefore, the sphere experiences a rightward translational acceleration and an anticlockwise angular acceleration simultaneously.

If the angular acceleration of the sphere is α , then its centre of mass has a backward acceleration $r\alpha$ relative to the plank. Hence, its net acceleration becomes less than that of the plank. Hence, option (2) is correct. Obviously, option (1) is incorrect.

Since there is no sliding between the surfaces of the sphere and the plank, therefore, no energy is lost against friction.

Since friction, acting on the sphere, is the only force which provides motion to it, therefore KE of the sphere at any instant is equal to work done by the friction acting on it. Hence, option (3) is correct.

Since no energy is lost against friction, therefore KE of the whole system is equal to work done by force F . Hence, option (4) is also correct.

21. (1), (3)

There will be exchange of linear velocities only. However, the two spheres cannot exert torques on each other, as their surfaces are frictionless and the angular velocities of the spheres do not change.

22. (2), (3), (4)

Using law of conservation of energy for wheel A,

$$mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} mv^2$$

$$mgh = \frac{1}{2} I \left(\frac{v}{r} \right)^2 + \frac{1}{2} mv^2$$

$$v^2 = \frac{mg}{\frac{1}{2} \left[\frac{I}{r^2} + m \right]} \cdot h$$

$$v \propto \sqrt{h}$$

$$v_A \propto \sqrt{h} \text{ and } v_B \propto \sqrt{4h}$$

$$v_B = 2v_A$$

$$\text{Also } v_A^2 = 2a_A \frac{h}{\sin \theta} \Rightarrow a_A = \frac{g \sin \theta}{1 + \frac{I}{mr^2}}$$

Acceleration of wheel is independent of h

$$a_A = a_B$$

$$\text{Also } \frac{h}{\sin \theta} = \frac{1}{2} a_A t_A^2 \Rightarrow t_A \propto \sqrt{h}$$

$$\frac{t_B}{t_A} = \sqrt{\frac{4h}{h}} = 2$$

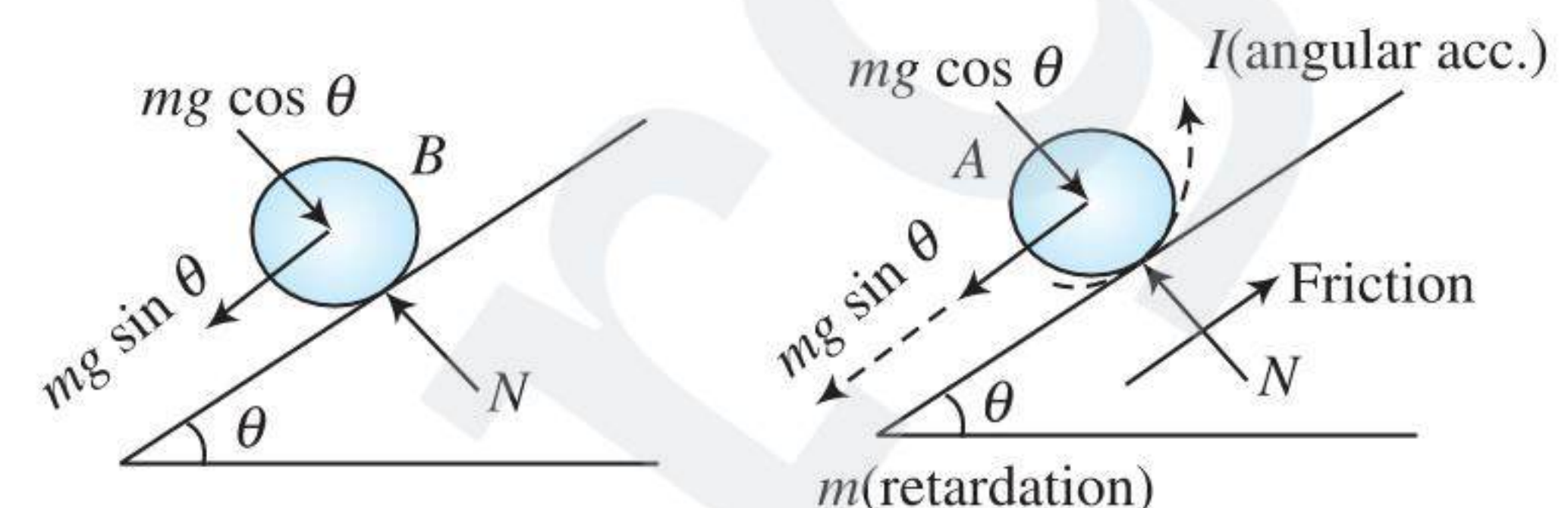
23. (1), (4)

Due to torque of friction about CM, ω eventually decreases to zero, initially there is no translation. Friction is sufficient for pure rolling therefore after sometime pure rolling begin. There is no external

force in x direction therefore momentum is conserved along x direction.

24. (2), (4)

If we consider free body diagram of the wheel B, the retarding force on wheel B is equal to $mg \sin \theta$. Therefore, its retardation is equal to $g \sin \theta$ and no moment is produced by these forces about its centroidal axis. Hence, the wheel B continues to rotate with a constant angular velocity and its translational velocity decreases during up the plane motion. Hence, at highest point it has a rotational KE.



While during ascending motion friction acts on wheel A. That is the friction which is responsible for angular retardation of wheel A as shown in the figure.

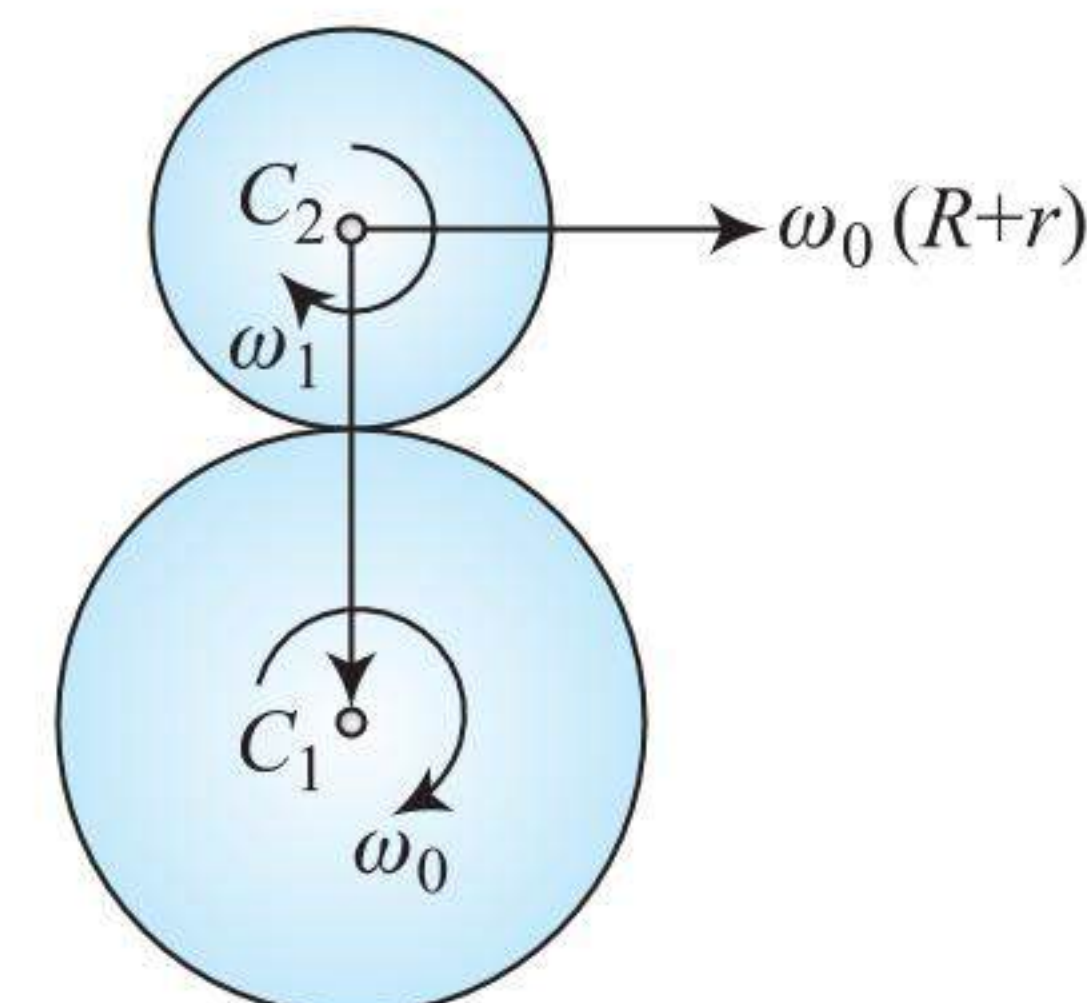
The resultant retarding force on wheel A is equal to $(mg \sin \theta - \text{friction})$. Hence, net retarding force on wheel A is less than $mg \sin \theta$. Therefore, it continues to move up the plane for longer time and comes to rest after moving more distance than wheel B.

Wheel A comes to rest when its total kinetic energy becomes equal to zero. It means, at highest points its potential energy will be equal to total initial KE, while at highest point, potential energy of wheel B is equal to its initial translational KE up. It means, maximum height ascended by wheel A is greater than that by B.

When wheel A descends, it tries to slide down the plane due to component $mg \sin \theta$ of its weight. Hence, the friction remains acting up the plane and produces a moment about centroidal axis. Hence, wheel experiences an angular acceleration in anti-clockwise direction.

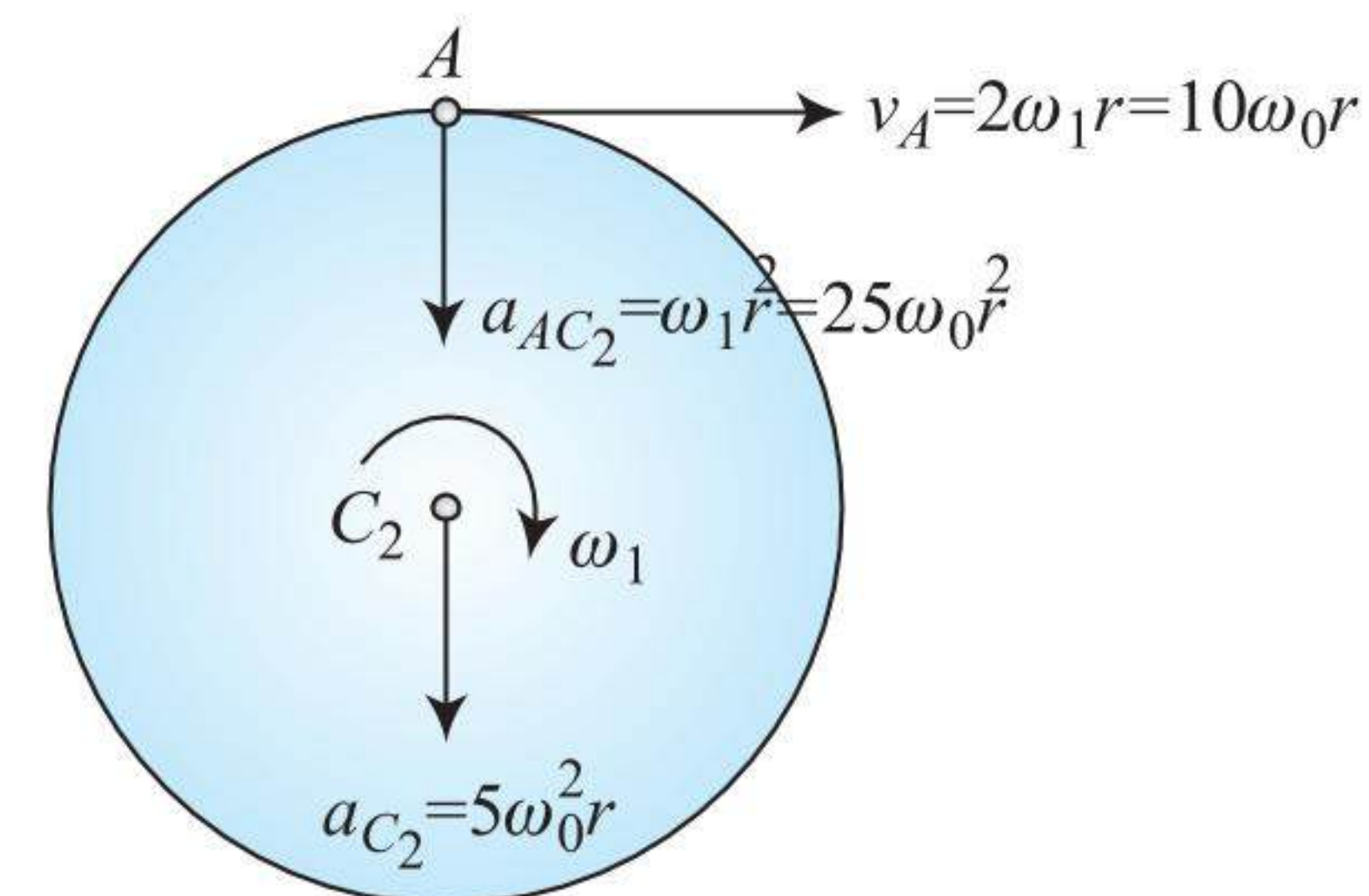
Since, all the remaining forces remains same, therefore, magnitude of friction also remains same.

25. (2), (3), (4)



$$\omega_1 r = \omega_0 (R + r) \Rightarrow \omega_1 = 5\omega_0$$

$$\text{KE} = \frac{1}{2} m (\omega_0 (R + r))^2 + \frac{1}{2} \left(\frac{mr^2}{2} \right) \omega_1^2 = \frac{75}{4} mr^2 \omega_0^2$$



$$\vec{a}_A = \vec{a}_{AC_2} + \vec{a}_{C_2} = 30\omega_0^2 r = \frac{(10\omega_0 r)^2}{\rho}$$

$$\rho = \frac{10r}{3}$$

26. (1), (4)

$$Mg \frac{L}{2} = \frac{1}{2} \left(\frac{ML^2}{3} \right) \omega_0^2$$

$$\omega_0 = \sqrt{\frac{3g}{2L}}$$

$$e = \frac{v - \omega(L-r)}{(L-r)\omega_0} = 1$$

Conservation of angular momentum

$$\frac{ML^2}{3} \omega_0 = \frac{ML^2}{3} \omega + mv(L-r)$$

27. (1), (4)

By conservation of linear momentum both will get same velocity of centre of mass

$$\therefore \frac{1}{2} mv_{cm}^2 = mgh$$

$$h_1 = h_2$$

But by conservation of angular momentum about centre of mass System 2 have angular velocity also

$$\therefore K_1 < K_2 \quad \left[\text{where } K = \frac{1}{2} mv_{cm}^2 + \frac{1}{2} I \omega^2 \right]$$

Linked Comprehension Type**For Problems 1–3**

1. (1) Since acceleration is same for all the three spheres, they cover equal distances in equal intervals of time in all the cases (A), (B) and (C).

Hence (1)

2. (3) From passage, for case (C);

$$\mu_{\min} = \frac{\tan \theta \left(\frac{k^2}{R^2} \right)}{\left(1 + \frac{k^2}{R^2} \right)} \rightarrow (\text{pure rolling})$$

Putting the values of k for different objects given in the table (in passage) we get:

$$\mu_{\min(\text{Ring})} = \frac{\tan \theta}{2}$$

$$\mu_{\min(\text{Disc})} = \frac{\tan \theta}{3}, \mu_{\min(\text{Solid sphere})} = \frac{2}{3} \tan \theta$$

$$\mu_{\min(\text{Hollow sphere})} = \frac{2}{7} \tan \theta$$

$$\Rightarrow \mu_{\min(\text{Ring})} \text{ is greater than either of } \mu_{\min(\text{Disc})}, \mu_{\min(\text{Solid Sphere})}, \mu_{\min(\text{Hollow sphere})}$$

Therefore, the pure rolling of ring will confirm pure rolling of all other bodies.

3. (3) As given in the equation of case (B);

$$\mu NR = Mk^2 \alpha \text{ and } N = Mg \cos \theta$$

As; θ, M, R, μ are same for all. α will be least for that object for which k and hence I is maximum.Therefore α for ring ($k = R$) and hence ω for ring at the bottom is minimum.

$$\text{Also, } Mg \sin \theta - \mu N = Ma$$

Since M, μ, θ, N are same for all objects, they have same linear acceleration and hence same linear velocity and hence same $\frac{1}{2} Mv_{cm}^2$.

$$\therefore \text{KE} = \left(\frac{1}{2} Mv_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 \right) \text{ is least for the ring.}$$

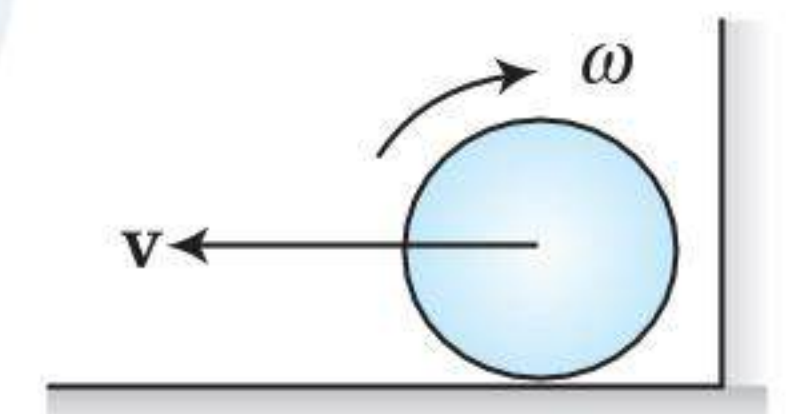
4. (2) For ring $a = \frac{g \sin \theta}{2}$ (for pure rolling) is less than that of disc.

For Problems 5–6

5. (2), 6. (1)

Just after collision

$$v = \omega r$$

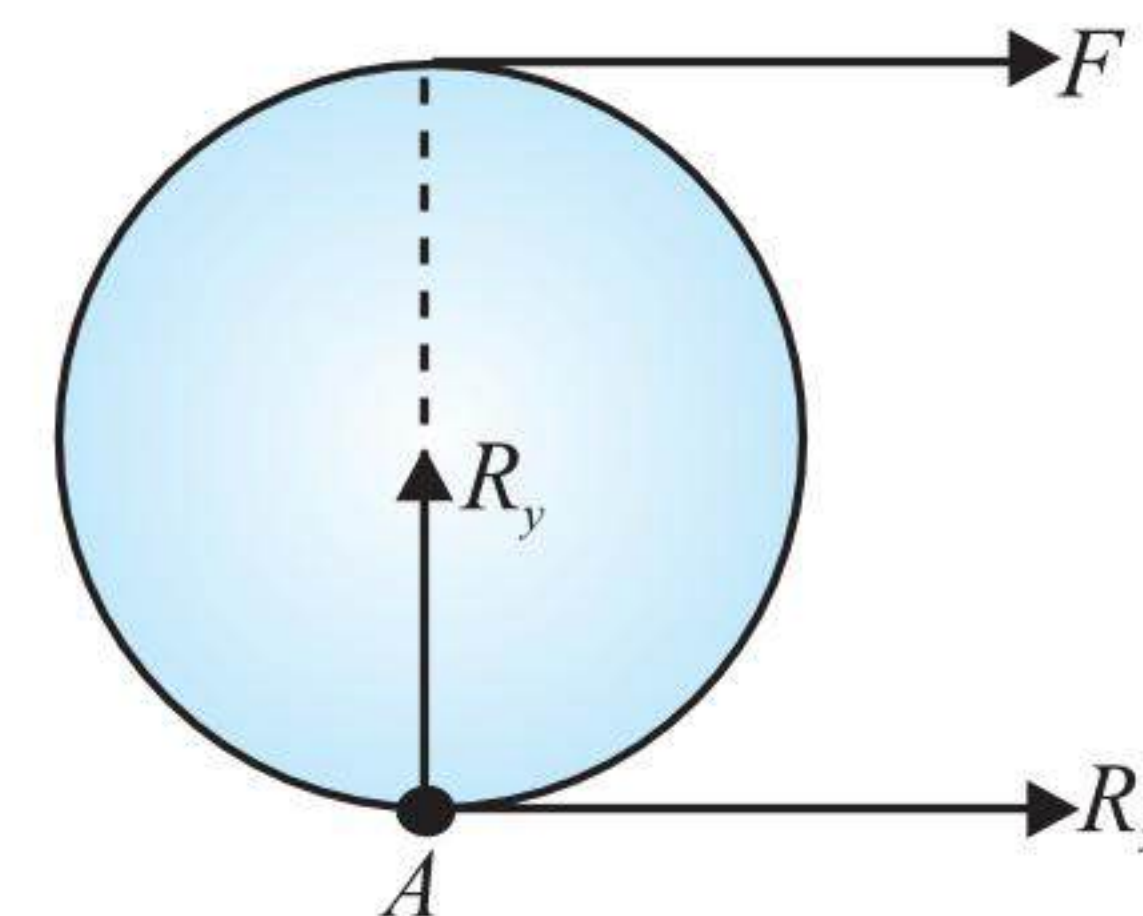
Velocity of lowest potential = $v + \omega r = 2v$ to the left.Change in angular momentum is due to translational part only, as ω does not change due to collision.**For Problems 7–9**

7. (4) Using torque equation about point A

$$\Rightarrow 2FR = I_A \alpha$$

$$\Rightarrow 2FR = \frac{3}{2} MR^2 \left(\frac{a}{R} \right) \Rightarrow a_t = \frac{4}{3} \frac{F}{m}$$

8. (2) $R_y = mg$



9. (3) Torque about the CM,

$$(F - R_x)R = I \alpha$$

$$\Rightarrow (F - R_x)R = \frac{MR^2 a_t}{2r}$$

$$\Rightarrow (F - R_x)R = \frac{2F}{3}$$

$$\Rightarrow F - \frac{2F}{3} = R_x \Rightarrow R_x = \frac{F}{3}$$

For Problems 10–12

10. (4), 11. (1), 12. (4)

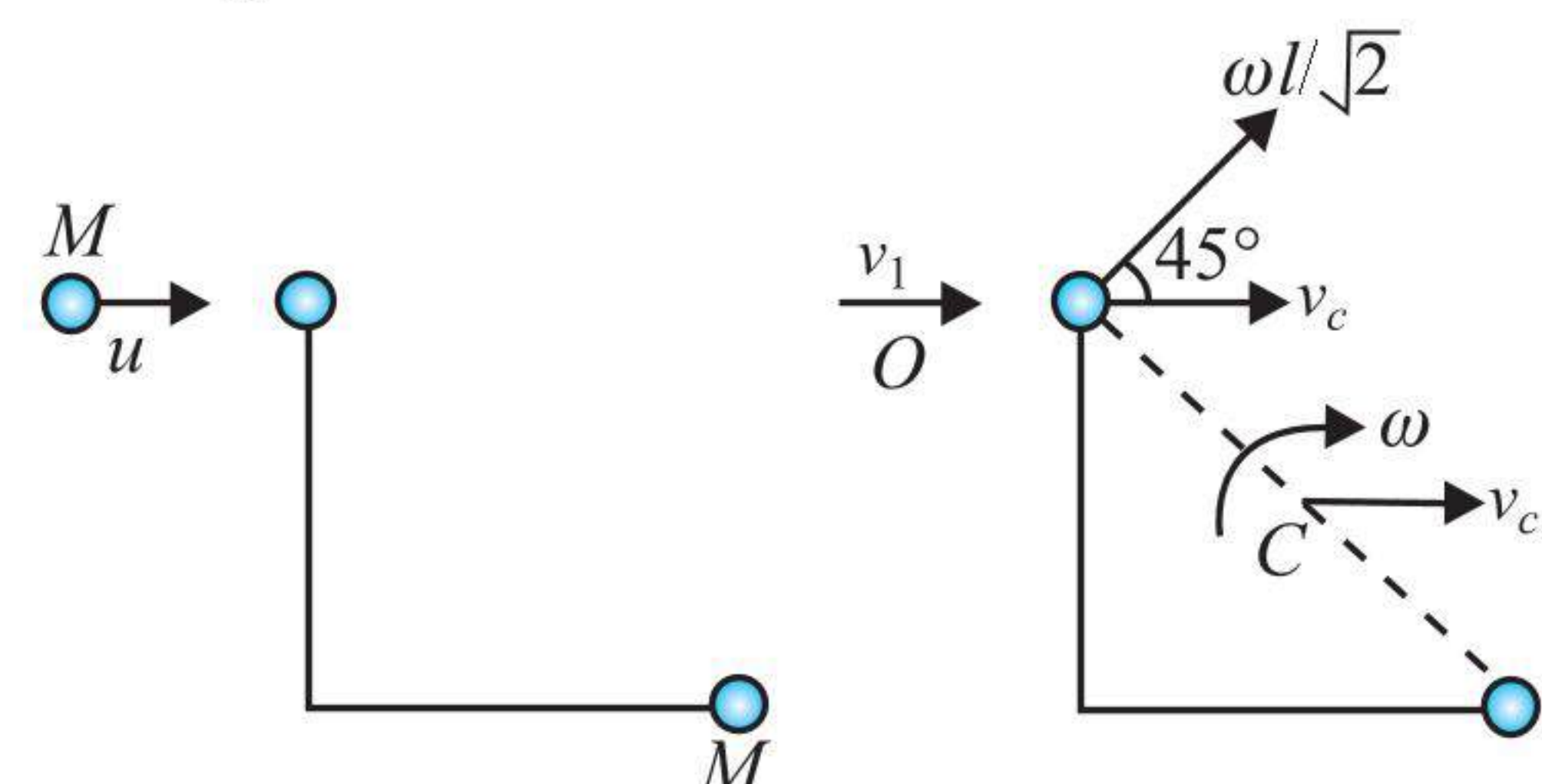
Applying conservation of momentum,

$$Mv_1 + 2Mv_c = Mu$$

$$v_1 + 2v_c = u \quad \dots(i)$$

$$e = 1 = \frac{v_c + \frac{\omega l}{2} \cos 45^\circ - v_1}{u - 0}$$

$$u = v_c + \frac{\omega l}{2} - v_1 \quad \dots(ii)$$



Applying conservation of angular momentum about C,

$$Mv_1 \frac{l}{2} + 2M \left(\frac{l}{\sqrt{2}} \right)^2 \omega = Mu \frac{l}{2}$$

$$v_1 + 2\omega l = u \quad \dots(\text{iii})$$

Solving Eqs. (i), (ii) and (iii), we get

$$v_1 = -\frac{u}{7}; \omega = \frac{4u}{7l}$$

Angle rotated, $\theta = \pi$

$$t = \frac{\theta}{\omega} = \frac{7\pi l}{4u}$$

For Problems 13–15

13. (3) Applying conservation of linear momentum,

$$(2 + 1 + 2)v_c = 2 \times 10 + 1 \times 10$$

$$v_c = 6 \text{ m/s}$$

14. (4) Applying conservation of angular momentum about the centre of the rod,

$$\left[\frac{2 \times (4)^2}{12} + 2 \times (1)^2 + 1 \times (1)^2 \right] \omega = 2 \times 10 \times 1 - 1 \times 10 \times 1$$

$$\omega = \frac{30}{17} \text{ rad/s}$$

15. (4) Linear momentum will decrease and angular momentum will increase.

For Problems 16–18

16. (1), 17. (2), 18. (1)

$$\text{KE} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{3}{4}mv^2 = 12 \text{ J}$$

$$\vec{a}_{A/C} = \vec{a}_A - \vec{a}_C$$

$$\Rightarrow \vec{a}_A = \vec{a}_{A/C} + \vec{a}_C = \omega^2 r \hat{i} + \alpha r \hat{j} + a \hat{i} = 60 \hat{i} + 10 \hat{j}$$

$$a_A = \sqrt{3600 + 100} = 10\sqrt{37} \text{ m/s}^2$$

$$\vec{a}_B = \omega^2 r \hat{j} + \alpha r (-\hat{i}) + a \hat{i} = 10 \hat{i} + 40 \hat{j}$$

$$a_B = 10\sqrt{17} \text{ m/s}^2$$

For Problems 19–21

19. (2), 20. (1), 21. (3)

$$F \cos \theta - f = ma \quad \dots(\text{i})$$

$$fR = \frac{1}{2}mR^2\alpha \Rightarrow fR = \frac{1}{2}mR^2 \frac{a}{R}$$

$$\Rightarrow f = \frac{ma}{2} \quad \dots(\text{ii})$$

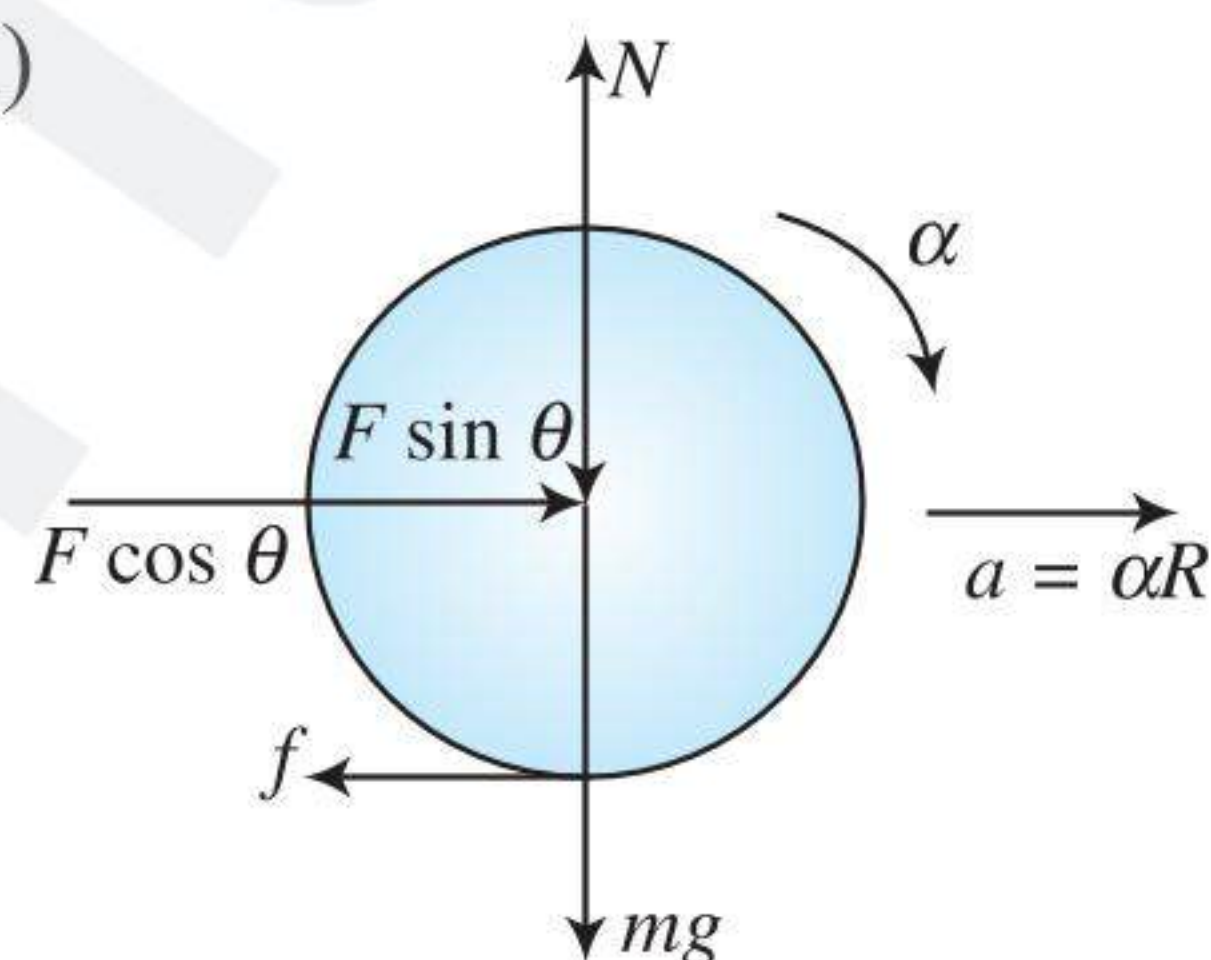
From (i) and (ii)

$$f = \frac{F \cos \theta}{3}$$

$$\text{Now } a = \frac{2f}{m} = \frac{2F \cos \theta}{3m}$$

$$f \leq f_l \Rightarrow \frac{F \cos \theta}{3} \leq \mu N$$

$$\Rightarrow \frac{F \cos \theta}{3} \leq \mu(mg + F \sin \theta) \Rightarrow F \leq \frac{3\mu mg}{\cos \theta - 3\mu \sin \theta}$$



For Problems 22–24

22. (4) As no external force is acting on the system (block + cylinder) in the horizontal direction, the linear momentum of the system remains conserved in the horizontal direction.

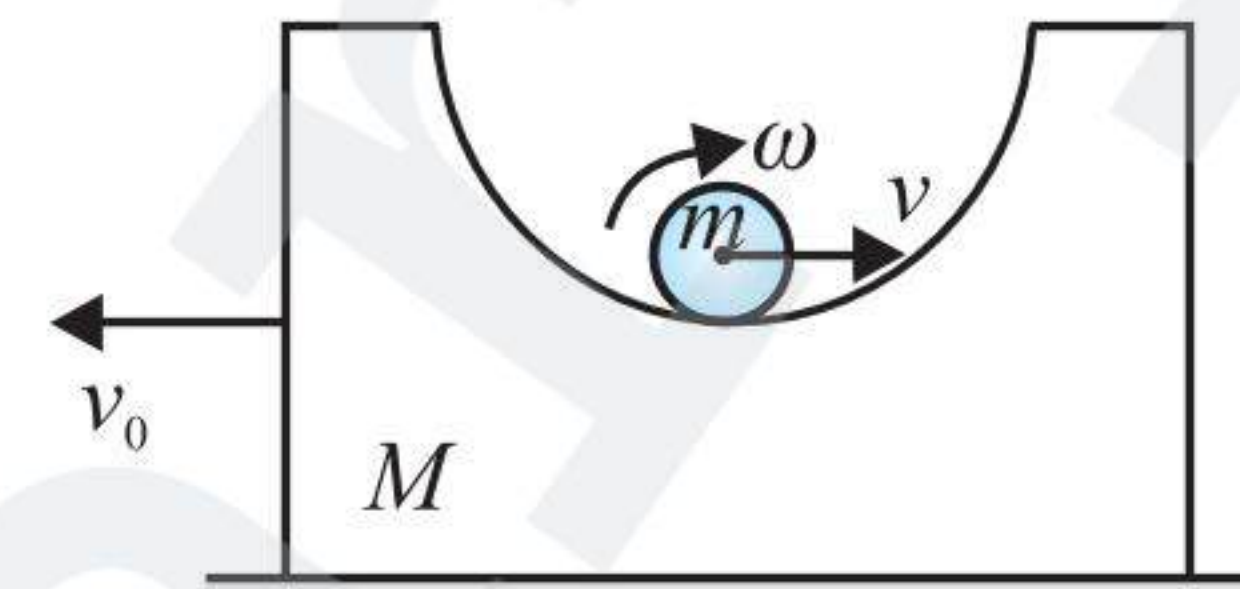
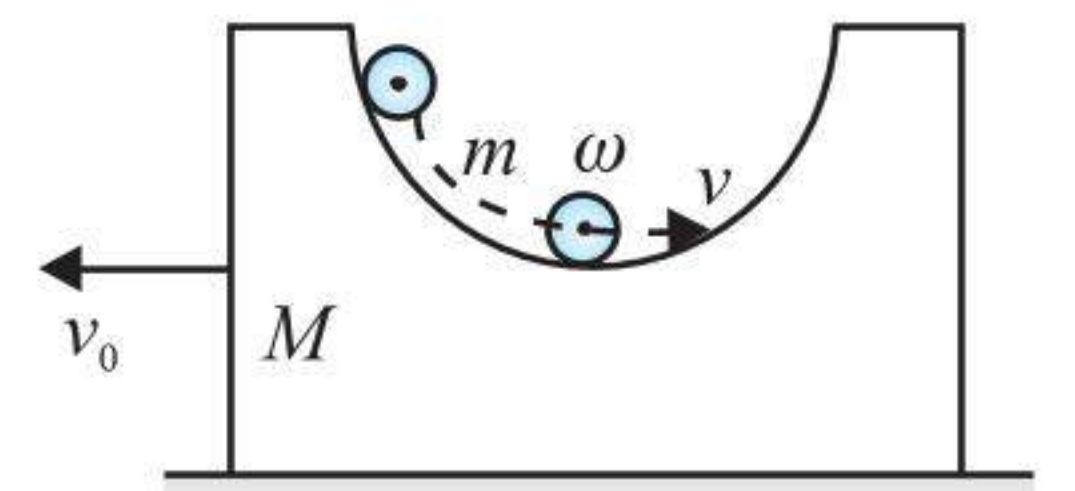
As the cylinder reaches the bottommost point of the track, its centre of mass covers a horizontal distance of 0.50 m ($R - r$) w.r.t. block towards the right. Let the block move by x in backward direction, then

$$Mx = m(0.5 - x)$$

$$x = \frac{2 \times 0.5}{2 + 3} = 0.2 \text{ m}$$

23. (4) v and v_0 are the velocities w.r.t. ground: $\omega = \frac{v + v_0}{r}$

24. (4) $mv = Mv_0$



$$mg(R - r) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_0^2$$

$$\Rightarrow v_0 = \frac{4}{\sqrt{11}} \text{ m/s}$$

$$v = \frac{6}{\sqrt{11}} \text{ m/s}$$

For Problems 25–27

25. (4) Acceleration of the bottom of the cylinder = acceleration of the plank = 2 m/s^2 towards the right

Acceleration of the top of the cylinder = acceleration of B = 6 m/s^2 towards the left

Let linear acceleration of the cylinder be a towards the left and angular acceleration of it α in an anticlockwise sense. Writing constraint, we get

$$a + R\alpha = 6 \text{ and } R\alpha - a = 2$$

$$\Rightarrow \alpha = 1 \text{ rad/s}^2, a = 2 \text{ m/s}^2.$$

26. (2) For block B; $m_B g - T = m_B a$... (i)

For cylinder; $T = m_{\text{cyl}} a$... (ii)

Solving Eqs. (i) and (ii), $m_{\text{cyl}} : m_B = 2 : 1$

27. (1) The acceleration of the top part of the cylinder (thread) with respect to CM of the cylinder is

$$a_{AO} = \alpha R = 1 \times 4 = 4 \text{ m/s}^2$$

$$\text{Length of the thread} = 20 + \frac{1}{2}(R\alpha)t^2 = 28 \text{ m.}$$

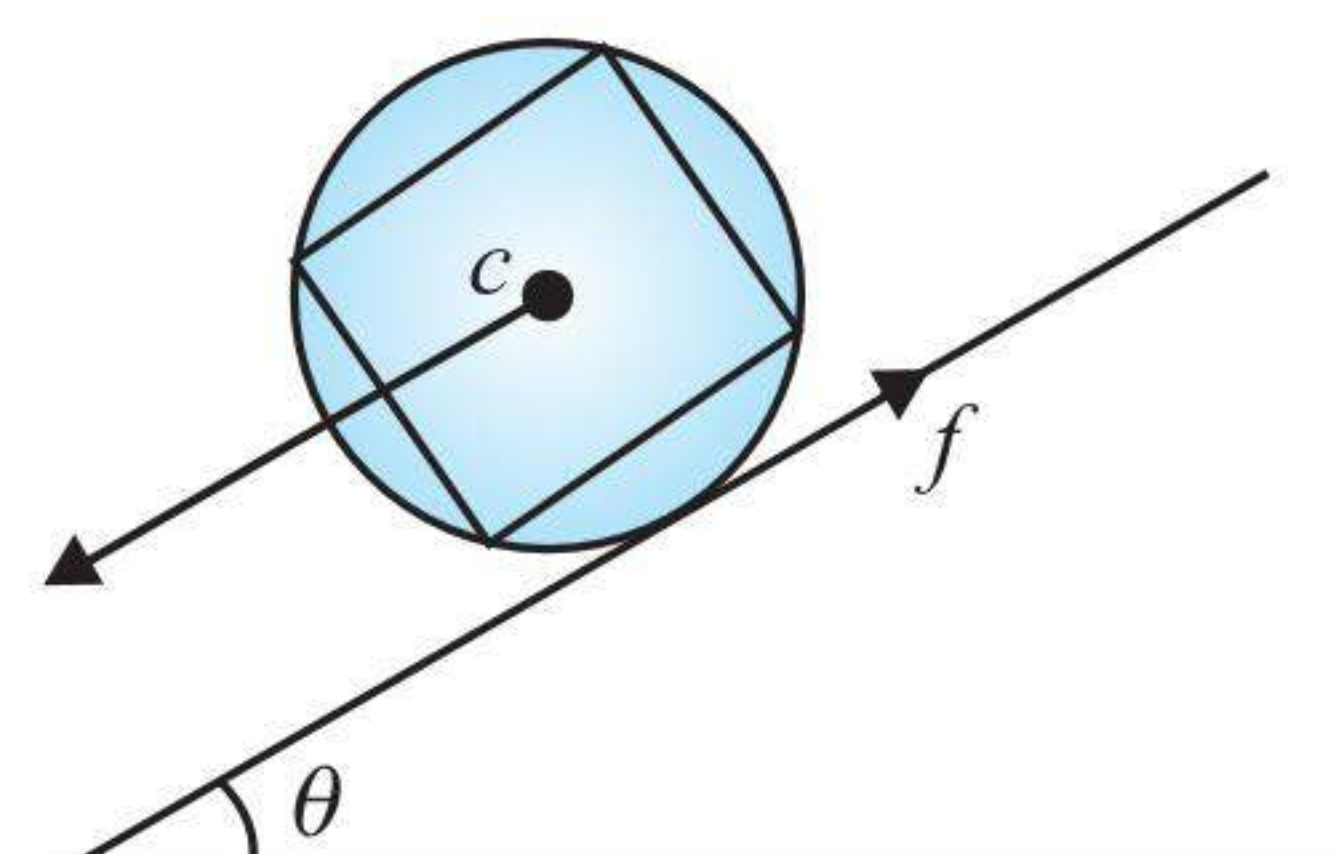
For Problems 28–30

28. (1), 29. (3), 30. (2)

$$I = \left\{ \frac{M(R\sqrt{2})^2}{12} + M \left(\frac{R}{\sqrt{2}} \right)^2 \right\} 4 + mR^2$$

$$= \frac{8}{3}MR^2 + mR^2 = 20 \text{ kg m}^2$$

$$(4M + m)g \sin \theta - f = (4M + m)a \quad \dots(\text{i})$$



$$fR = I \frac{a}{R} \quad \dots(ii)$$

Solving Eqs. (i) and (ii),

$$a = \frac{7g}{24}$$

$$f = 20a = \mu \times 28g \cos 30^\circ$$

$$\mu = \frac{5}{12\sqrt{3}}$$

For Problems 31–33

31. (3) By conservation of angular momentum on the man–table system,

$$L_i = L_f$$

$$0 + 0 = I_m \omega_m + I_t \omega_t$$

$$\Rightarrow \omega_t = -\frac{I_m \omega_m}{I_t} \text{ where } \omega_m = \frac{v}{r} = \frac{1}{2} \text{ rad/s}$$

$$\Rightarrow \omega_t = -\frac{100(2)^2 \times \left(\frac{1}{2}\right)}{4000}$$

$$\Rightarrow \omega_t = -\frac{1}{20} \text{ rad/s}$$

Thus, the table rotates clockwise (opposite to the man) with angular velocity 0.05 rad/s.

32. (4) If the man completes one revolution relative to the table, then

$$\theta_m = 2\pi$$

$$\Rightarrow 2\pi = \theta_m - \theta_t$$

$$2\pi = \omega_m t - \omega_t t \text{ (where } t \text{ is the time taken)}$$

$$t = \frac{2\pi}{\omega_m - \omega_t} = \frac{2\pi}{0.5 - (-0.05)} = \frac{2\pi}{0.55} \text{ s}$$

Angular displacement of the table is

$$\theta_t = \omega_t t = -0.05 \times \left(\frac{2\pi}{0.55}\right) = -\left(\frac{2\pi}{11}\right) \text{ radians}$$

The table rotates through $2\pi/11$ radians clockwise.

33. (1) If the man completes one revolution relative to the earth, then

$$\theta_m = 2\pi$$

$$\text{Time} = \frac{2\pi}{\theta_m} = \frac{2\pi}{0.5}$$

During this time, the angular displacement of the table,

$$\theta_t = \omega_t (\text{time}) = -0.05 \times \left(\frac{2\pi}{0.5}\right) = -\frac{\pi}{5} \text{ radians}$$

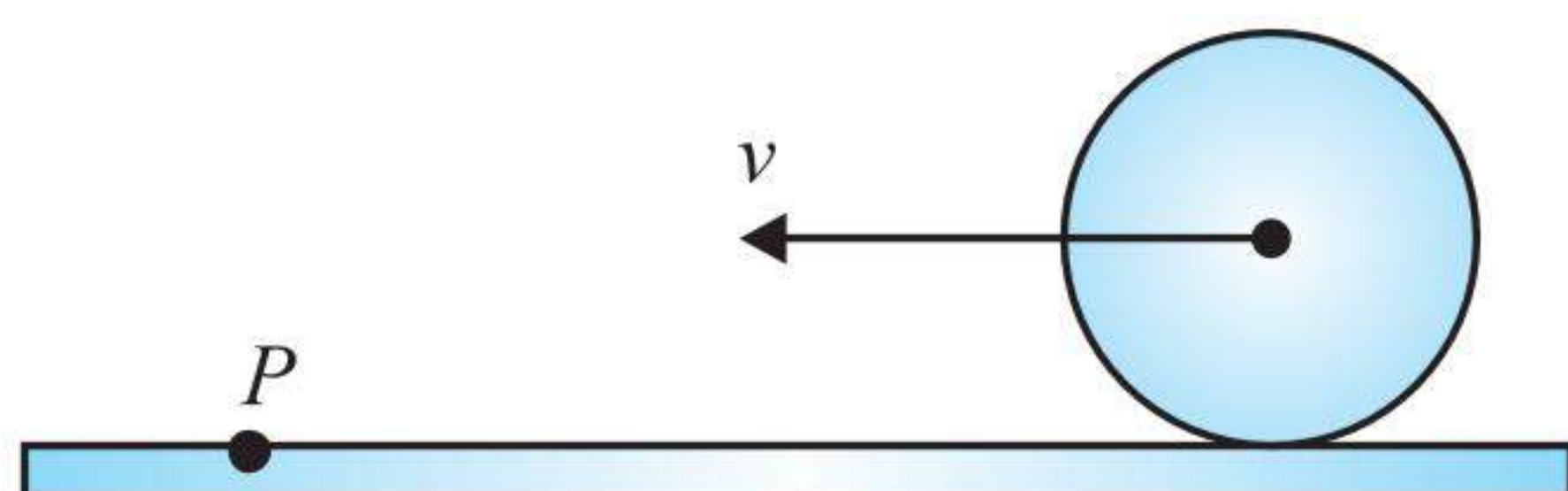
$$\theta_t \equiv 36^\circ \text{ in clockwise direction}$$

For Problems 34–35

34. (2), 35. (2)

In reference frame of truck, the angular momentum is conserved about P , i.e.,

$$MvR = Mv_0R + \frac{2}{5}MR^2\omega_0 \quad \dots(i)$$



Rolling constraint

$$v = R\omega \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get $v = \frac{5}{7}v_0$

Note that v_0 is the speed in truck frame; in ground frame, the velocity is $v_0 - (5v_0/7) = 2v_0/7$ towards the right.

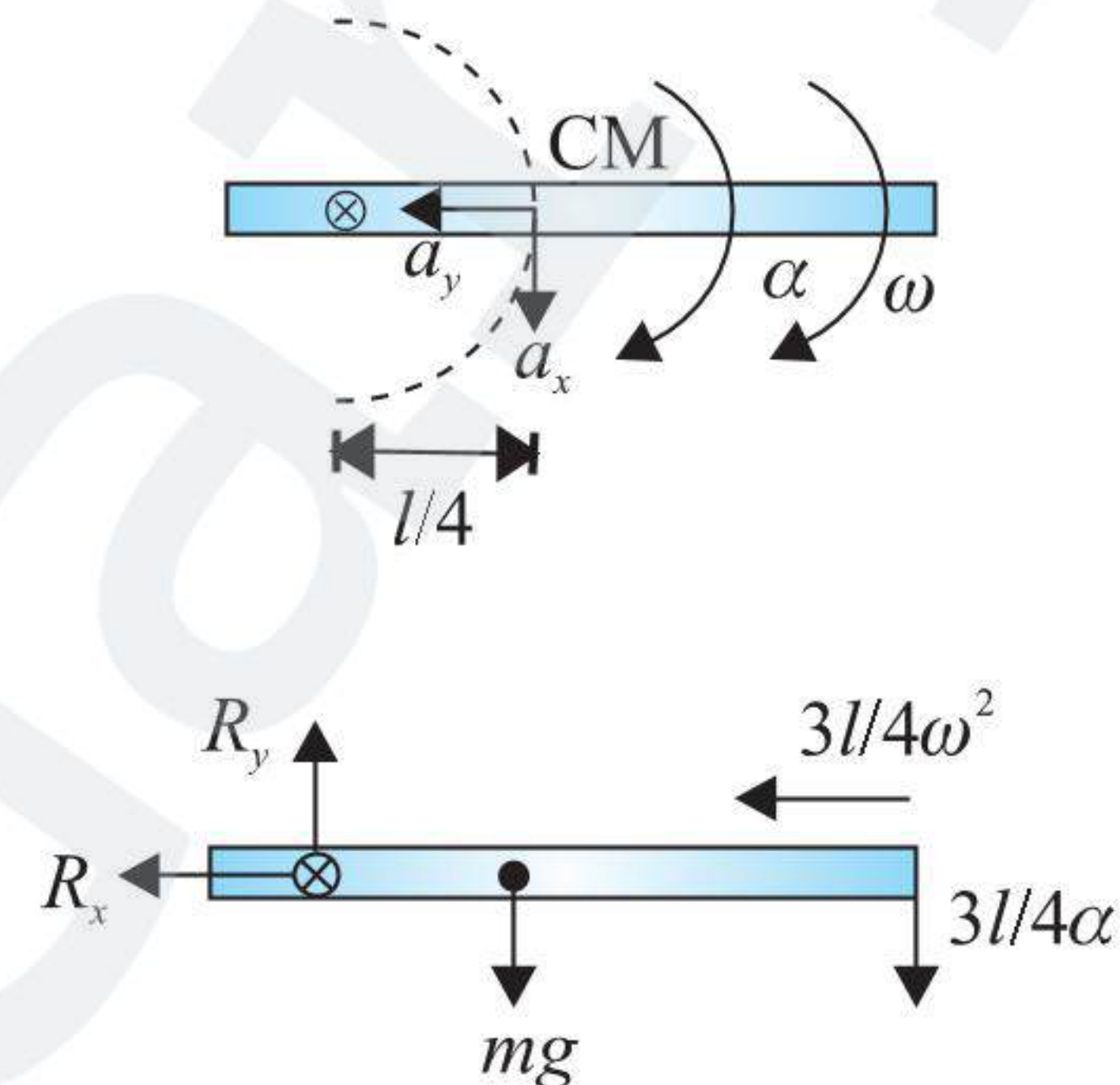
The answer is independent of M or R .

For Problems 36–38

36. (1), 37. (4), 38. (3)

From Conservation of energy, we get

$$\frac{mgl}{4} = \frac{1}{2}m \left[\frac{l^2}{12} + \frac{l^2}{16} \right] \omega^2; \omega^2 = \frac{24g}{7l}$$



From Newton's law,

$$\frac{mgl}{4} = \left(\frac{ml^2}{12} + \frac{ml^2}{16} \right) \alpha \quad \dots(i)$$

$$mg - R_y = m \left(\frac{l}{4} \alpha \right) \quad \dots(ii)$$

$$R_x = \frac{ml}{4} \omega^2 \quad \dots(iii)$$

$$\alpha = \frac{12g}{7l}; \vec{a}_{CM} = -\frac{\omega^2 l}{4} \hat{i} - \frac{l}{4} \alpha \hat{j} = -\frac{6g}{7} \hat{i} - \frac{3g}{7} \hat{j}$$

$$\vec{a}_B = -\omega^2 \left(\frac{3l}{4} \right) \hat{i} - \left(\frac{3l}{4} \right) \alpha \hat{j} = \frac{-18g}{7} \hat{i} - \frac{9g}{7} \hat{j}$$

$$\vec{R}_x = -\frac{6mg}{7} \hat{i}; \vec{R}_y = +\frac{4mg}{7} \hat{j}$$

For Problems 39–42

39. (1), 40. (2), 41. (1), 42. (3)

Conservation of linear momentum,

$$mv_0 = M \frac{v_0}{8} - m \frac{v_0}{4}; 8m = M - 2m$$

$$\Rightarrow 10m = M$$

$$\Rightarrow \frac{M}{m} = 10$$

Conservation of angular momentum about point of collision.

$$0 = \frac{M(8l)^2}{12} \omega - M \frac{v_0}{8} l$$

$$\Rightarrow \omega = \frac{3v_0}{128l}$$

Using coefficient of restitution equation,

$$(v_2 - v_1) = e(u_1 - u_2)$$

$$\left[\left(\frac{v_0}{8} + \omega l \right) - \left(-\frac{v_0}{4} \right) \right] = e[v_0 - 0]$$

$$\Rightarrow e = \frac{51}{128}$$

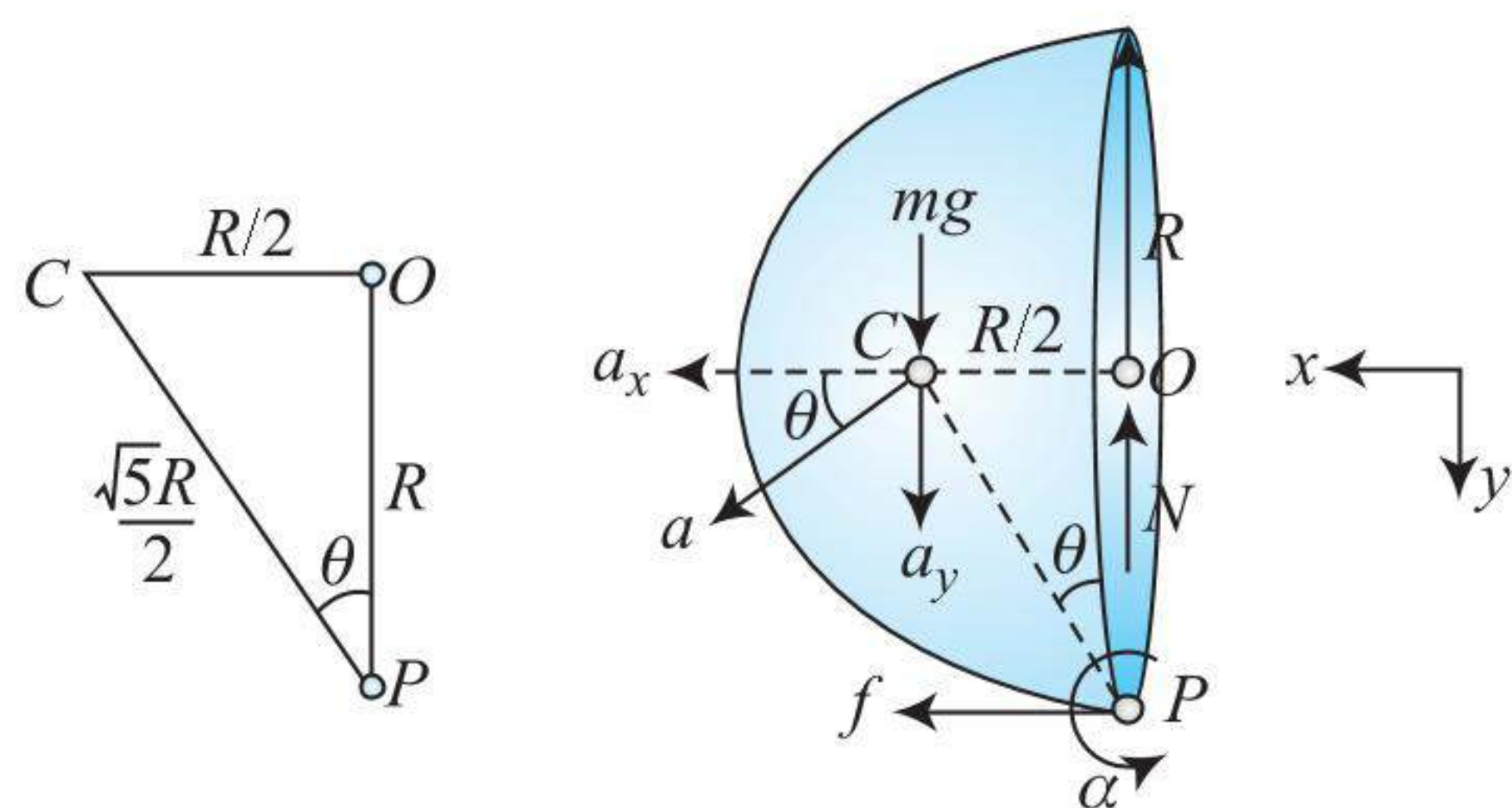
$$\text{Velocity at end } A, v_A = \frac{v_0}{8} + \omega(4l) = \frac{28}{128} v_0$$

$$\Rightarrow v_A = \frac{7}{32} v_0$$

$$\text{Velocity at end } B, v_B = \frac{v_0}{8} - \omega(4l) = \frac{4}{128} v_0 = \frac{1}{32} v_0$$

For Problems 43–44

43. (2) $mg - N = ma_y$
 $mg - N = ma \sin \theta$
 Torque about 'P'



$$mg \frac{R}{2} = \frac{5}{3} m R^2 \alpha \Rightarrow \alpha = \frac{3}{10} \frac{g}{R}$$

$$a = \left(\frac{\sqrt{5}R}{2} \right) \alpha = \frac{3\sqrt{5}}{20} g$$

$$\text{So, } N = mg - ma \sin \theta = mg - m \left(\frac{3\sqrt{5}}{20} g \right) \cdot \frac{1}{\sqrt{5}}$$

$$\Rightarrow N = \frac{17}{20} mg$$

In x-direction, $f = ma_x$

For no sliding, $f \leq \mu N$

$$ma \cos \theta \leq \mu N \quad \text{or} \quad \mu \geq \frac{6}{17}$$

$$\text{Hence } \mu_{\min} = \frac{6}{17}$$

44. (1) For slipping,

$$f = ma_x = \mu N \quad \dots(i)$$

$$mg - N = ma_y \quad \dots(ii)$$

$$\text{from (i) and (ii), } N \left(\frac{1}{2} - \mu \right) = \left(\frac{5}{12} m \alpha R \right) \quad \dots(iii)$$

$$\alpha r \sin \theta = a_y$$

$$\Rightarrow a_y = \frac{\alpha R}{2}$$

$$\frac{mg - N}{m} = \frac{N}{2} \left(\frac{1}{2} - \mu \right) \times \frac{12}{5m}$$

$$N = \frac{5mg}{8 - 6\mu}$$

$$\text{From } \mu = \frac{3}{17} \Rightarrow N = \frac{85}{118} mg$$

Matrix Match Type

1. i. $\rightarrow$ b., d.; ii. $\rightarrow$ a.; iii. $\rightarrow$ a., b., c., d.; iv. $\rightarrow$ b., d.

$$Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$$

But as in rolling, $v = r\omega$

$$mgh = \frac{1}{2} mv^2 \left[1 + \frac{I}{mr^2} \right] \quad \left[\text{Putting } \beta = 1 + \frac{I}{mr^2} \right]$$

$$= \frac{1}{2} \beta mv^2$$

$$\Rightarrow v = \sqrt{\frac{2gh}{\beta}}$$

Solving for acceleration, we have $a = \frac{g \sin \theta}{\beta}$

$$\text{and time taken, } t = \frac{1}{\sin \theta} \sqrt{\beta \left(\frac{2h}{g} \right)}$$

2. i. $\rightarrow$ c.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a., b., d.; iv. $\rightarrow$ a., b., d.

$$a = \frac{g \sin \theta}{1 + \frac{I}{MR^2}}$$

For a solid cylinder: $I = \frac{1}{2} MR^2$

$$\text{So } a_1 = \frac{2}{3} g \sin \theta$$

For a thin shell: $I = MR^2$

$$a_2 = \frac{g \sin \theta}{2}$$

So, $a_1 > a_2$, hence the solid cylinder reaches earlier. Now $v = \sqrt{2al}$, acceleration for the solid cylinder is more, so v on reaching the bottom is more for the solid cylinder.

F_{tran} is maximum and KE_{rot} is minimum for the solid cylinder.

KE_{tran} is minimum and KE_{rot} maximum for the thin cylinder.

3. i. $\rightarrow$ d.; ii. $\rightarrow$ b., c.; iii. $\rightarrow$ d.; iv. $\rightarrow$ a., c.

We know that angular momentum = linear momentum $\times$ perpendicular distance. So about O , angular momentum will remain constant. But about E it will keep on changing. It is maximum about E when the particle is at A , because then the perpendicular distance is maximum.

We know that angular velocity (ω) = v/r . So about O , angular velocity will remain constant. But about E it will keep on changing. It is minimum about E when the particle is at A , because then the perpendicular distance (r) is maximum.

4. i. $\rightarrow$ a.; ii. $\rightarrow$ b., d.; iii. $\rightarrow$ a.; iv. $\rightarrow$ b., d.

a. Speed of point P changes with time.

b. Acceleration of point P is equal to $\omega^2 x$ (ω = angular speed of disc and $x = OP$). The acceleration is directed from P towards O .

c. The angle between acceleration of P (constant in magnitude) and velocity of P changes with time. Therefore, tangential acceleration of P changes with time.

d. The acceleration of the lowest point is directed towards the centre of the disc and remains constant with time.

5. i. $\rightarrow$ a., b., c.; ii. $\rightarrow$ a., b., c.; iii. $\rightarrow$ a., b.; iv. $\rightarrow$ a., b., c.

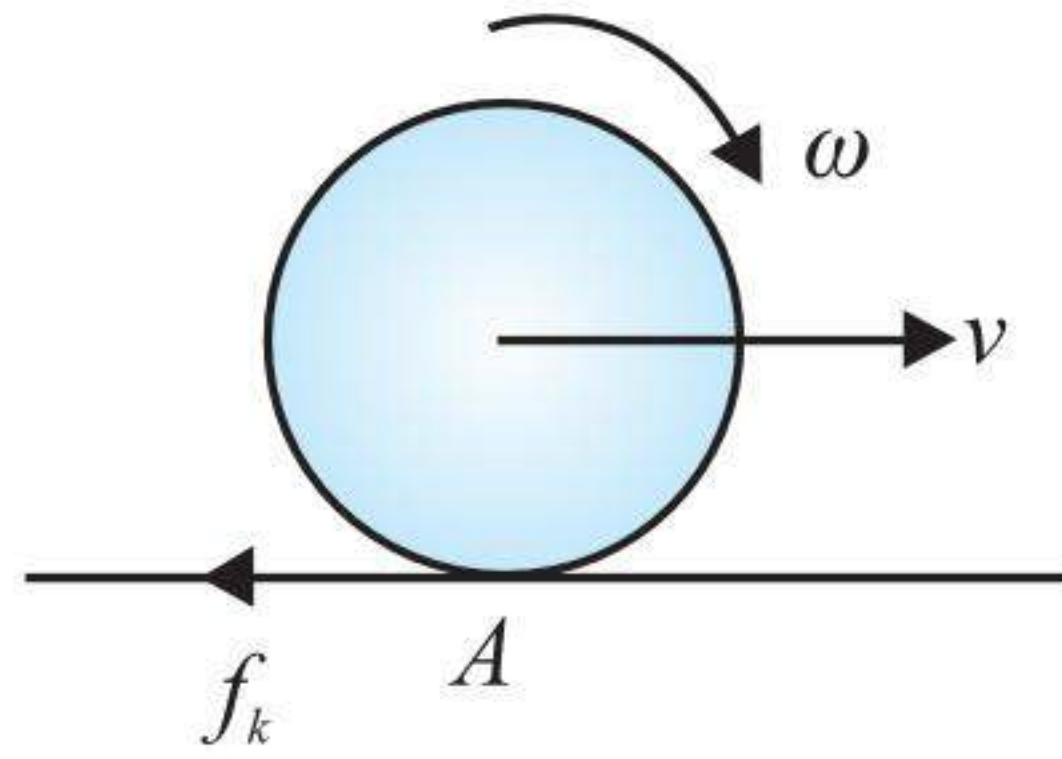
Since all forces on the disc pass through the point of contact with the horizontal surface, the angular momentum of the disc about the point on the ground in contact with disc is conserved. Also the angular momentum of the disc in all cases is conserved about any point on the line passing through the point of contact and parallel to the velocity of the centre of mass.

The KE of the disc is decreased in all cases due to work done by friction. From the calculation of velocity of the lowest point on disc, the direction of friction in case (i), (ii) and (iv) is towards the left and in case (iii) it is towards the right.

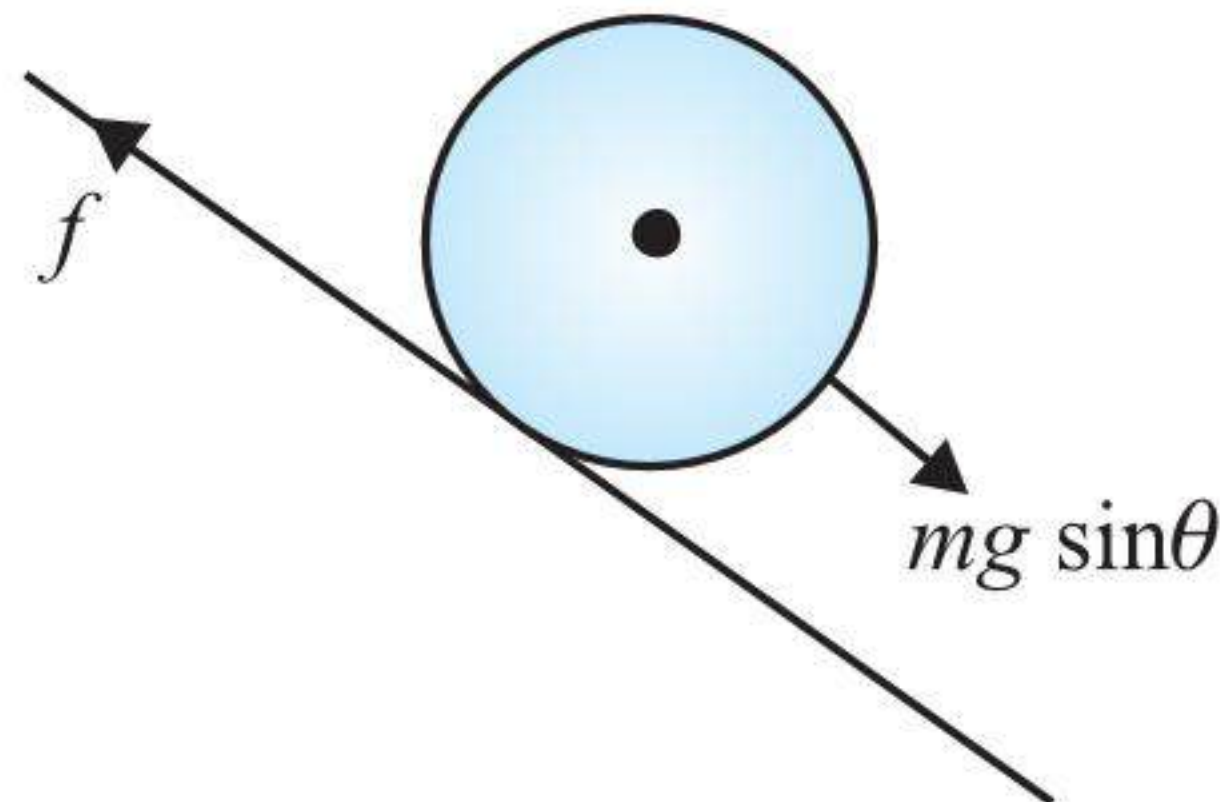
The direction of frictional force cannot change in any given case

6. i. $\rightarrow$ c.; ii. $\rightarrow$ a., b., c., d., e.; iii. $\rightarrow$ a., b., c., d., e.; iv. $\rightarrow$ e.

- a. When the sphere rolls on the rough surface, its angular momentum about contact point A remains constant because torque of friction f_k about this point is zero.

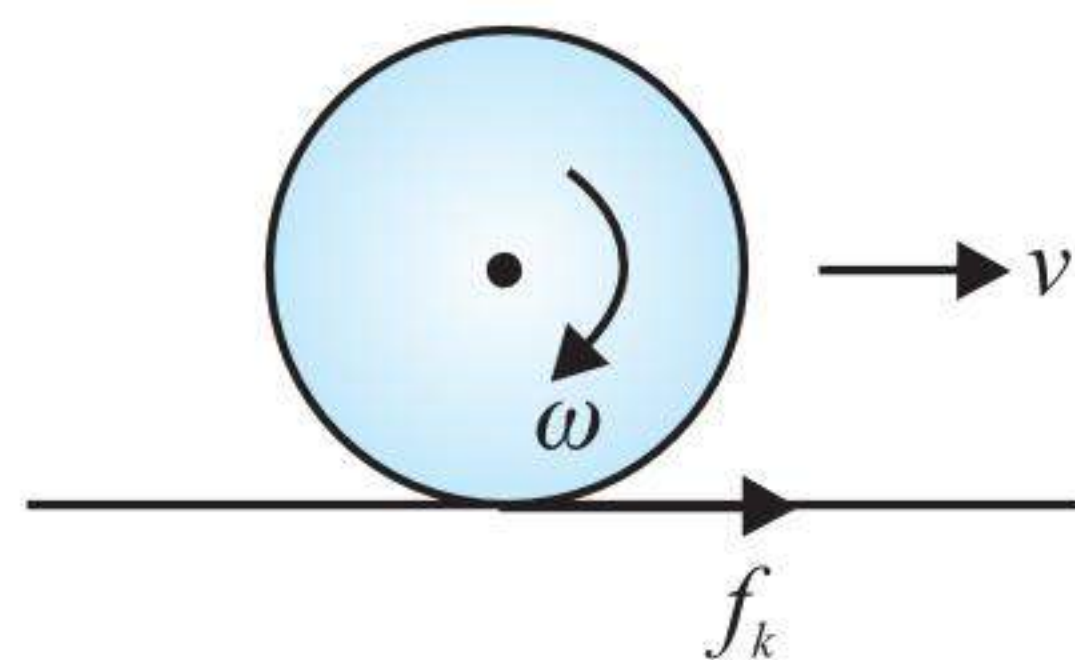


- b. Here no friction will act on the sphere. So here everything will remain the same.
 c. Here also no friction will act, so everything will remain the same.
 d. Here KE will increase. Potential energy will decrease but the total mechanical energy will remain constant. Angular momentum will not be conserved, because $mg \sin \theta$ will provide the torque about contact point.

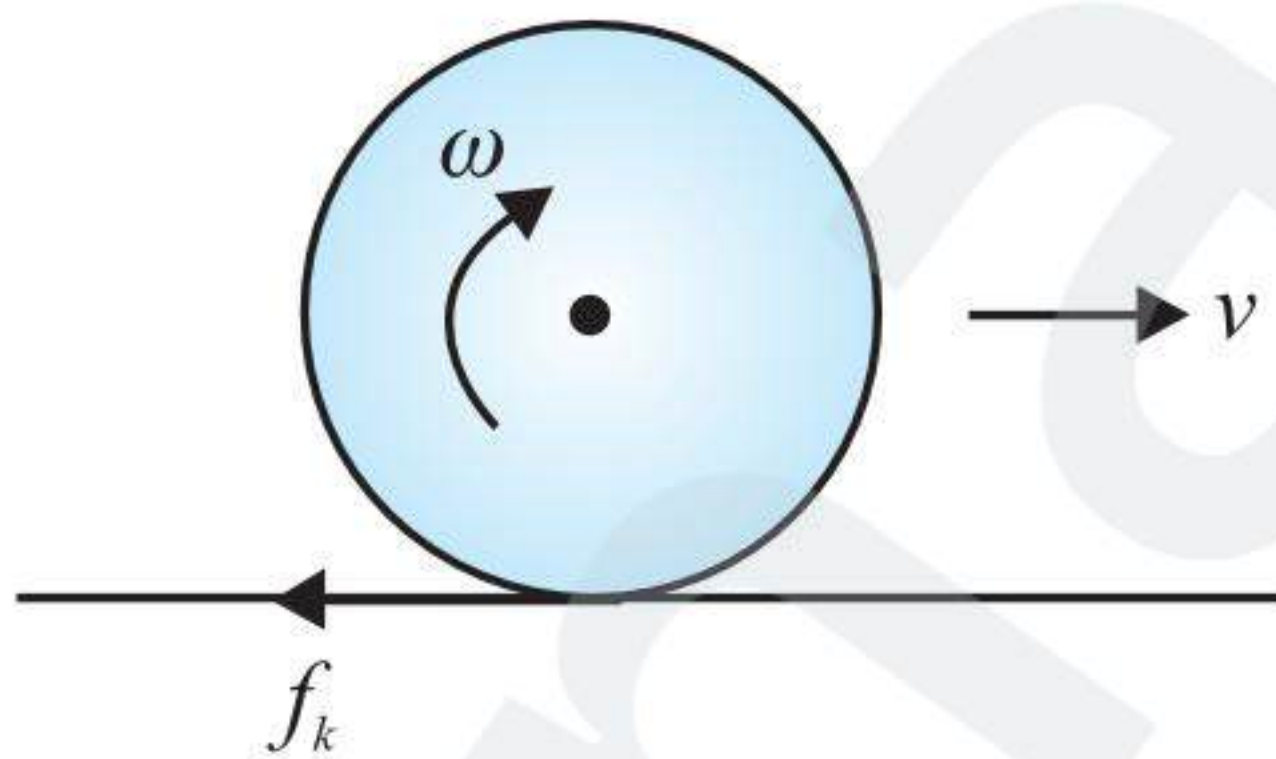


7. i. $\rightarrow$ a., b., c.; ii. $\rightarrow$ a.; iii. $\rightarrow$ d.; iv. $\rightarrow$ d.

- a. v increases, ω decreases, hence translational work by friction is positive and rotational work is negative. But overall work will be negative because the friction is kinetic.

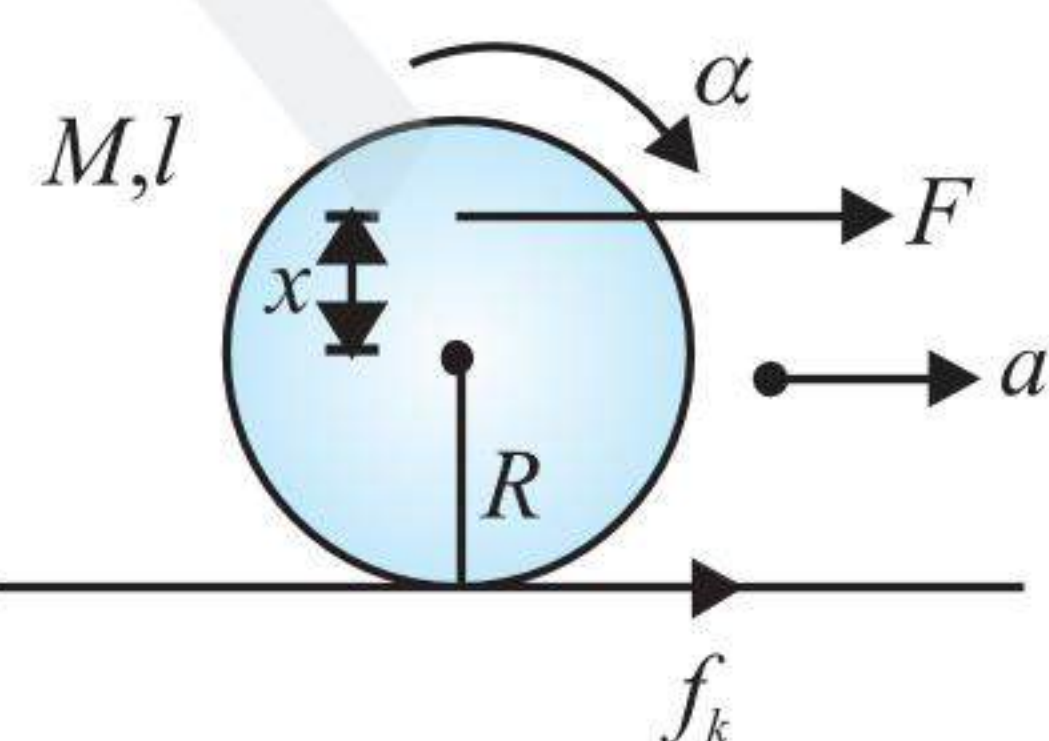


- b. v decreases, ω increases, overall work by friction will be negative.



- c. This is pure rolling with constant velocity. No friction will act.
 d. Here F will pass through the contact point. It will not move. Hence, work by friction is zero.

8. i. $\rightarrow$ a., b., d.; ii. $\rightarrow$ a., b., c.; iii. $\rightarrow$ a., b., d.; iv. $\rightarrow$ a., b., c.



$$F + f = ma$$

$$Fx - fR = I\alpha, \alpha = \frac{a}{R}$$

$$\text{Solve to get } f = \frac{F(MxR - I)}{I + MR^2}$$

$$f = 0, \text{ if } x = \frac{I}{MR}$$

$$f > 0, \text{ if } x > \frac{I}{MR}$$

So friction will act in the forward direction.

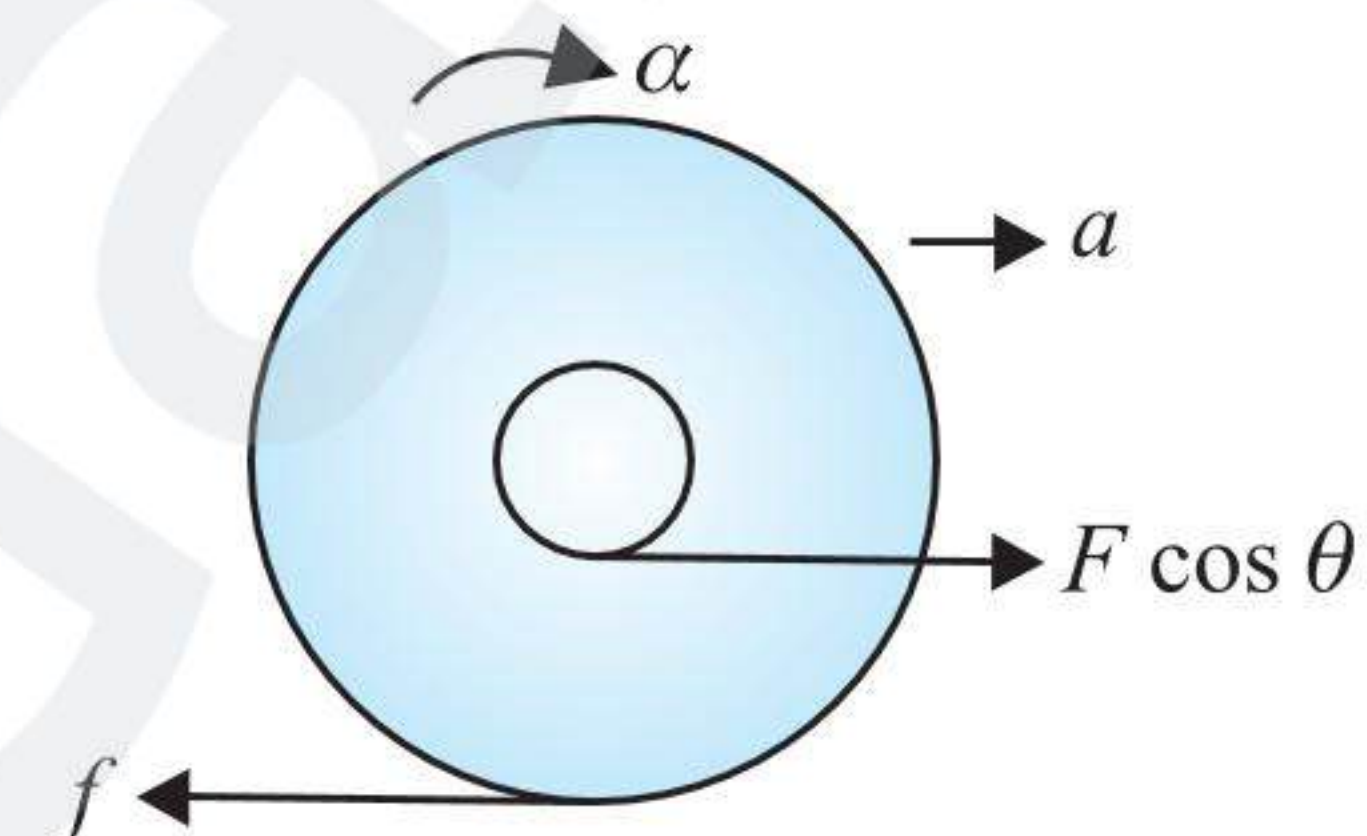
$F < 0$ if $x < I/MR$, so friction will act in the backward direction.

$$\text{Also solved to get } a = \frac{FR(x + R)}{I + MR^2}$$

A is positive for all cases, hence body will accelerate in the forward direction in all cases.

It means α will be in the clockwise direction. Hence, bodies will rotate in the clockwise direction.

9. i. $\rightarrow$ b., c.; ii. $\rightarrow$ a., c.; iii. $\rightarrow$ c.



$$F \cos \theta - f = ma$$

$$fR - Fr = \frac{Ia}{R}$$

$$F \cos \theta - F \frac{r}{R} = a(m + \frac{I}{R^2})$$

If $\cos \theta > r/R$, then a is positive

$$a = \frac{F(\cos \theta - \frac{r}{R})}{(m + \frac{I}{R^2})}$$

If $\cos \theta < r/R$, then a is negative.

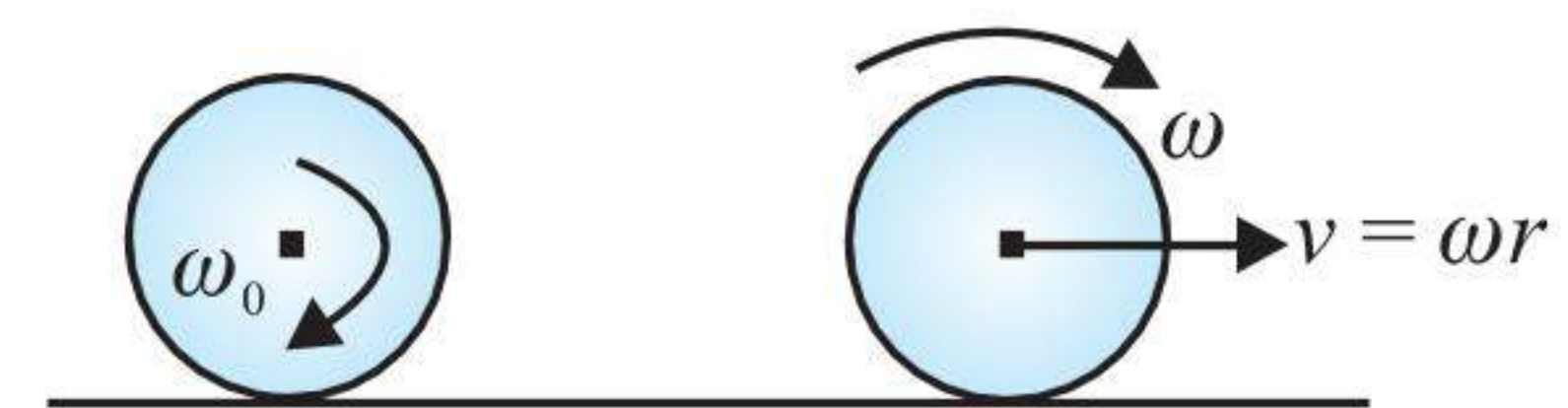
In (i), since a is positive, α should be clockwise hence f is towards the left.

Similarly for others.

10. i. $\rightarrow$ a., b., d.; ii. $\rightarrow$ a., b.; iii. $\rightarrow$ a., b., c.; iv. $\rightarrow$ a., b.

In each case, the friction force will act in the forward direction. This friction will increase translational velocity and decrease angular velocity.

Apply conservation of angular momentum about the lowest point,



$$I\omega_0 = I\omega + mvr$$

$$I\omega_0 = \frac{Iv}{r} + mvr$$

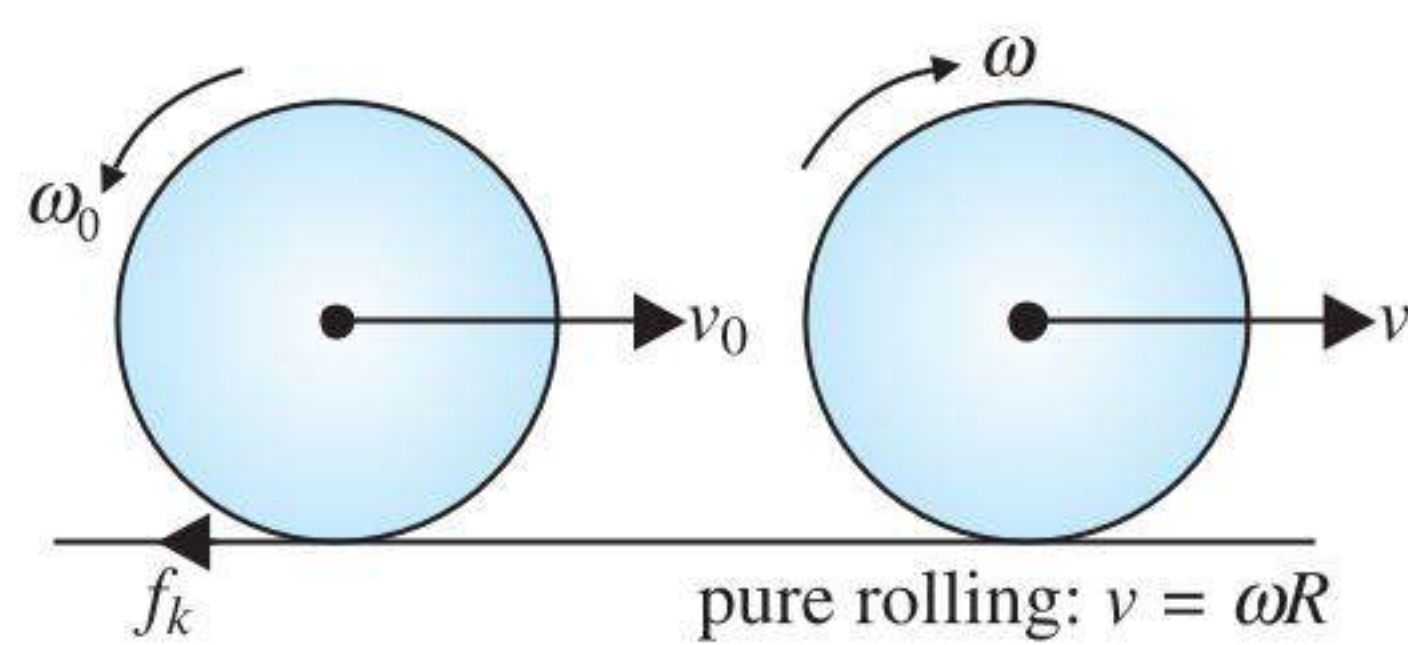
$$v = \frac{I\omega_0 R}{I + mr^2} = \frac{\omega_0 r}{1 + \frac{mr^2}{I}}$$

I is minimum for the solid sphere. So v is minimum for the solid sphere.

Similarly, v will be maximum for the ring. From $v = u + at$, time taken by the ring will be maximum, because acceleration is the same for all.

11. i. → a.; ii. → a.; iii. → a.; iv. → c.

Here angular momentum will be conserved about bottommost point, because no external torque acts about this point.



$$\begin{aligned}\vec{L}_i &= \vec{L}_f \Rightarrow Mv_0R - I_{\text{cm}}\omega_0 = MvR + I_{\text{cm}}\omega \\ \Rightarrow Mv_0R - \frac{MR^2}{2}\omega_0 &= MvR + \frac{MR^2}{2}\frac{v}{R} \\ \Rightarrow v &= \frac{2}{3}\left[v_0 - \frac{\omega_0 R}{2}\right]\end{aligned}$$

If $v > 0$, finally cylinder moves in forward direction.

If $v < 0$, finally cylinder moves in backward direction.

If $v = 0$, finally cylinder stops.

12. i. → a., c., d.; ii. → b., c., d.; iii. → b.; iv. → b., c., d.

i. Here $x > \frac{I}{mR}$, so friction acts towards right.

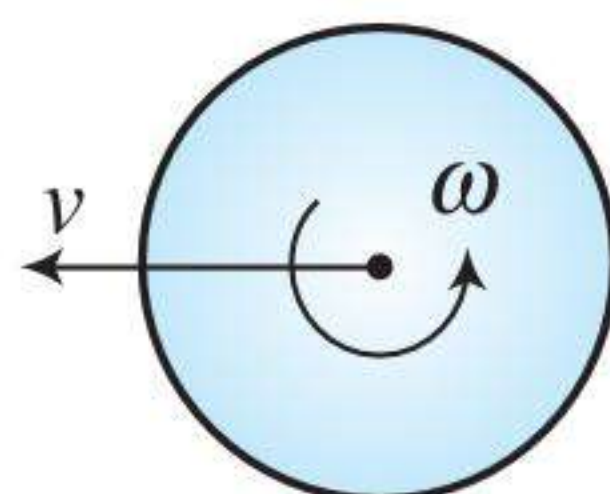
ii. Here $x = 0$, so friction acts towards left.

iii. Here α will be anticlockwise, bottommost point tends to move forward, so friction acts in backward direction.

iv. Similar to 2nd part.

13. i. → a., b.; ii. → b., c.; iii. → a., b.; iv. → a., b.

For (i)



$\frac{2v}{R}$ Angular momentum about O = $mVR + I\omega$. Hence final angular momentum should also be in $\curvearrowright$.

For (ii): Initial angular momentum = $mvR - I\omega$ for ring $L_i = mvR - mR^2 \times \frac{2v}{R} = mvR$; final angular momentum is to be $\curvearrowright$; for solid sphere $L_i = mvR - \frac{2}{5}mR^2 \frac{2v}{R} = \frac{mvR}{5}$

For (iii): $L_i = mVR + I\omega$

For ring $L_i = mVR + MR^2 \times \frac{v}{2R} = \frac{3mVR}{2}$

For sphere $L_i = mVR + \frac{2mR^2}{5} \frac{V}{2R} = \frac{6mVR}{5}$

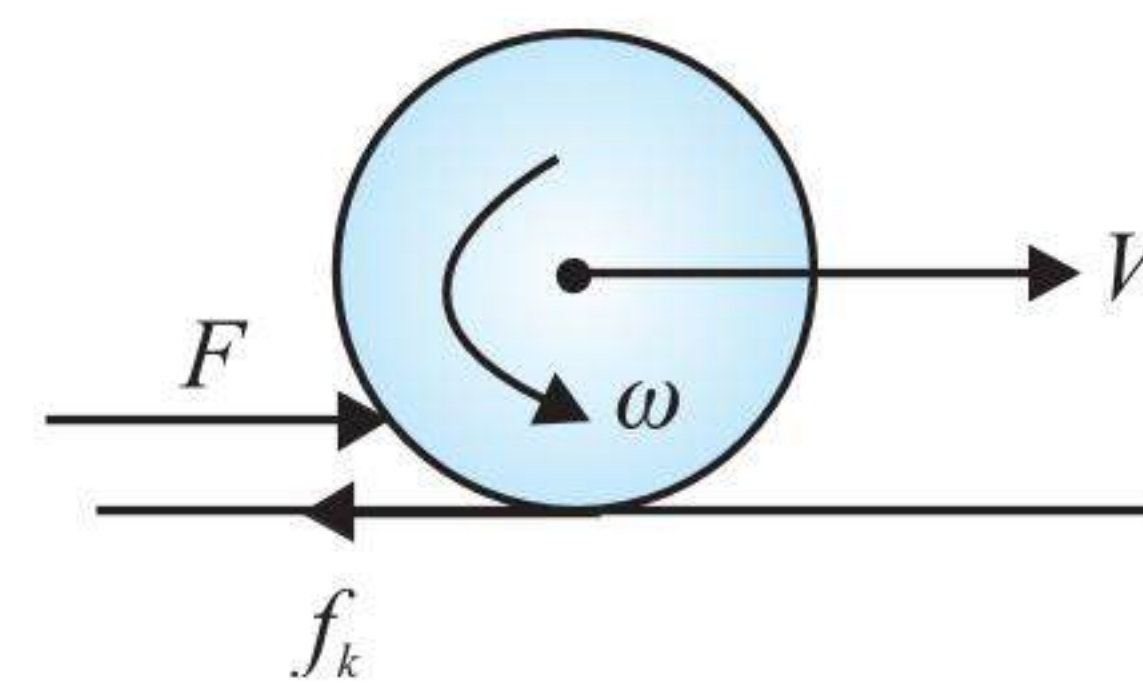
For (iv): $L_i = mVR - I\omega$

For ring $L_i = mVR - mR^2 \frac{V}{2R} = \frac{mVR}{2}$

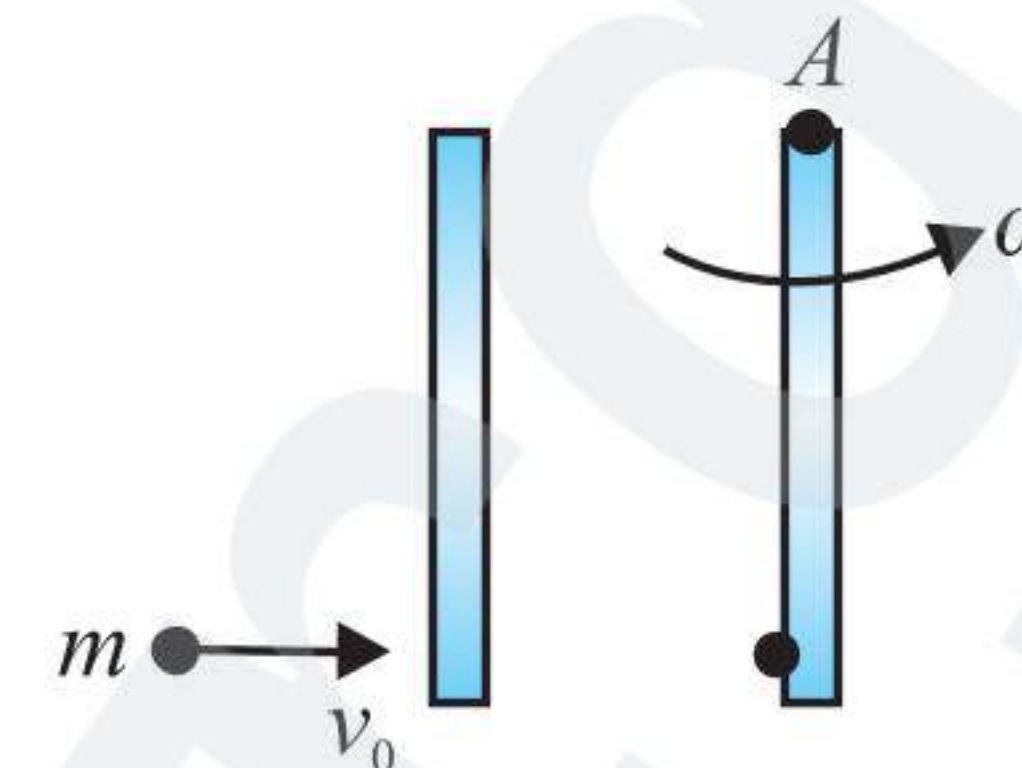
For sphere $L_i = mVR - \frac{2}{5}mR^2 \frac{V}{2R} = \frac{4mVR}{5}$

14. i. → d.; ii. → b., d.; iii. → a., b., c.; iv. → b., c., d.

a. Immediately the sphere will gain velocity, so momentum increases.



- b. Here angular momentum about the hinged point will remain constant.



Applying conservation of angular momentum about A,

$$mv_0L = \left(mL^2 + \frac{ML^2}{3}\right)\omega$$

$$\text{Final momentum, } \omega = \frac{mv_0}{\left(m + \frac{M}{3}\right)^2}$$

$$m\omega L + M\omega \frac{L}{2} = \left(m + \frac{M}{2}\right) \frac{mv_0}{\left(m + \frac{M}{3}\right)}$$

This is greater than mv_0 .

- c. Here no external force or external torque acts. So all remain conserved, as the collision is elastic.

- d. Here also momentum increases. As the collision is elastic, so energy is also conserved. Angular momentum is conserved about the hinged point.

15. i. → a., b., c., d.; ii. → a., d.; iii. → a., b., c., d.; iv. → b., c., d.

- a. Since impulse is given to the body, so its momentum will increase and translation will occur. The impulse J does not pass through the centre of mass. Hence, the dumb-bell will rotate and its angular momentum will increase.

- b. Here J passes through centre of mass. So rotation does not occur and angular momentum remains zero.

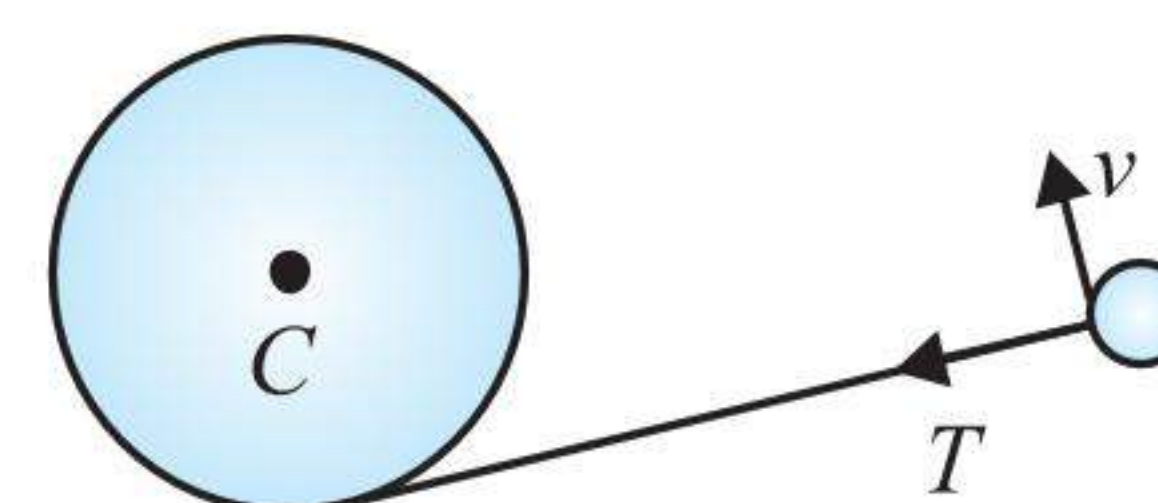
- c. This situation is similar to part (a).

- d. Here body is hinged. So translation does not occur, rotation will be there so angular momentum will increase.

Linear momentum increases as centre of mass gains some velocity.

16. i. → b., c.; ii. → a., b., c.; iii. → a., d.; iv. → a., d.

- a. Angular momentum will not be conserved as T will provide torque about C. Kinetic energy will remain conserved as tension is perpendicular to velocity and it does not do any work. Obviously, the total mechanical energy also remains conserved.

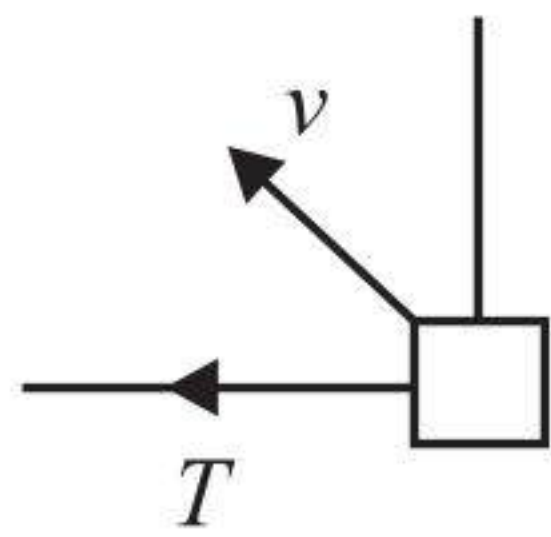


- b. Angular momentum remains conserved as no external torque acts. Mechanical energy and kinetic energy of (rod + sleeve) system will remain the same because no work is done by external forces and there is no dissipating force in the system.
- c. Angular momentum will remain conserved, because there is no external force (friction is zero because of ice).

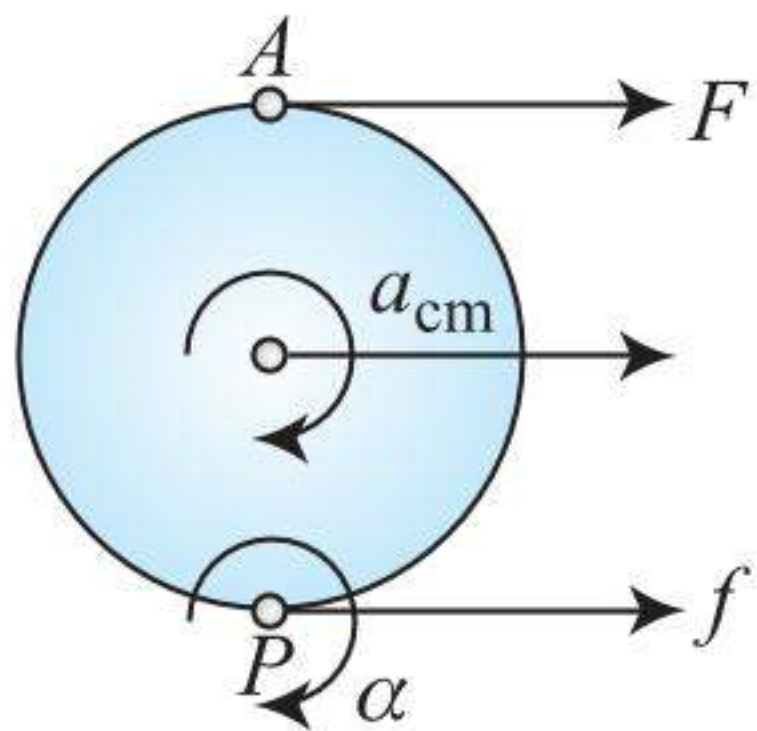
Kinetic energy and mechanical energy will decrease because of work done by internal forces (due to muscles) between the ice skaters.

- d. Angular momentum will remain conserved, because torque of T will be zero.

T will do positive work on the block due to which kinetic energy and mechanical energy of the block will increase.



17. (2)



Point P is ICR, applying torque equation about P

$$F \times 2R = I_P \alpha \Rightarrow 2FR = (mk^2 + mR^2) \alpha$$

$$\Rightarrow \alpha = \frac{2F}{mR \left(1 + \frac{k^2}{R^2} \right)}$$

Hence acceleration of CM; $a_{cm} = \alpha R = \frac{2F}{m \left(1 + \frac{k^2}{R^2} \right)}$

From FBD: $F + f = ma_{cm}$

which gives $f = F \left[\frac{1 - \frac{k^2}{R^2}}{1 + \frac{k^2}{R^2}} \right]$

For solid sphere: $\frac{k^2}{R^2} = \frac{2}{5}$; $a_{cm} = \frac{10}{7} \frac{F}{m}$

and $f = F \left[\frac{1 - \frac{2}{5}}{1 + \frac{2}{5}} \right] = \frac{3F}{7}$

Similarly, For ring: $\frac{k^2}{R^2} = 1$

$a_{cm} = \frac{F}{m}$ and $f = 0$

For hollow sphere: $\frac{k^2}{R^2} = \frac{2}{3}$

$a_{cm} = \frac{6}{5} \frac{F}{m}$ and $f = \frac{F}{5}$

For disc: $\frac{k^2}{R^2} = \frac{1}{2}$

$a_{cm} = \frac{4}{3} \frac{F}{m}$ and $f = \frac{F}{3}$

Numerical Value Type

1. (2) Applying conservation of energy, we get

$$mgs(\sin \alpha - \sin \beta) = \frac{1}{2} I \omega^2 + 2 \left(\frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 \right)$$

where $\omega = v/r$ and $I = mr^2/2$

Putting values and solving, we get $v = 2$ m/s.

2. (0) Let f be the friction force acting on the disc in the backward direction. Then $2F - f = ma$ and $(F + f)r = I\omega$

$$\Rightarrow (F + f)r = \frac{1}{2} mr^2 \frac{a}{r}$$

$\therefore$ Solving the above equations, we get $f = 0$.

3. (6) Since $10 < mg$, the cylinder rolls on ground without lifting up

$$KE = \text{Workdone} = F \times \text{Displacement} = 10 \times \frac{3}{5} = 6 \text{ J}$$

Note: Point of application of F will move upward by same distance as the cylinder moves on the ground.

4. (5) $a_c = \frac{g \sin \theta}{1 + \frac{I}{mR^2}} = \frac{g \sin \theta}{1 + \frac{1}{2}} = \frac{2}{3} g \sin \theta$

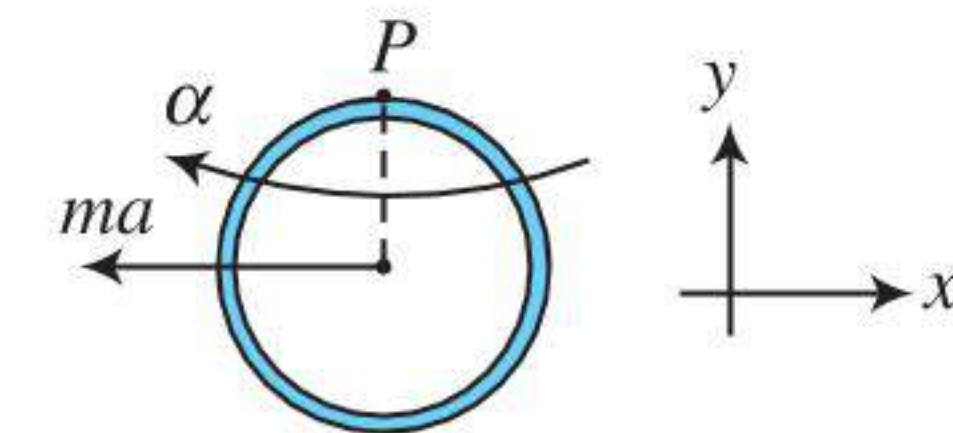
$$a_P = \frac{g \sin \theta}{1 + \frac{I}{mR^2}} = \frac{g \sin \theta}{2}$$

$$a_{rel} = a_C - a_P = \frac{g \sin \theta}{6} = \frac{10 \times \frac{1}{2}}{6} = \frac{5}{6}$$

$$s_{rel} = \frac{1}{2} \times \frac{5}{6} \times (2\sqrt{3})^2 = 5 \text{ m}$$

5. (2) The pseudo force acting at the CM of the ring is $\vec{F}_{ps} = -ma\hat{i}$ which produces a torque about P given as

$$\vec{\tau}_{ps} = -(ma)(R)\hat{k}$$



This produces an angular acceleration

$$\vec{\alpha} = \frac{\vec{\tau}_{ps}}{I_P} = \frac{-maR\hat{k}}{2mR^2} \quad (\because I_P = mR^2 + mR^2 = 2mR^2)$$

$$= -\frac{a}{2R}\hat{k} = \frac{4}{2 \times 1} = 2 \text{ rad/s}^2$$

6. (4) FBD: The forces acting on the body are $F \rightarrow$, $mg \downarrow$ and $N \uparrow$ and let us assume that f is backward ($\leftarrow$).

Force equation: $F - f = ma$... (i)

$N - mg = 0$... (ii)

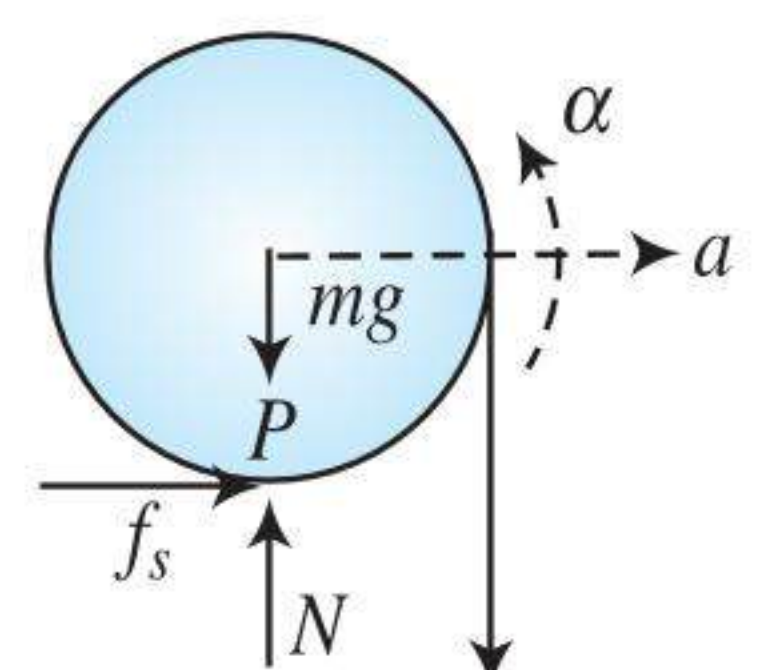
Torque equation: $f \cdot R = \frac{2}{5} mR^2 \alpha$... (iii)

Condition for rolling $a = R\alpha$... (iv)

Putting a from Eq. (i), α from Eq. (iii) in Eq. (iv), we have

$$\frac{F - f}{m} = R \cdot \frac{5f}{2mR} \Rightarrow f = \frac{2}{7} F = 4 \text{ N}$$

7. (1) FBD: Let the static friction f_s act towards right. It pushes the body with an acceleration a , say and rotate the body with an angular acceleration α , say. The other forces acting on the body are $N \uparrow$ and $mg \downarrow$.



Force equation: $f_s = ma$... (i)

$N = mg$... (ii)

Torque equation:

$$\tau_c = f_s R = mk^2 \alpha$$
 ... (iii)

Rolling: $a_p = a_0$ where $a_p = a + R\alpha$

$$a + R\alpha = a_0 \quad \dots(\text{iv})$$

Substituting a from Eq. (i), α from Eq. (iii) in Eq. (iv), we have

$$\frac{f_s}{m} + R\left(\frac{f_s R}{mk^2}\right) = a_0$$

$$f_s = \frac{ma_0}{1 + \frac{R^2}{k^2}} = \frac{4 \times 1}{(1+3)} = 1 \text{ N}$$

$$8. (7) \quad \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2 = Mgh$$

$$\frac{1}{2} Mv^2 + \frac{1}{2} \left(\frac{2}{5} MR^2 \right) \left(\frac{v^2}{R^2} \right) = Mgh$$

$$\frac{7}{10} Mv^2 = Mgh$$

$$h = \frac{7v^2}{10g} = \frac{7(10)^2}{10(10)} = 7 \text{ m}$$

$$9. (6) \quad \text{Total kinetic energy of loop} = \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2$$

$$= \frac{1}{2} (MR^2\omega^2) + \frac{1}{2} Mv^2 = Mv^2 = 8 \text{ J (given)}$$

$$\text{Total kinetic energy of disc} = \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2$$

$$= \frac{1}{2} \left(\frac{1}{2} MR^2 \right) \left(\frac{v^2}{R^2} \right) + \frac{1}{2} Mv^2 = \frac{3}{4} Mv^2 = \frac{3}{4} (8) = 6 \text{ J}$$

$$10. (5) \quad K = K_R + K_T = \frac{1}{2} \left[mR^2 + m \frac{(2R)^2}{12} \right] \left[\frac{V}{R} \right]^2 + \frac{1}{2} (2m) V^2$$

$$= \frac{5}{3} mv^2 = \frac{5}{3} (3)(1)^2 = 5 \text{ J}$$

$$11. (20) \quad F + f = ma \quad \dots(\text{i})$$

Torque equation about point of contact

$$F2R = I_p \alpha$$

$$F2R = \left(\frac{2}{5} mR^2 + mR^2 \right) \alpha$$

$$F = \frac{7}{10} mR\alpha \quad \dots(\text{ii})$$

$$\text{For no sliding case } a = R\alpha \quad \dots(\text{iii})$$

$$\text{From (ii) and (iii), } a = \frac{10F}{7m} \quad \dots(\text{iv})$$

$$\text{From (i) and (iv), } F + f = m \cdot \frac{10F}{7m} \Rightarrow f = \frac{3}{7} F$$

i.e., friction will act in backward direction. For no sliding

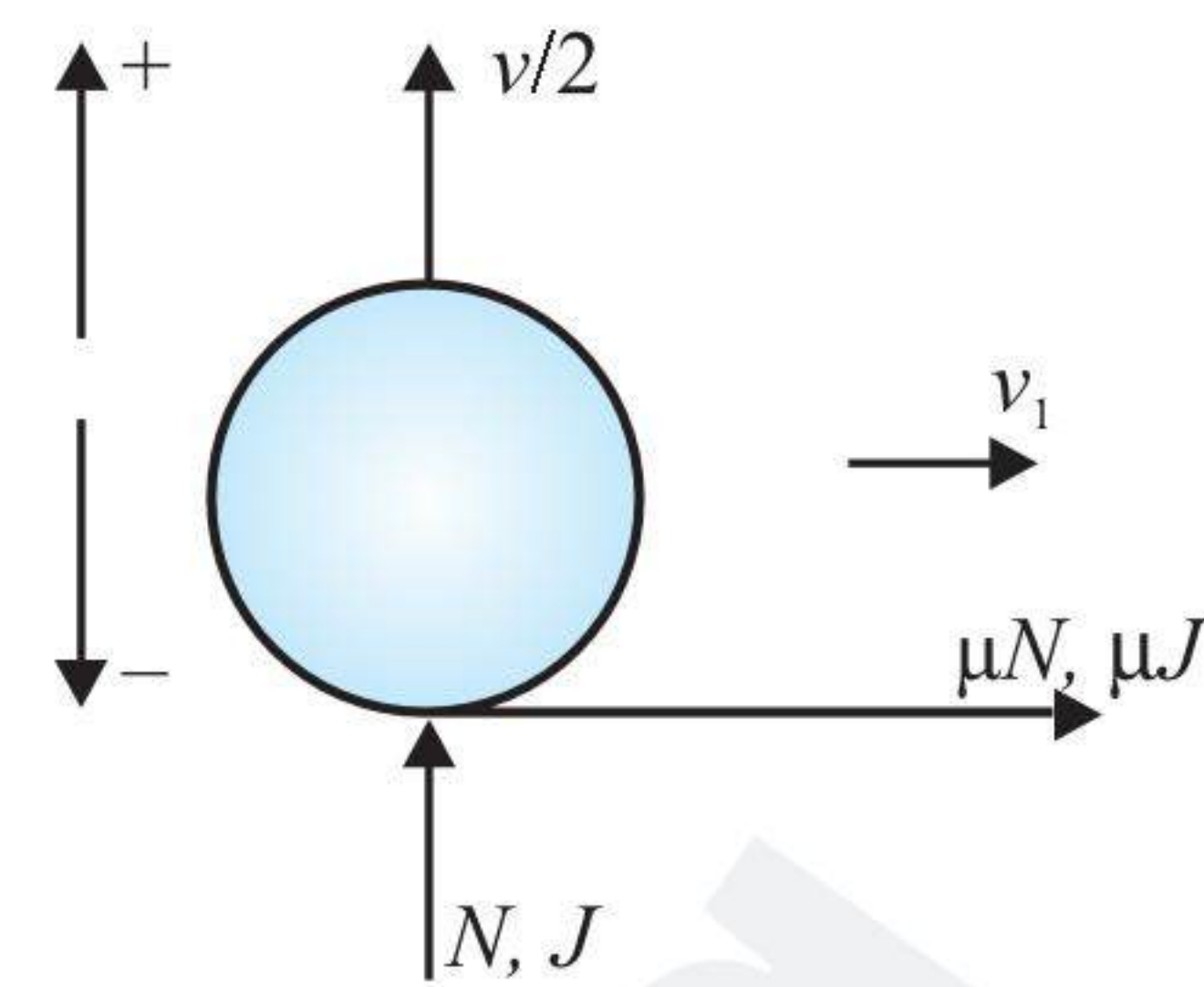
$$f \leq \mu N$$

$$\frac{3}{7} F \leq \mu mg \Rightarrow F \leq \frac{7}{3} \mu mg$$

$$F \leq \frac{7}{3} \times \frac{2}{7} \times 3 \times 10 \Rightarrow F \leq 20 \text{ N}$$

$$12. (2) \quad J = \int N dt = m \frac{v}{2} - (-mv) = \frac{3}{2} mv \quad \dots(\text{i})$$

$$\mu JR = \int \mu (N dt) R = \left(\frac{2}{5} mR^2 \omega - 0 \right) = \frac{2}{5} mR^2 \omega \quad \dots(\text{ii})$$



From Eqs. (i) and (ii), we get

$$\frac{3}{2} mv R\mu = \frac{2}{5} mR^2 \omega \quad \dots(\text{iii})$$

Let V and V_1 be the speeds of the plank and the sphere, respectively, in the horizontal direction.

$$\mu J = \int \mu N dt = MV = mV_1 \quad \dots(\text{iv})$$

From Eqs. (i) and (iv), $\mu \left(\frac{3}{2} \right) mv = MV$

$$V = \frac{3}{2} \frac{\mu mv}{M} = \frac{3}{2} \frac{4}{15} \frac{mR\omega}{M} = \frac{2}{5} \frac{m}{M} R\omega$$

$$\text{and } V_1 = \frac{2}{5} R\omega = 2 \text{ m/s}$$

$$13. (4.00) \quad fr\Delta t = I_{cm}\omega \quad \dots(\text{i})$$

$$-f\Delta t = m(0 - v_0) \quad \dots(\text{ii})$$

$$\omega r = v_1 \quad \dots(\text{iii})$$

$$\Rightarrow mv_0 r = \frac{2}{5} mr^2 \omega$$

$$\Rightarrow \omega = \frac{5v_0}{2r} \Rightarrow v_1 = \frac{5v_0}{2}$$

$$\Rightarrow v_0 = \frac{2v_1}{5} \Rightarrow v_0 = \frac{2 \times 10}{5} = 4 \text{ m/s}$$

14. (5.00) From COAM about CM,

$$\text{We get, } v_0 \frac{l_0}{2} = \frac{(l_0 + x)}{2} v,$$

where x = maximum elongation = 0.4 m

natural length = 2.4 m

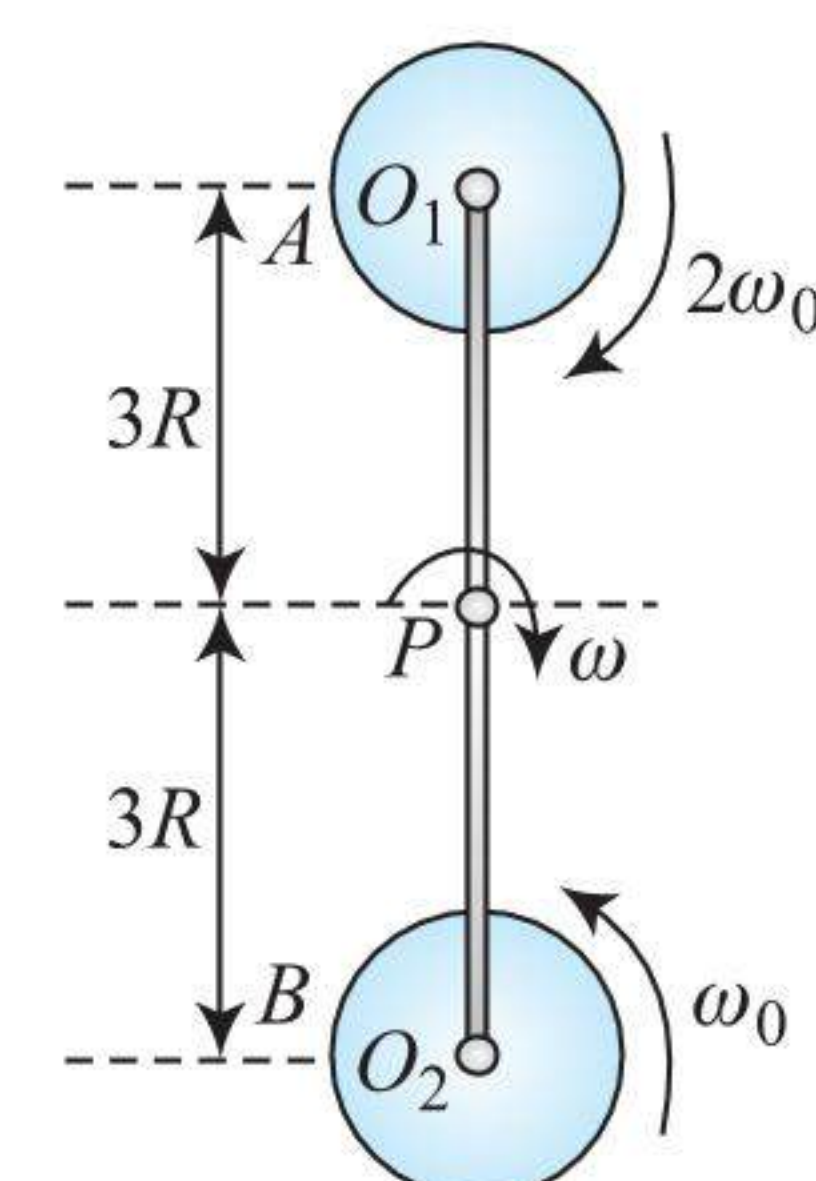
v = speed of the sphere in CM frame at maximum elongation

From conservation of energy,

$$2 \times \frac{1}{2} mv_0^2 = \frac{1}{2} (2m) \left(\frac{\sqrt{2}v_0}{2} \right)^2 + 2 \frac{1}{2} mv^2 + \frac{1}{2} kx^2$$

Solving, we get, $v_0 = 5 \text{ m/s}$

15. (2.00) The angular velocities of both the disc will remain unchanged



From conservation of mechanical energy

$$\Delta K + \Delta U = 0 \Rightarrow \Delta K = -\Delta U$$

$$\left(\frac{1}{2}I_P\omega^2 - 0\right) = -(Mg3R - 2Mg3R)$$

$$I_P = \frac{M(6R)^2}{12} + M(3R)^2 + 2M(3R)^2 = 30MR^2$$

$$\Rightarrow 3MgR = 15MR^2\omega^2 \text{ or } \omega = \sqrt{\frac{g}{5R}}$$

$$\text{On substituting the given values we get } \omega = \sqrt{\frac{10}{5 \times 0.5}} = 2 \text{ rad/s}$$

16. (20.00) The centre of mass of the ring rolls in a circle of radius $(3R/4)$.

$\Rightarrow$ Velocity of centre of mass at the topmost point,

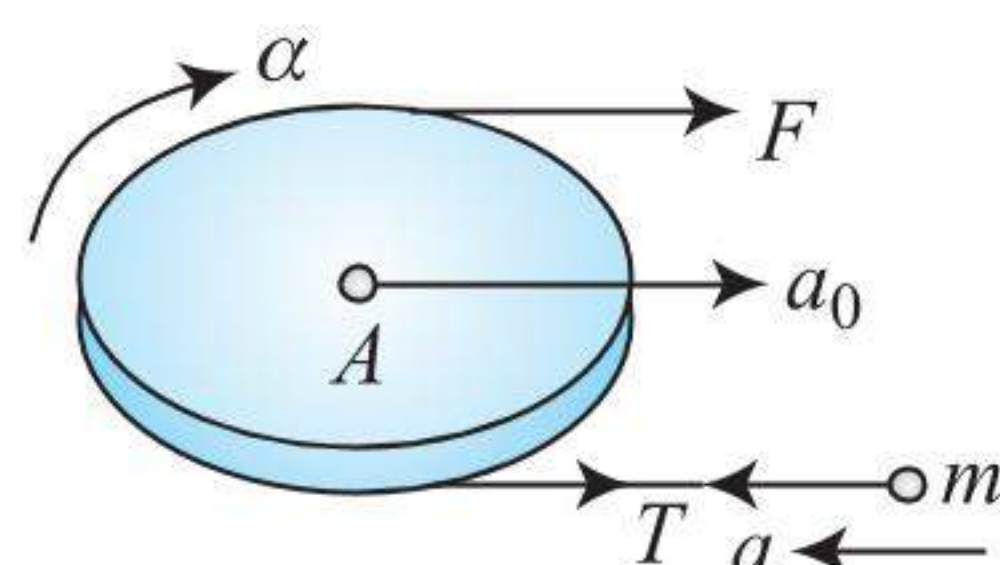
$$v = \sqrt{g \times \frac{3R}{4}}$$

$$\Rightarrow \omega = \sqrt{\frac{3gR}{4}} / \frac{R}{4} = \sqrt{\frac{12g}{R}} = 2\sqrt{\frac{3g}{R}}$$

On substituting the given values we get

$$\omega = 2\sqrt{\frac{3 \times 10}{0.3}} = 20 \text{ rad/s}$$

17. (5.00) For no slipping condition of thread acceleration of point A on the circumference of the disc = acceleration of the small block.



$$\Rightarrow R\alpha = a_0 + a \quad \dots(i)$$

Other equations are

$$T = ma \quad \dots(ii)$$

$$F + T = ma_0 \quad \dots(iii)$$

$$\text{and } (F - T)R = \frac{1}{2}mR^2 \cdot \alpha$$

$$\Rightarrow F - T = \frac{1}{2}mR\alpha \quad \dots(iv)$$

$$\text{Using (i) in (iv) } F - T = \frac{1}{2}m(a + a_0)$$

$$\text{Using (ii) in this } F = \frac{3}{2}ma + \frac{1}{2}ma_0 \quad \dots(v)$$

$$\text{From (ii) and (iii) } F = ma_0 - ma \quad \dots(vi)$$

$$\text{Solving (v) and (vi) } a = \frac{F}{4m}$$

$$\text{Hence } a = \frac{20}{4 \times 1} = 5 \text{ m/s}^2$$

18. (1.20) If we take 'plate and particle' as system. There is no external torque about the axis passing through O. Hence angular momentum of the system should be conserved.

Let ω = angular speed of square plate when the particle moves out from point B. Let it is in clockwise sense.

Angular momentum conservation about rotation axis passing through O,

$$mv_A \frac{l}{2}(-\hat{k}) = mv_B \frac{l}{2}(\hat{k}) + I_O\omega(-\hat{k})$$

$$-1 \times 2 \times 1 = 1 \times v_B \times 1 - 4\omega$$

$$\Rightarrow \omega = \frac{v_B + 2}{4} \quad \dots(i)$$

Conserving mechanical energy

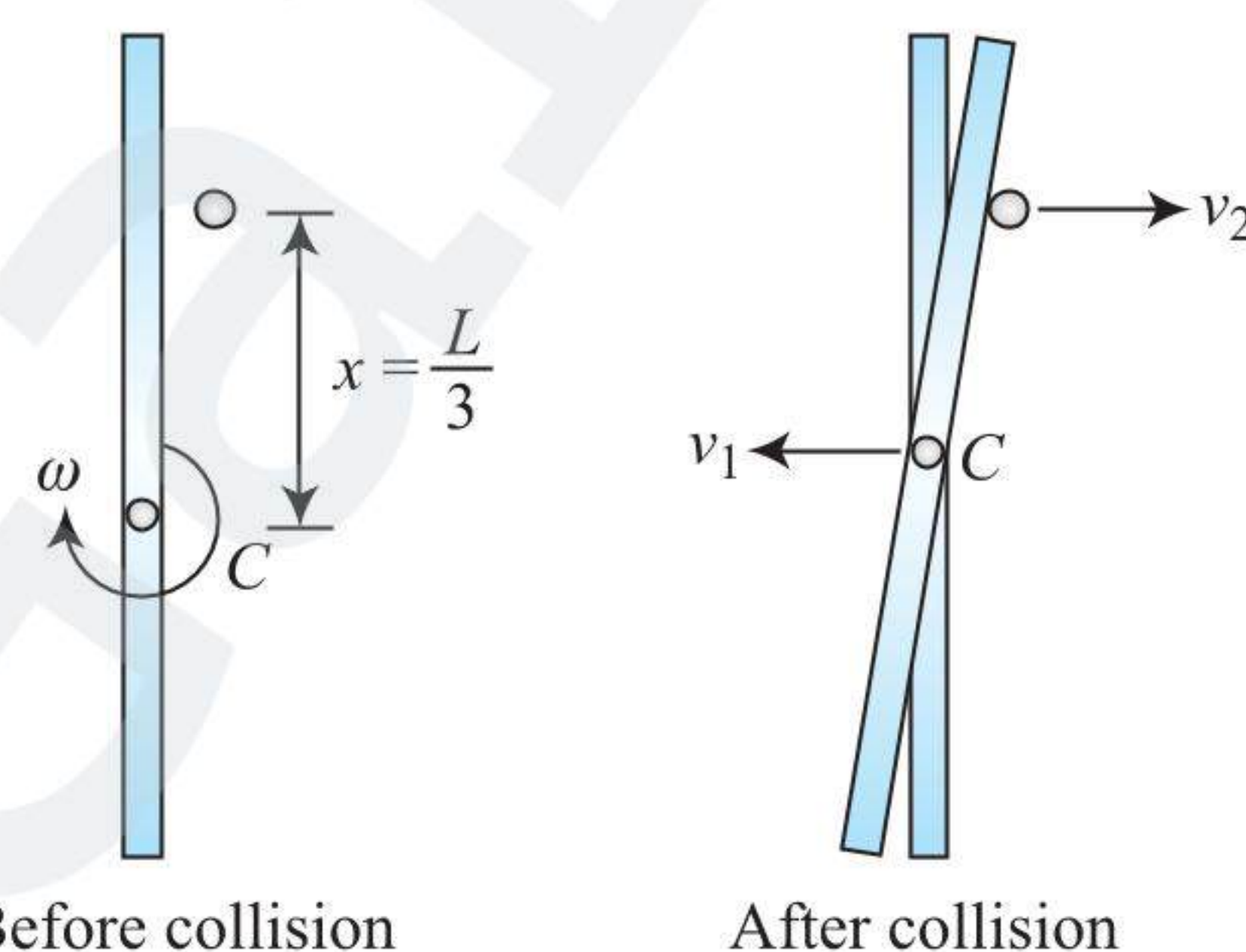
$$\frac{1}{2} \times 1 \times 2^2 = \frac{1}{2} \times 1 \times v_B^2 + \frac{1}{2} \times 4 \times \omega^2$$

$$v_B^2 + 4\omega^2 = 4 \quad \dots(ii)$$

$$\text{From (i) and (ii), } v_B^2 + 4\left(\frac{v_B + 2}{4}\right)^2 = 4$$

$$5v_B^2 + 4v_B - 12 = 0 \Rightarrow v_B = \frac{6}{5} \text{ m/s} = 1.20 \text{ m/s}$$

19. (3.00) Let the velocity of particle after collision be v_1 and that of centre of the stick be v_2 (as shown)



$$\text{Momentum conservation gives: } mv_1 = Mv_2 \quad \dots(i)$$

Angular momentum conservation about C:

$$\frac{ML^2}{12}\omega = mv_1 \frac{L}{3} \Rightarrow v_1 = \frac{ML\omega}{4m} \quad \dots(ii)$$

Collision is elastic, i.e., $e = 1$

$$\frac{v_1 - (-v_2)}{\omega \frac{L}{3} - 0} = 1 \Rightarrow v_1 + v_2 = \frac{\omega L}{3}$$

Substituting for v_1 and v_2 from (i) and (ii)

$$\frac{M\omega L}{4m} + \frac{\omega L}{4} = \frac{\omega L}{3} \quad \dots(iii)$$

$$\Rightarrow \frac{m}{M} = 3$$

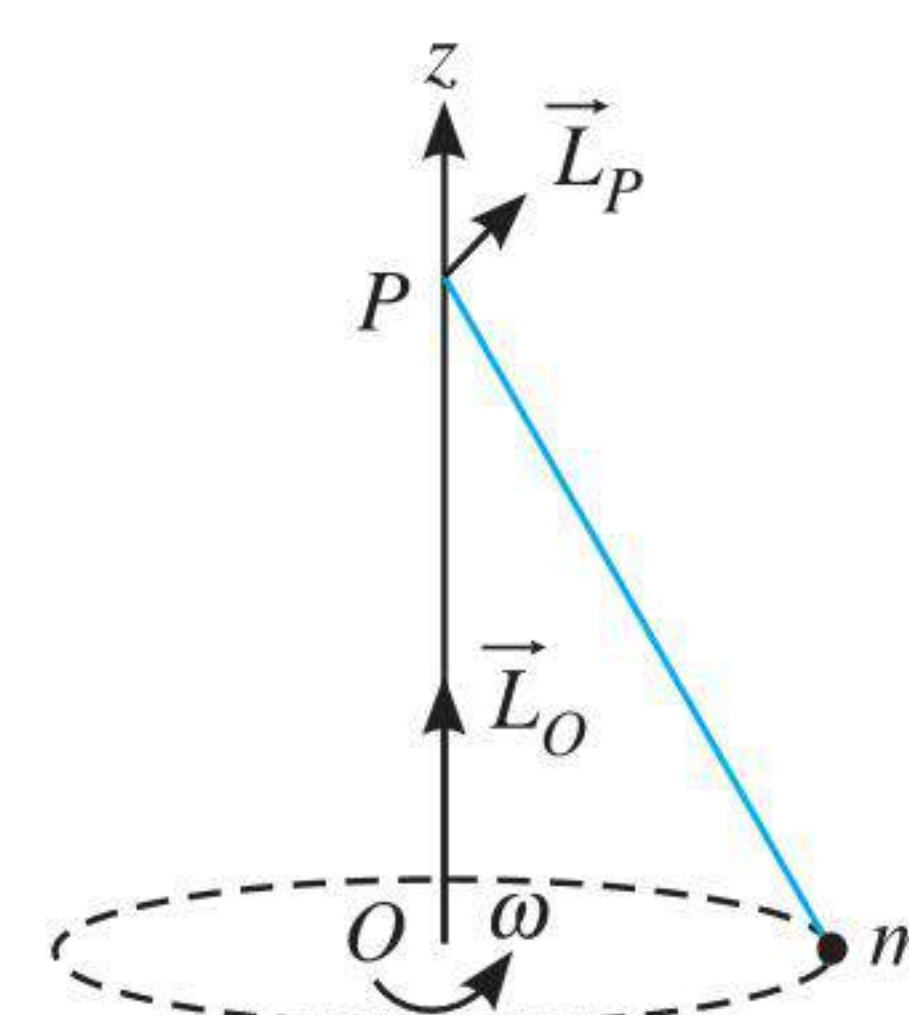
Archives

JEE Advanced

Single Correct Answer Type

- (1) Linear momentum remains constant if net external force on the system of particles is zero.
- (3) Magnitude and direction of $\vec{L}_O$ remain constant.

Magnitude of $\vec{L}_P$ remains constant but direction of $\vec{L}_P$ changes.



3. (4) $I_P > I_Q$

$$a_P = \frac{g \sin \theta}{1 + \frac{I_P}{mR^2}}, \quad a_Q = \frac{g \sin \theta}{1 + \frac{I_Q}{mR^2}}$$

$$a_P < a_Q \Rightarrow V = u + at \Rightarrow t \propto \frac{1}{a}$$

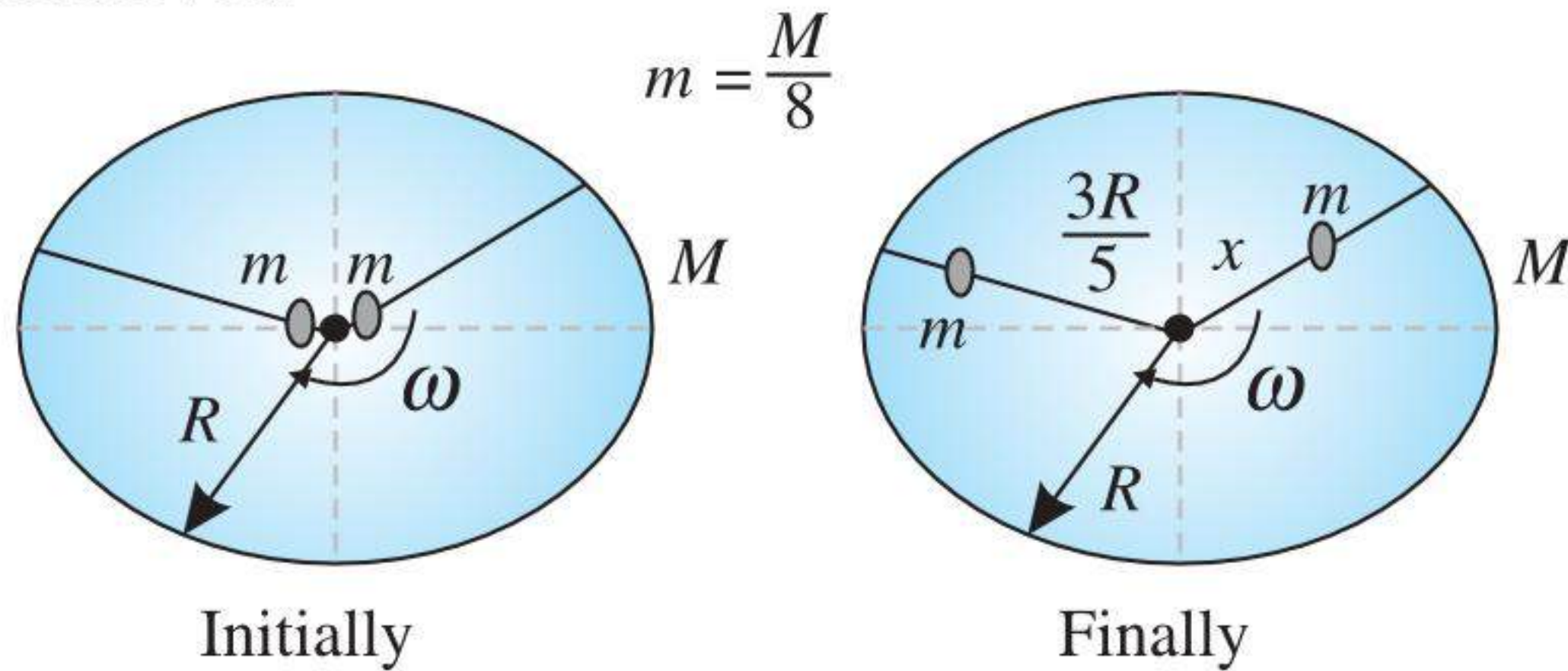
$$t_P > t_Q$$

$$V^2 = u^2 + 2as \Rightarrow V \propto a \Rightarrow V_P < V_Q$$

$$\text{Translational KE, } \frac{1}{2}mV^2 \Rightarrow \text{TR KE}_P < \text{TR KE}_Q$$

$$V = \omega R \Rightarrow \omega \propto V \Rightarrow \omega_P < \omega_Q$$

4. (4) Angular momentum of ring and mass system should be conserved.



$$I\omega = \text{constant}$$

Initial moment of inertia of system (ring + masses)

$$I_{\text{initial}} = MR^2$$

Final moment of inertia

$$MR^2 \cdot \omega = \left(MR^2 + \frac{M}{8} \frac{9}{25} R^2 + \frac{M}{8} x^2 \right) 8\omega$$

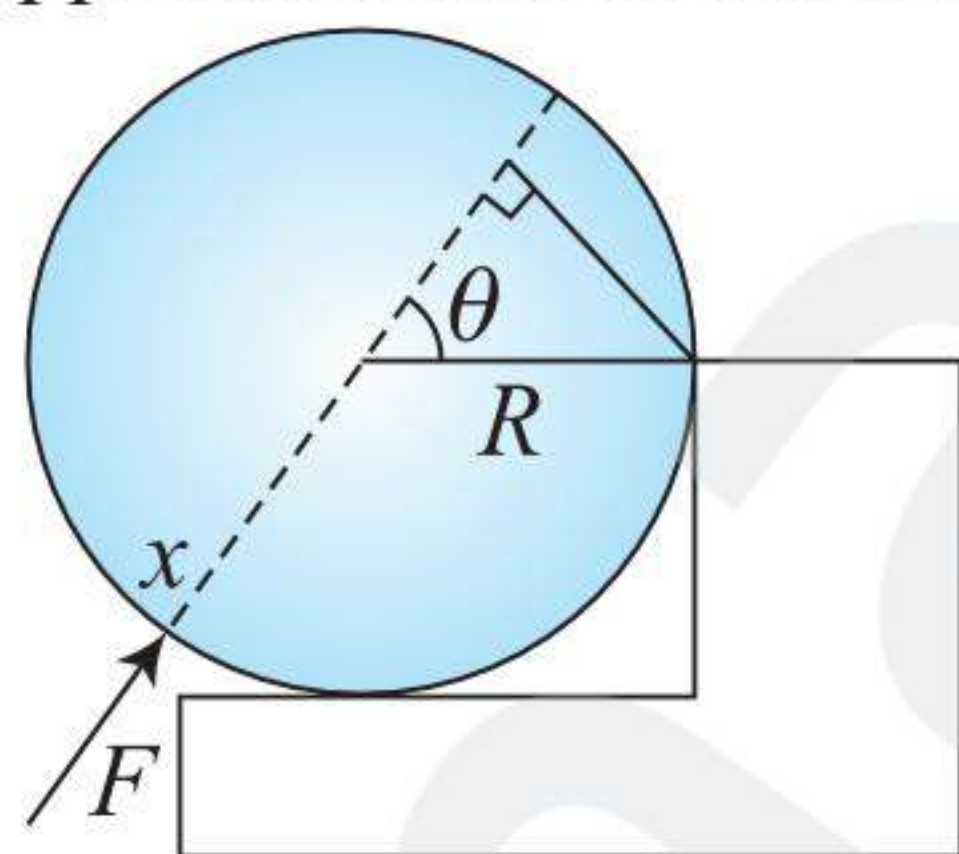
$$\frac{9}{8} MR^2 = MR^2 + \frac{9}{25} \frac{MR^2}{8} + \frac{M}{8} x^2$$

$$\frac{Mx^2}{8} = MR^2 \left(\frac{1}{8} - \frac{9}{25 \times 8} \right)$$

$$\frac{M}{8} x^2 = \frac{2}{25} MR^2$$

$$\Rightarrow x^2 = \frac{16}{25} R^2 \Rightarrow x = \left(\frac{4}{5} \right) R$$

5. (3) If the force is applied normal to the circumference at point X



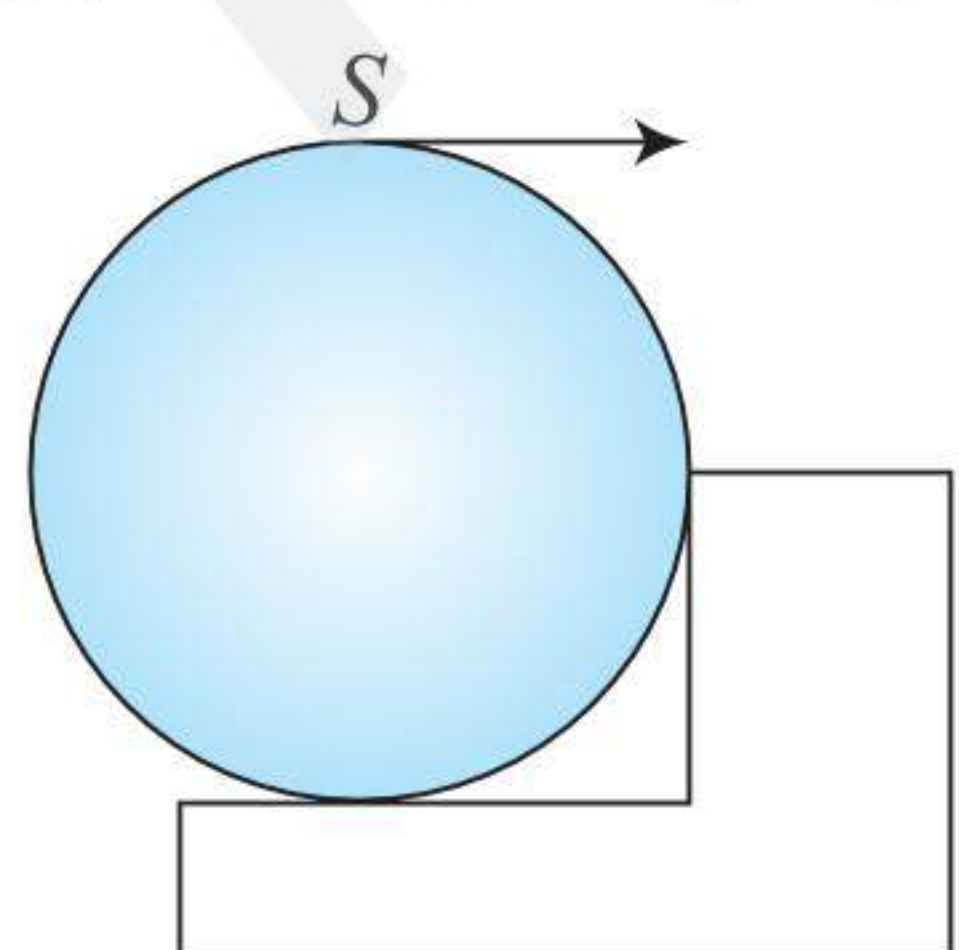
If force is applied normal to surface at point X

$$\tau = FR \sin \theta$$

$$\tau \propto \sin \theta$$

Since when θ increases, τ decreases. Thus τ depends on θ and it is not constant. Hence option (1) is incorrect.

If the force is applied tangentially at point S

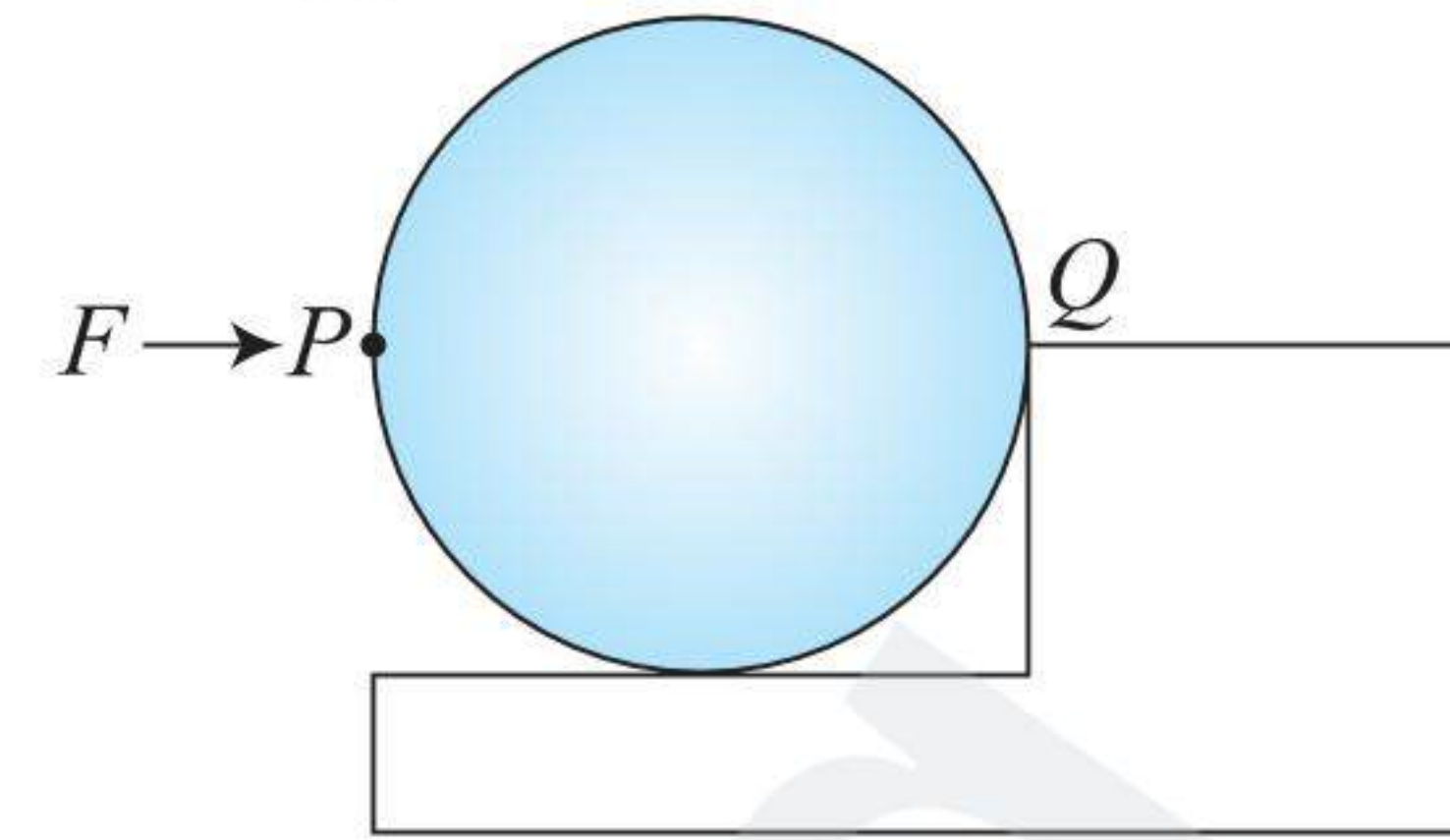


If force applied tangentially at S, then

$$\tau = F \times R \neq 0$$

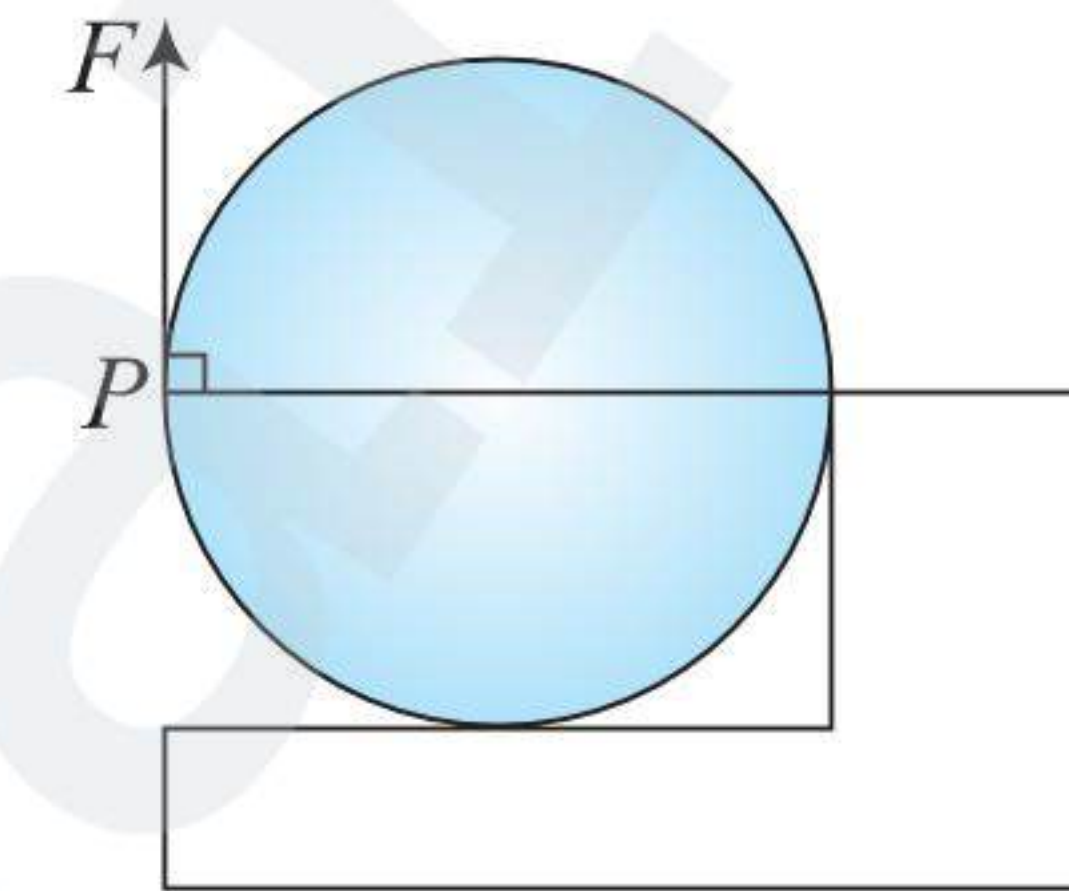
Wheel can climb, so option (2) is incorrect.

If the force is applied normal to the circumference at point P

If force is applied normal to surface at P then line of action of force will pass from Q and thus $\tau = 0$.

So option (3) is correct.

If the force is applied at point P tangentially



$$\tau = F \times 2R = \text{constant, so option (4) is incorrect.}$$

6. (1)

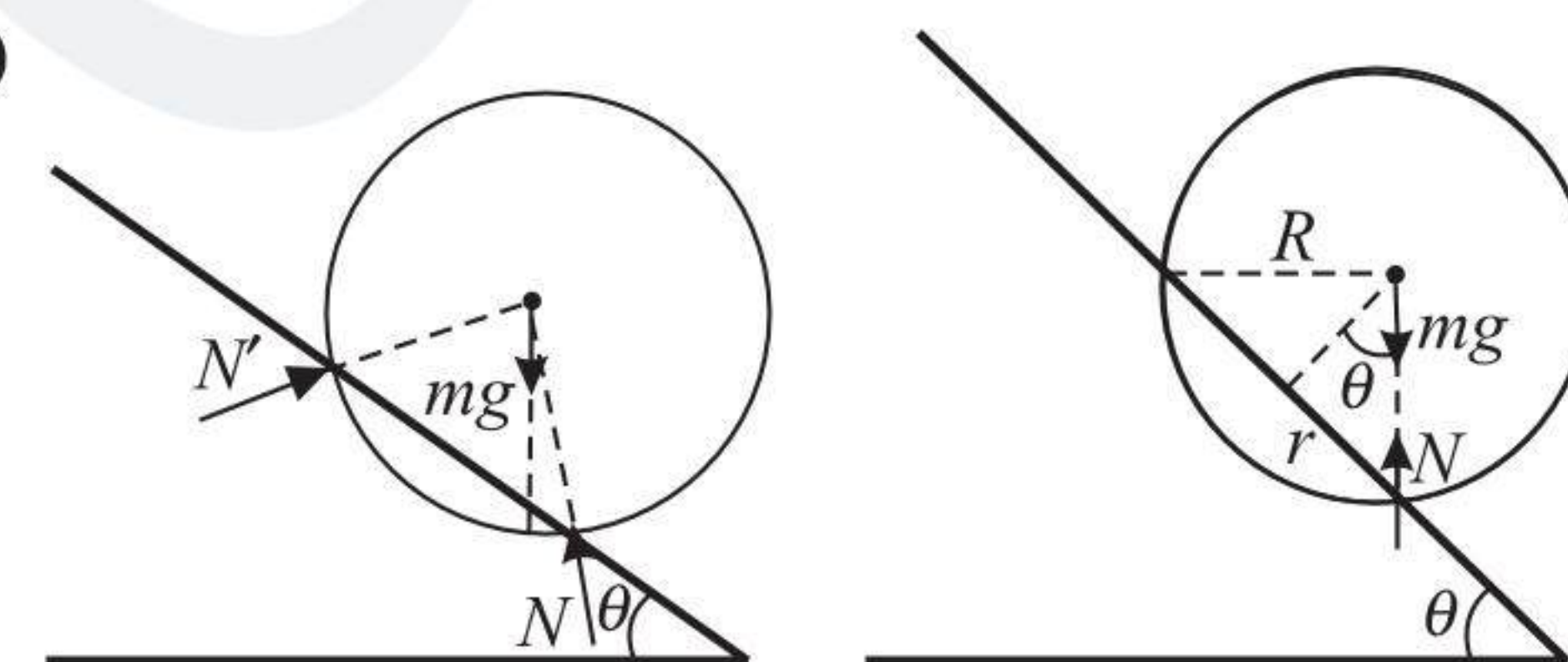


Fig. (a)

Fig. (b)

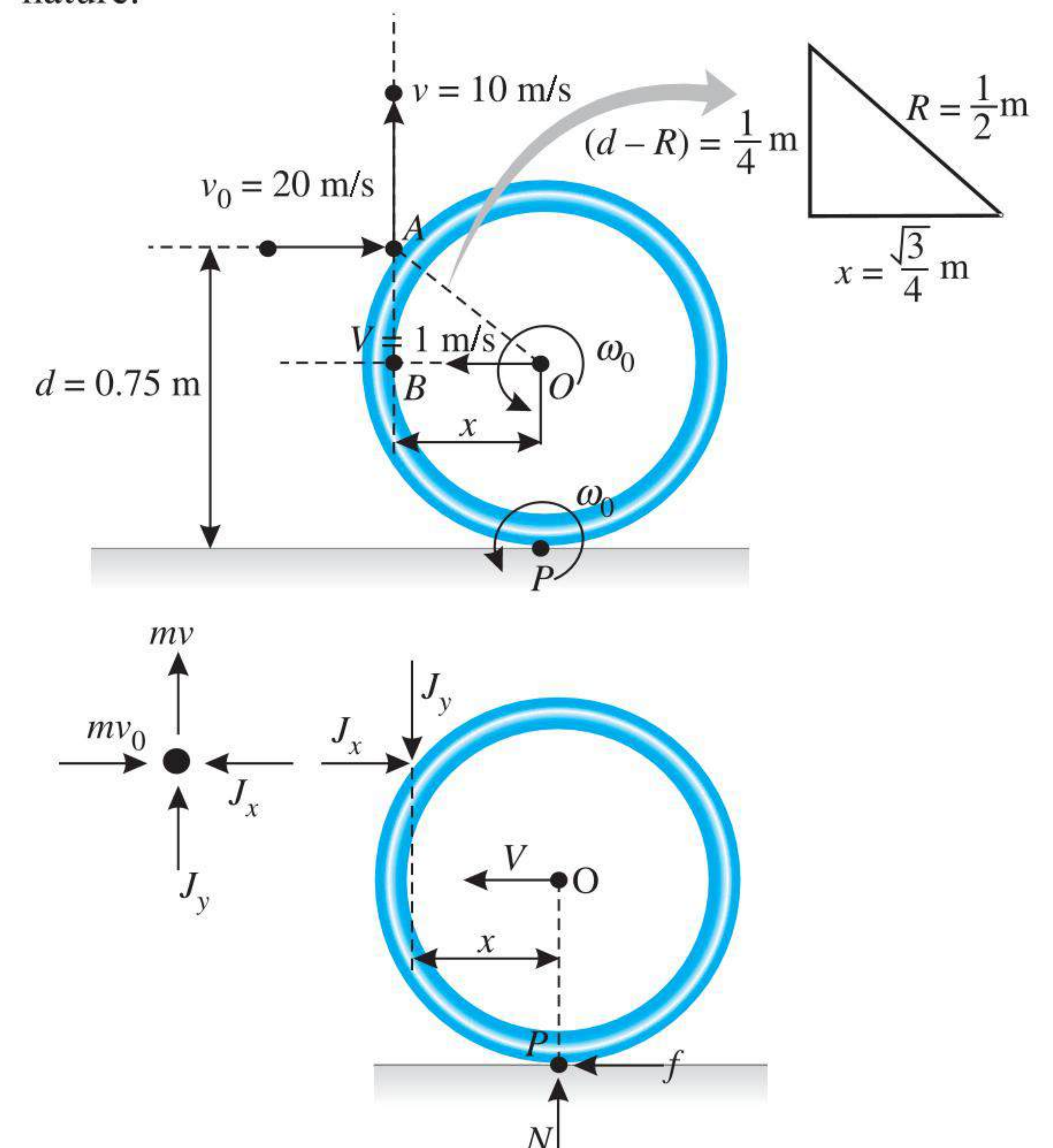
For maximum value of θ , the football is about to roll, then $N' = 0$ and all the forces (mg and N) must pass through lower contact point. From Fig. (b)

$$\sin \theta = \frac{r}{R}$$

Multiple Correct Answers Type

1. (1), (3)

The friction and normal reaction on the ring is impulsive in nature.



Considering the motion of ball,

$$mv_0 - J_x = 0 \text{ or } J_x = mv_0$$

Hence, $J_x = 0.1 \times 20 = 2 \text{ kgm/s}$

and $J_y = mv = 0.1 \times 10 = 1 \text{ kgm/s}$

Now considering the motion of ring.

$$J_x = MV_f - (-MV)$$

$$2 = MV_f + 2 \times 1 \Rightarrow V_f = 0$$

Hence, the linear velocity of ring becomes zero just after collision.

Conserving angular momentum of system (ring + ball) about point of contact (P).

$$I_P \omega_0 (\hat{k}) + mv_0 d (-\hat{k}) = I_0 \vec{\omega} + mvx (-\hat{k})$$

$$I_0 \vec{\omega} = I_P \omega_0 \hat{k} + m[vx - v_0 d] \hat{k}$$

As ring is rolling $\omega_0 = \frac{V}{R} = \frac{1}{1/2} = 2 \text{ rad/s}$

$$2 \times \left(\frac{1}{2}\right)^2 \vec{\omega} = \frac{3}{2} \times 2 \times \left(\frac{1}{2}\right)^2 \times 2\hat{k} + 0.1 \left[10 \times \frac{\sqrt{3}}{4} - 20 \times \frac{1}{4}\right] \hat{k}$$

$$2 \times \left(\frac{1}{2}\right)^2 \vec{\omega} = \frac{3}{2} \hat{k} + 0.1 \times \frac{10}{4} [3 - 2] \hat{k}$$

Hence, $\omega \neq 0$ and will be in counter clockwise direction. Now friction will be kinetic in nature and will act in leftward direction.

2. (1), (2), (4)

$$\vec{r} = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 3\alpha t^2 \hat{i} + 2\beta t \hat{j}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 6\alpha t \hat{i} + 2\beta \hat{j}$$

At $t = 1$

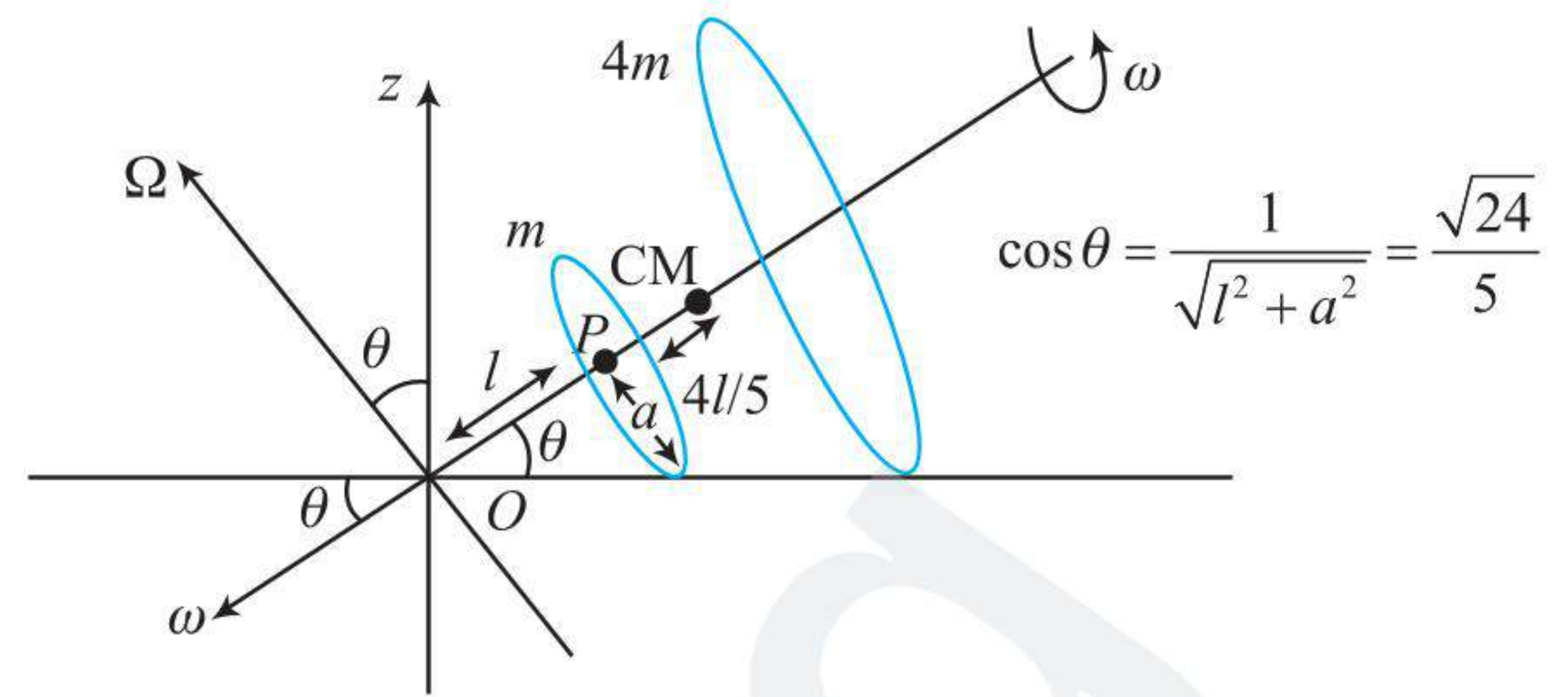
$$(a) \vec{v} = 3 \times \frac{10}{3} \times 1 \hat{i} + 2 \times 5 \times 1 \hat{j} = 10\hat{i} + 10\hat{j}$$

$$(b) \vec{L} = \vec{r} \times \vec{p} \\ = \left(\frac{10}{3} \times 1 \hat{i} + 5 \times 1 \hat{j}\right) \times 0.1(10\hat{i} + 10\hat{j}) \\ = -\frac{5}{3} \hat{k}$$

$$(c) \vec{F} = m \times \left(6 \times \frac{10}{3} \times 1 \hat{i} + 2 \times 5 \hat{j}\right) = 2\hat{i} + \hat{j}$$

$$(d) \vec{\tau} = \vec{r} \times \vec{F} \\ = \left(\frac{10}{3} \hat{i} + 5 \hat{j}\right) \times (2\hat{i} + \hat{j}) = +\frac{10}{3} \hat{k} + 10(-\hat{k}) = -\frac{20}{3} \hat{k}$$

3. (2), (4)



$$(a) L_Z = L_{CM-O} \cos \theta - L_{D-CM} \sin \theta$$

$$= \frac{81\sqrt{24}}{5} a^2 m \omega \times \frac{\sqrt{24}}{5} - \frac{17ma^2 \omega}{2} \\ \times \frac{1}{\sqrt{24}} = \frac{81 \times 24 ma^2 \omega}{25} - \frac{17ma^2 \omega}{2\sqrt{24}}$$

$$(b) L_{D-CM} = \frac{ma^2}{2} \omega + \frac{4m(2a)^2}{2} \omega = \frac{17ma^2 \omega}{2}$$

$$(c) L_{CM-O} = (5m) \left[\frac{9l}{5} \Omega \right] \frac{9l}{5} = \frac{81ml^2 \Omega}{5} = \frac{81ml^2}{5} \times \frac{a\omega}{l}$$

$$L_{CM-O} = \frac{81mla\omega}{5} = \frac{81\sqrt{24}a^2 m \omega}{5}$$

(d) Velocity of point P : $a\omega = l\Omega$ then

$$\Omega = \frac{a\omega}{l} = \text{Angular velocity of CM w.r.t. point O.}$$

Angular velocity of CM w.r.t. z axis = $\Omega \cos \theta$

$$\omega_{CM-z} = \frac{a\omega}{l} \frac{\sqrt{24}}{5} = \frac{a\omega}{\sqrt{24}a} \frac{\sqrt{24}}{5}$$

$$\omega_{CM-z} = \frac{\omega}{5}$$

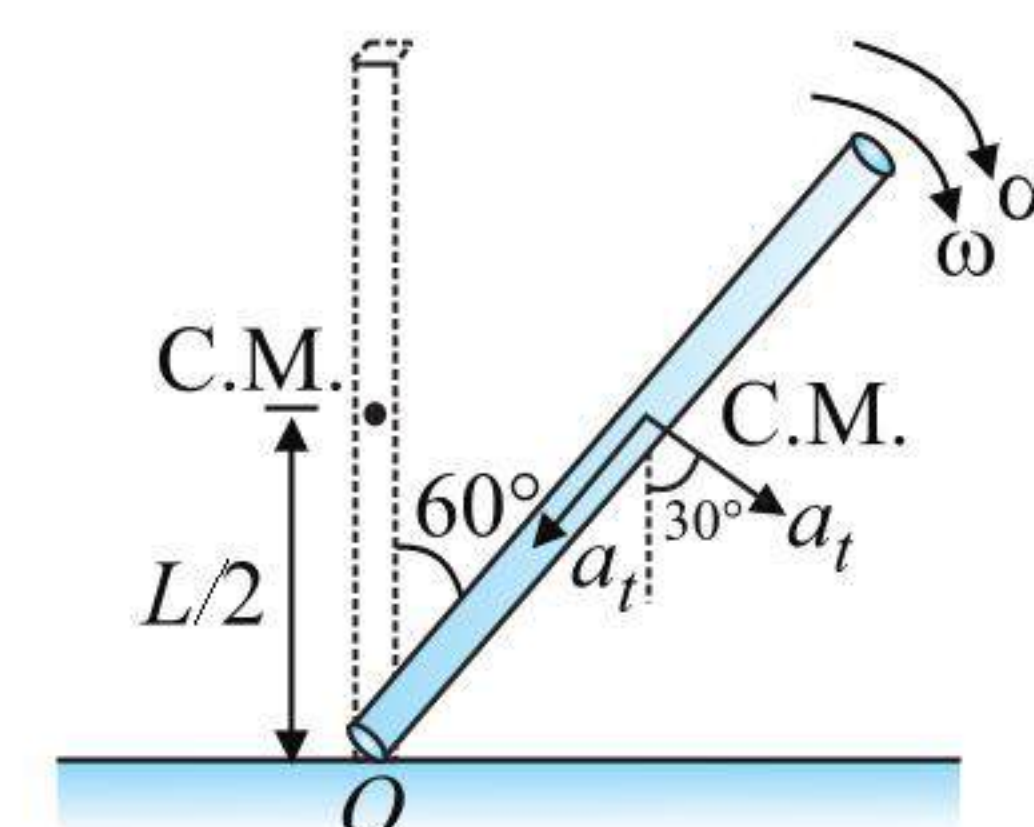
4. (1),(3),(4)

We can treat contact point as hinged. Applying work energy theorem

$$W_g = \Delta \text{KE}$$

$$mg \frac{L}{4} = \frac{1}{2} \left(\frac{mL^2}{3} \right) \omega^2$$

$$\omega = \sqrt{\frac{3g}{2L}}$$



$$\text{Radial acceleration of C.M. of rod} = \left(\frac{L}{2} \right) \omega^2 = \frac{3g}{4}$$

Using $\tau = I \alpha$ about contact point

Net vertical acceleration of C.M. of rod

$$\frac{mgL}{2} \sin 60^\circ = \frac{mL^2}{3} \alpha \Rightarrow \alpha = \frac{3\sqrt{3}}{4L} g$$

Net vertical acceleration of C.M. of rod

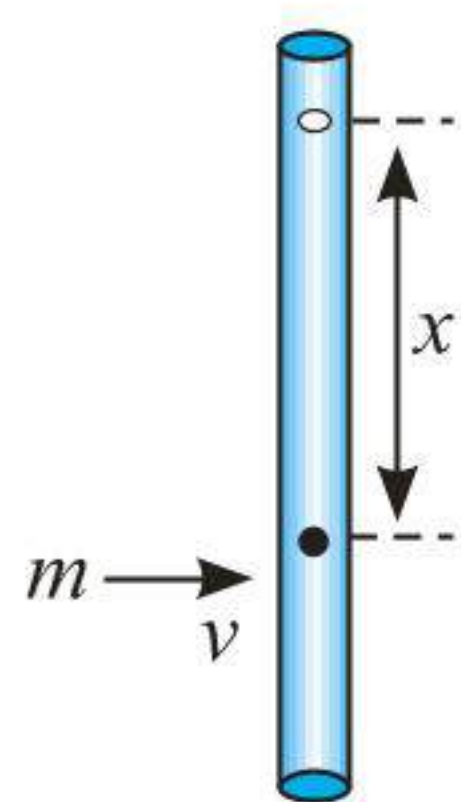
$$\begin{aligned} a_v &= a_r \cos 60^\circ + a_t \cos 30^\circ \\ &= \left(\frac{3g}{4}\right)\left(\frac{1}{2}\right) + \left(\alpha \frac{L}{2}\right) \cos 30^\circ \\ &= \frac{3g}{8} + \frac{3\sqrt{3}g}{4L} \left(\frac{L}{2}\right) \left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{3g}{8} + \frac{9g}{16} = \frac{15}{16}g \end{aligned}$$

Applying $F_{\text{net}} = ma$ in vertical direction on rod as system

$$mg - N = ma_v = m\left(\frac{15}{16}g\right)$$

$$\Rightarrow N = \frac{mg}{16}$$

5. (1),(3),(4)



Applying the angular momentum conservation about hinge.

$$mvx = \left(\frac{mL^2}{3} + mx^2\right)\omega$$

$$\therefore \omega = \frac{mvx}{\frac{mL^2}{3} + mx^2} = \frac{3vx}{L^2 + 3x^2} \quad (\text{Option 1})$$

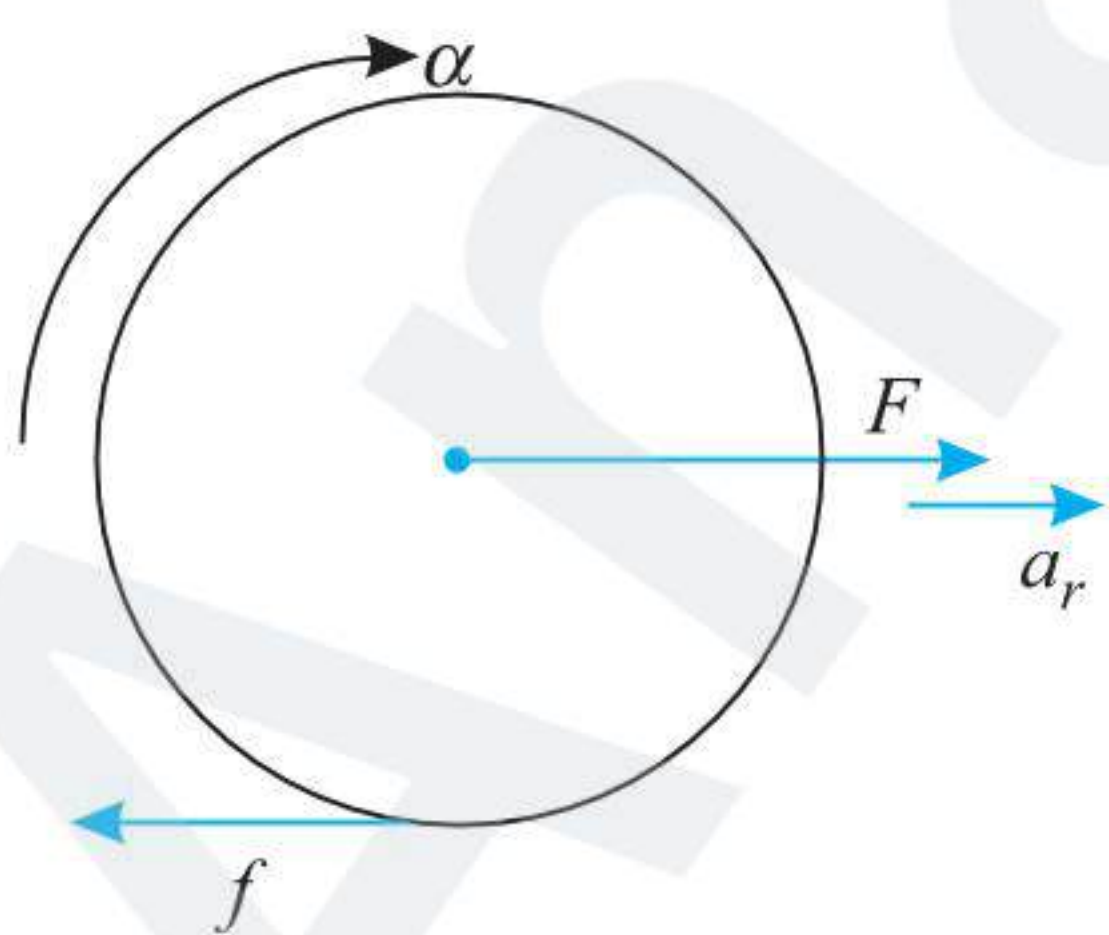
For maximum $\omega \Rightarrow \frac{d\omega}{dx} = 0$

$$\therefore x_M = \frac{L}{\sqrt{3}} \quad (\text{Option 3})$$

So the $\omega = \frac{V}{2L}\sqrt{3}$ (Option 4)

6. (2),(4)

Considering FBD of the cylinder,



Force equation, $F - f = ma_C$... (i)

Torque equation about O, $fR = I_C \alpha$... (ii)

$$F - I_C \frac{a_C}{R} = ma_C$$

From (i) and (ii)

The object is rolling $a_C = \alpha R \Rightarrow \alpha = \frac{a_C}{R}$... (iii)

$$F - I_C \frac{a_C}{R^2} = ma_C$$

$$\Rightarrow a_C = \frac{F}{\frac{I_C}{R^2} + m} \quad \dots (iv)$$

$$f = \frac{I_C \alpha}{R} = \frac{I_C}{R^2} a_C = \frac{I_C}{R^2} \left[\frac{F}{\frac{I_C}{R^2} + m} \right]$$

$$f = \frac{F}{\left(m + \frac{I_C}{R^2}\right)}$$

Thin walled hollow cylinder, $I_C = mR^2$

From equation (iv), $a_C = \frac{F}{2m}$

From equation (ii), $fR = I_C \alpha = \frac{I_C a_C}{R}$

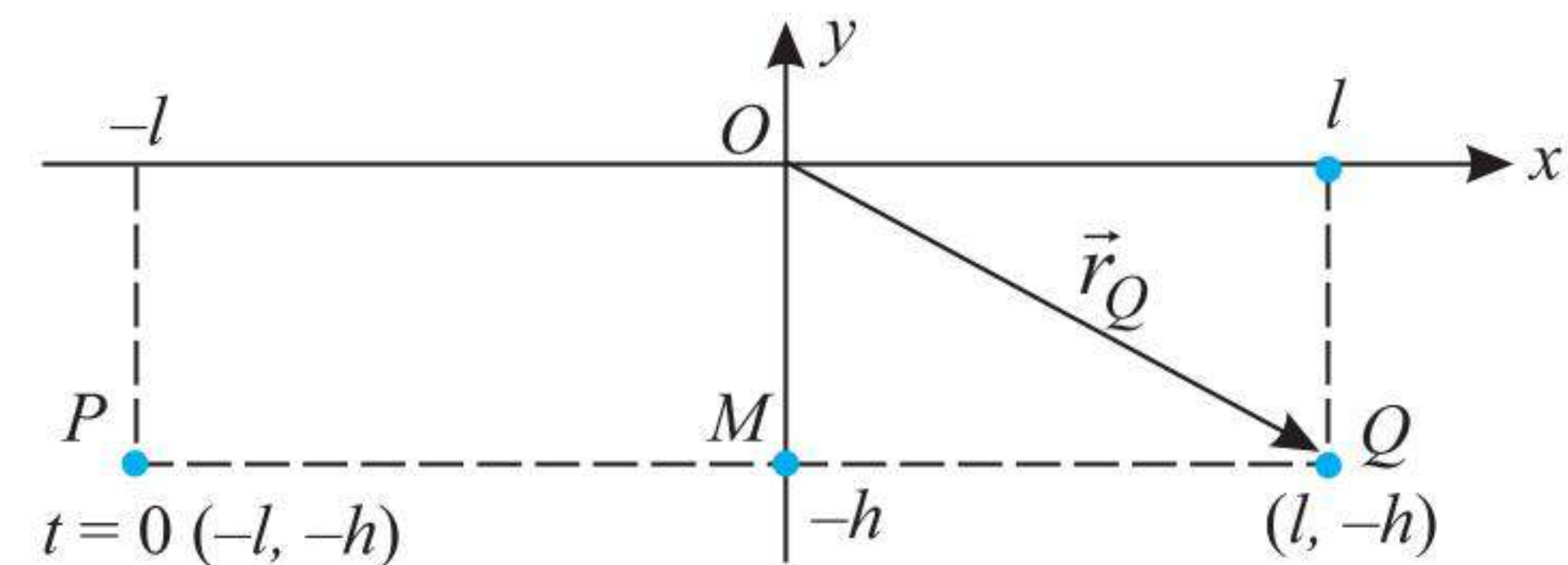
$$f = \frac{I_C a_C}{R^2} \leq \mu mg \Rightarrow a_C \leq \frac{\mu mg R^2}{I_C}$$

For solid cylinder $I_C = \frac{mR^2}{2}$

From equation (iv), $a_C \leq 2\mu g$

It means $(a_C)_{\text{max}} = 2\mu g$

7. (1),(2),(3)



(1) At Q displacement of the particle = $2l = 20$ m

$$\text{Using } S = \frac{1}{2}at^2$$

$$\Rightarrow 20 = \frac{1}{2} \times 10t^2 \Rightarrow t = 2 \text{ sec}$$

Hence option (1) is correct.

(2) Position vector of the particle at Q

$$\vec{r}_Q = (10\hat{i} - \hat{j})\text{m and } \vec{F} = m\vec{a} = 0.2 \times 10\hat{i} = 2\hat{i} \text{ N}$$

Torque about O, $\vec{\tau}_0 = \vec{r} \times \vec{F}$

$$\Rightarrow \vec{\tau}_0 = (10\hat{i} - \hat{j}) \times 2\hat{i} = 2\hat{k} \text{ (N-m)}$$

Hence option (2) is correct.

(3) $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

Velocity of the particle at Q, $\vec{v} = 0 + 10 \times 2 = 20\hat{i} \text{ m/s}$

$$\vec{L}_0 = 0.2[(10\hat{i} - \hat{j}) \times 20\hat{i}] = 4\hat{k} \text{ (N-m/s)}$$

Hence option (3) is correct.

(4) At position M , $\vec{r}_M = -\hat{j}(m)$

$$\vec{\tau}_0 = \vec{r}_M \times \vec{F} = (-\hat{j}) \times (2\hat{i}) = 2\hat{k}(\text{N-m})$$

Linked Comprehension Type

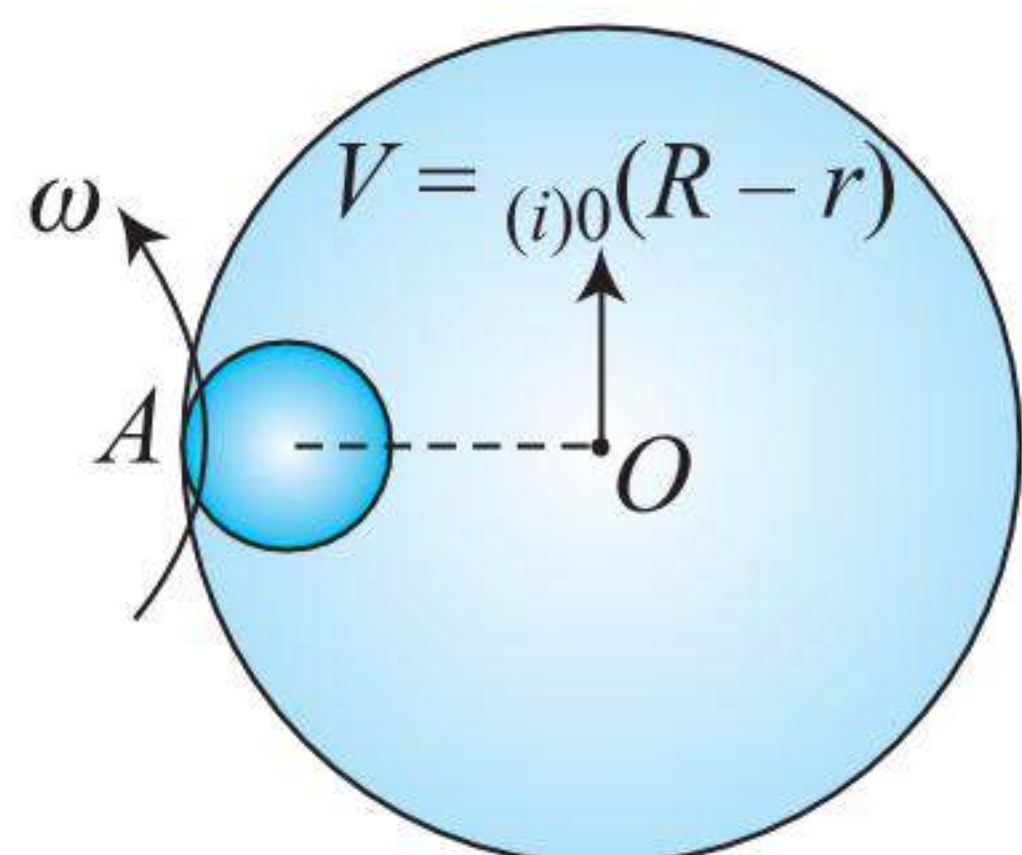
For Problems 1–2

1. (2) A will be instantaneous centre of rotation

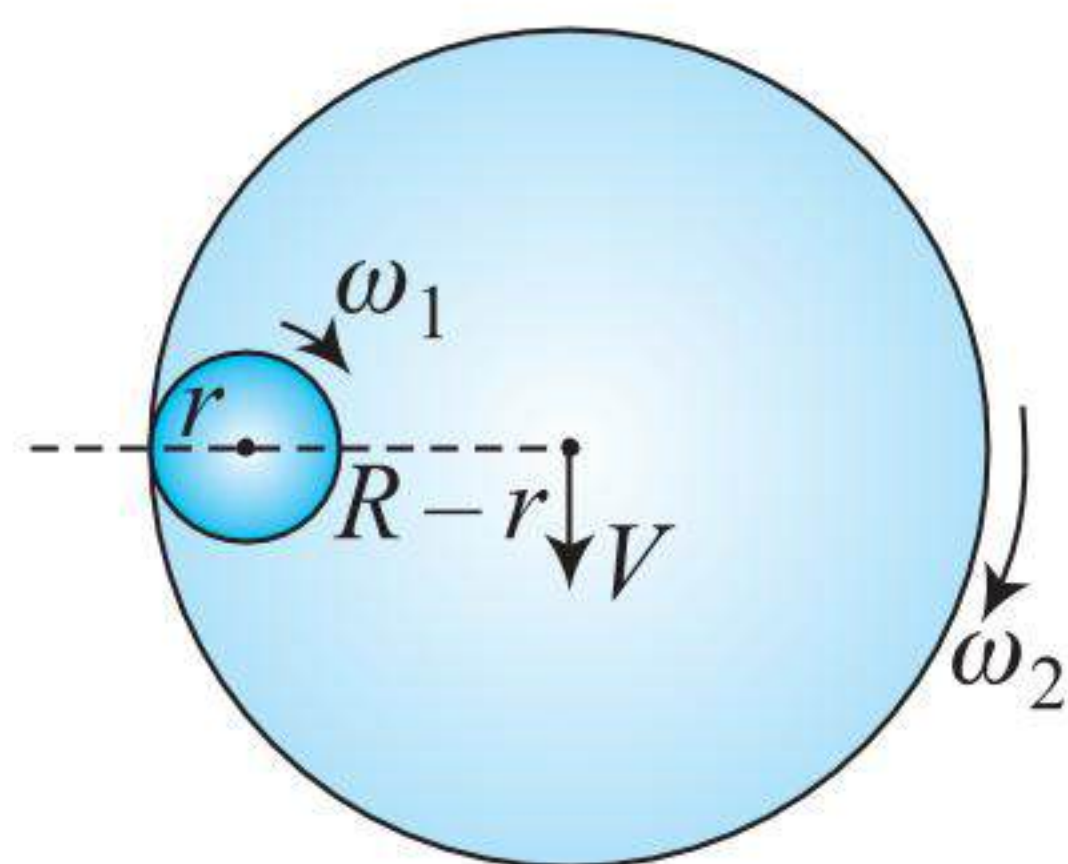
$$\omega R = \omega_0 (R - r)$$

$$\omega = \omega_0 \left(\frac{R - r}{R} \right)$$

$$\text{KE} = \frac{1}{2} (2mR^2) \omega^2 = m\omega_0^2 (R - r)^2$$



2. (2) The motion can be visualized as a larger ring spinning around a smaller rotating disk, without slipping. Let angular speeds of smaller and larger ring be ω_1 and ω_2 . In addition to rotation about its own axis centre of larger ring is moving say with a speed v . Centre of larger ring is moving in a circle of radius $(R - r)$.



$$\text{So, } V = (R - r) \omega_2 \quad \dots(i)$$

Also, no slipping at the point of contact gives

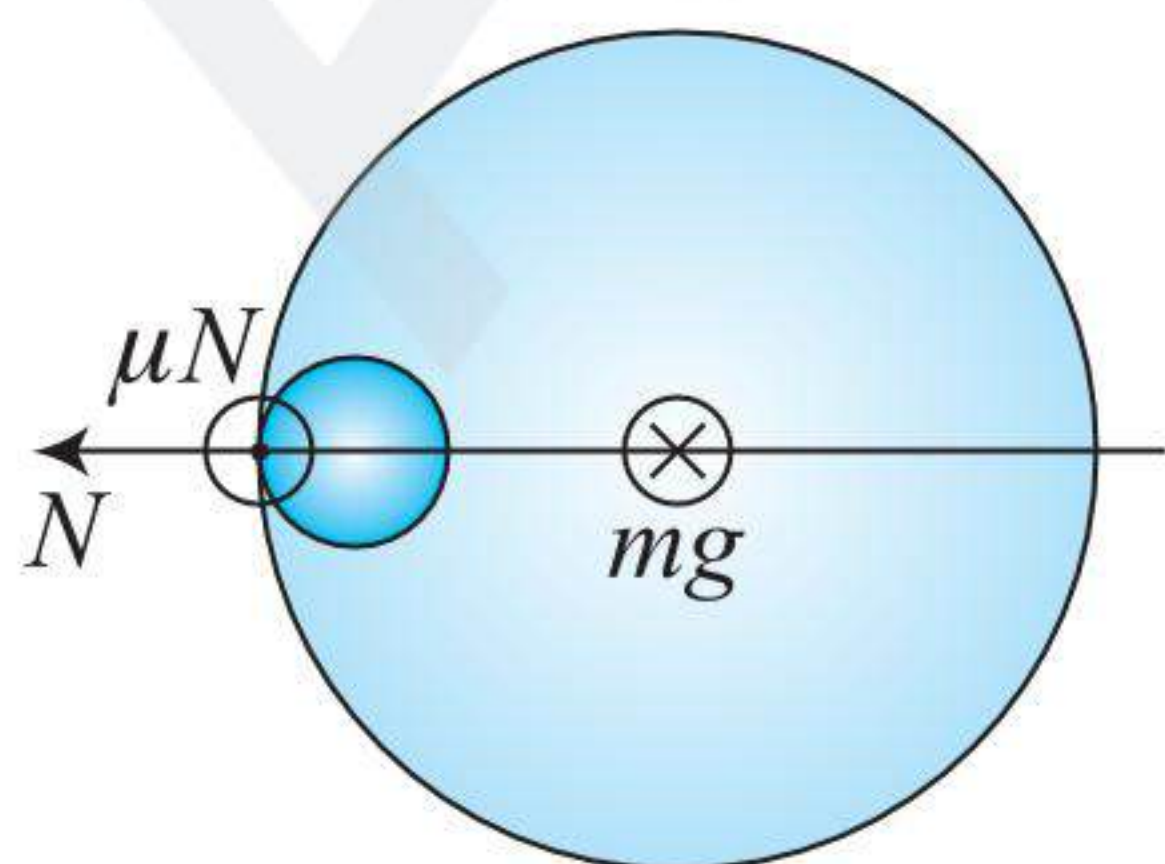
$$r\omega_1 = R\omega_2 - V \quad \dots(ii)$$

(i) and (ii) give $\omega_1 = \omega_2 \rightarrow \omega_0$

Now draw the FBD of ring

$$N = ma_c = m\omega_0^2 (R - r) \quad \dots(iii)$$

Balancing forces in vertical direction for translational equilibrium, $mg = \mu N$



$$\therefore mg = \mu m\omega_0^2 (R - r) \quad \text{i.e. } \omega_0 = \sqrt{\frac{g}{\mu(R - r)}}$$

Matrix Match Type

1. (1) (P) $\vec{r}(t) = \alpha t \hat{i} + \beta t \hat{j}$

$$\vec{v} = \frac{d\vec{r}(t)}{dt} = \alpha \hat{i} + \beta \hat{j} \{\text{constant}\}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 0$$

$$\vec{p} = m\vec{v} \text{ (remain constant)}$$

$$K = \frac{1}{2} m v^2 \text{ (remain constant)}$$

$$\vec{F} = - \left[\frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} \right] = 0$$

$$\Rightarrow U \rightarrow \text{constant}$$

$$E = K + U$$

$$\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r} \times \vec{F} = 0$$

$$\vec{L} = \text{constant}$$

(Q) $\vec{r} = \alpha \cos(\omega t) \hat{i} + \beta \sin(\omega t) \hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = -\alpha \omega \sin(\omega t) \hat{i} + \beta \omega \cos(\omega t) \hat{j}$$

$$\begin{aligned} \vec{a} &= \frac{d\vec{v}}{dt} = -\alpha \omega^2 \cos(\omega t) \hat{i} - \beta \omega^2 \sin(\omega t) \hat{j} \\ &= -\omega^2 [\alpha \cos(\omega t) \hat{i} + \beta \sin(\omega t) \hat{j}] \end{aligned}$$

$$\vec{a} = -\omega^2 \vec{r}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = 0 \quad \{\vec{r} \text{ and } \vec{F} \text{ are parallel}\}$$

$$\Delta U = - \int \vec{F} \cdot d\vec{r} = + \int_0^r m\omega^2 \cdot r \cdot dr$$

$$\Delta U = m\omega^2 \left[\frac{r^2}{2} \right]$$

$$U \propto r^2$$

$$r = \sqrt{\alpha^2 \cos^2(\omega t) + \beta^2 \sin^2(\omega t)} \quad r \text{ is a function of time } (t)$$

U depends on r hence it will change with time.

Total energy remains constant because force is central.

(R) $\vec{r}(t) = \alpha (\cos \omega t \hat{i} + \sin \omega t \hat{j})$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$$

$$= \alpha [-\omega \sin(\omega t) \hat{i} + \omega \cos(\omega t) \hat{j}]$$

$$|\vec{v}| = \alpha \omega \text{ (Speed remains constant)}$$

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt}$$

$$= \alpha [-\omega^2 \cos(\omega t) \hat{i} - \omega^2 \sin(\omega t) \hat{j}]$$

$$= -\alpha\omega^2 [\cos(\omega t)\hat{i} + \sin(\omega t)\hat{j}]$$

$$\vec{a}(t) = -\omega^2 (\vec{r})$$

$$\vec{\tau} = \vec{F} \times \vec{r} = 0$$

$|\vec{r}| = \alpha$ (remains constant) Force is central in nature and distance from fixed point is constant.

Potential energy remains constant. Kinetic energy is also constant (speed is constant) energy remains constant

$$(S) \vec{r} = \alpha t \hat{i} + \frac{\beta}{2} t^2 \hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} \text{ (Speed of particle depends on 't')}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \beta \hat{j} \{\text{constant}\}$$

$$\vec{F} = m\vec{a} \{\text{constant}\}$$

$$\Delta U = -\int \vec{F} \cdot d\vec{r} = -m \int_0^t \beta \hat{j} \cdot (\alpha \hat{i} + \beta t \hat{j}) dt$$

$$U = \frac{-m\beta^2 t^2}{2}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(\alpha^2 + \beta^2 t^2)$$

$$E = K + U$$

$$= \frac{1}{2}m\alpha^2 [\text{remains constant}]$$

Numerical Value Type

1. (4) Since net torque about centre of rotation is zero, so we can apply conservation of angular momentum of the system about center of disc

$$L_i = L_f$$

$$0 = I\omega + 2mv(r/2); \text{ comparing magnitude}$$

$$\therefore \left(\frac{0.45 \times 0.5 \times 0.5}{2} \right) \omega = 0.05 \times 9 \times \frac{0.5}{2} \times 2$$

$$\therefore \omega = 4$$

2. (7) Kinetic energy of a purely rolling disc

$$= \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{m}{2}\right)V^2 = \frac{3}{4}mV^2$$

Using conservation of energy on both discs and equating their final energies

$$\frac{3}{4}mV_1^2 + mg \cdot 30 = \frac{3}{4}mV_2^2 + mg \cdot 27$$

$$3mg = \frac{3}{4}m(V_2^2 - V_1^2)$$

$$4g = V_2^2 - 9$$

$$V_2^2 = 49$$

$$V_2 = 7$$

3. (6) Moment of inertia of a hollow sphere is $I_0 = \frac{2}{3}mR^2$

Here, for sphere A:

$$I_A = \int_0^R \frac{2}{3}r^2 \cdot 4\pi r^2 dr K \frac{r}{R}$$

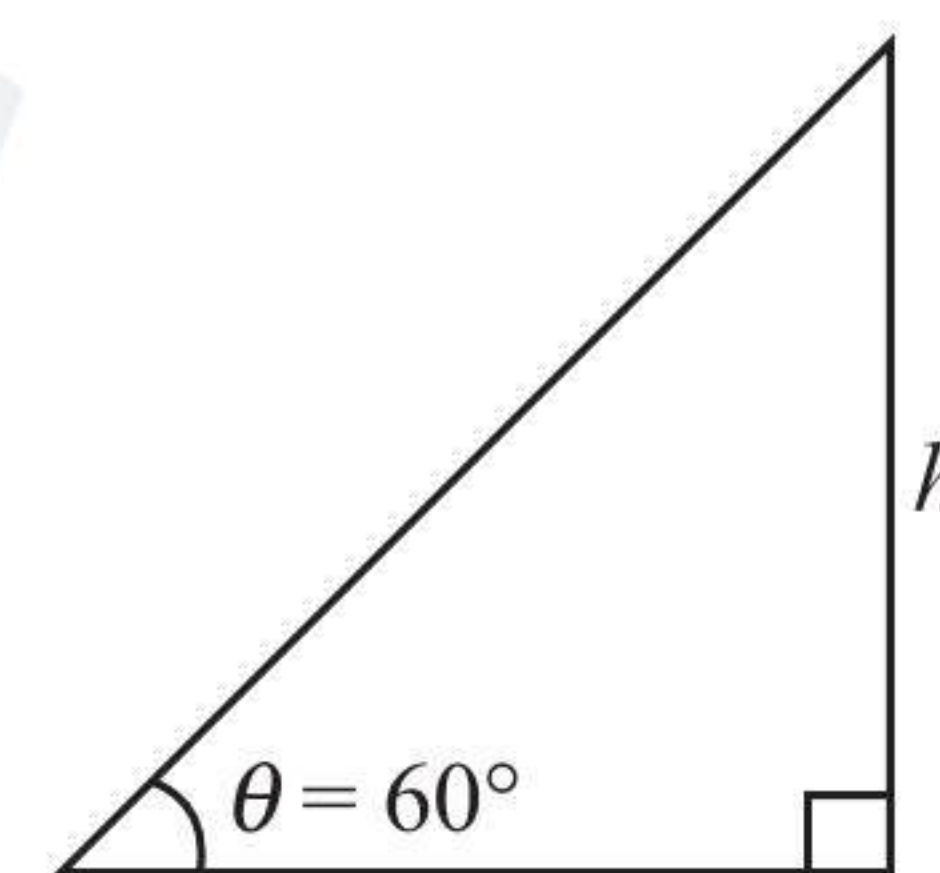
$$= \frac{8\pi K}{3R} \int_0^R r^5 dr = \frac{8\pi K}{3R} \frac{R^6}{6}$$

Also, for sphere B:

$$I_B = \int_0^R \frac{2}{3}r^2 \cdot 4\pi r^2 dr \frac{K}{R^5} \cdot r^5 = \frac{8\pi K}{3R^5} \cdot \frac{R^{10}}{10}$$

$$\text{Ratio, } \frac{I_B}{I_A} = \frac{6}{10}$$

4. (0.75) The acceleration of a rolling object in an inclined plane.



$$a_c = \frac{g \sin \theta}{1 + \frac{I_C}{MR^2}}$$

$$\text{For ring } I_C = MR^2 \Rightarrow a_{\text{ring}} = \frac{g \sin \theta}{2}$$

$$\text{For disc } I_C = \frac{MR^2}{2} \Rightarrow a_{\text{disc}} = \frac{2g \sin \theta}{3}$$

$$\text{For ring: } \frac{h}{\sin \theta} = \frac{1}{2} \left(\frac{g \sin \theta}{2} \right) t_1^2$$

$$\Rightarrow t_1 = \sqrt{\frac{4h}{g \sin^2 \theta}} = \sqrt{\frac{16h}{3g}}$$

$$\text{For disc: } \frac{h}{\sin \theta} = \frac{1}{2} \left(\frac{2g \sin \theta}{3} \right) t_2^2$$

$$\Rightarrow t_2 = \sqrt{\frac{3h}{g \sin^2 \theta}} = \sqrt{\frac{4h}{g}}$$

The difference of time of reaching the ground.

$$\Rightarrow \sqrt{\frac{16h}{3g}} - \sqrt{\frac{4h}{g}} = \frac{2 - \sqrt{3}}{\sqrt{10}}$$

$$\text{or } \sqrt{h} \left[\frac{4}{\sqrt{3}} - 2 \right] = 2 - \sqrt{3}$$

$$\Rightarrow \sqrt{h} = \frac{(2-\sqrt{3})\sqrt{3}}{(4-2\sqrt{3})} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow h = \frac{3}{4} = 0.75 \text{ m}$$

5. (49)

Total angular momentum about O ,

$$L = \frac{Ma^2}{3}\Omega + M\left(\frac{3a}{4}\right)^2\Omega + \frac{M\left(\frac{a}{4}\right)^2 4\Omega}{2}$$

$$L = \frac{49}{48}Ma^2\Omega \Rightarrow n = 49$$